Mathematical modeling of the tennis match Djoković -Federer

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Abstract

The paper deals with a session with gifted sixteen-year-old students from the Jovan Jovanović-Zmaj Grammar School in Novi Sad, Serbia, in November 2012 on mathematical modelling of the probability for a tennis player to win a match. The modelling was based on the statistics of the match between Djoković and Federer at the Barclays ATP World Tour Finals 2012 in London.

Keywords

Modeling, combinatorics, probability, tennis, interdisciplinary connections

1. Introduction

Sports are a part of everyday life, especially for many students. They enjoy different sport activities and understand their rules and corresponding statistics. Now, these widespread interests for different sports can be used to catch students' attention in teaching and learning, in particular mathematical modelling of sports activities based on corresponding roles.

Tennis is a very popular sport in Serbia. The famous Monica Seles, the Number One woman tennis player in the nineties, was born in our home town Novi Sad. Nowadays, we in Serbia are very proud of our successful tennis players, let us mention only Ana Ivanović, Jelena Janković, Janko Tipsarević, Viktor Troicki, Nenad Zimonjić.

It is a well known that Novak Djoković ranked World No. 1 by the ATP and is the year-end World No. 1 for both 2011 and 2012. The final match between Novak Djoković and Roger Federer at Barclays ATP World Tour Finals 2012 was played on Monday afternoon, November 12th, in London. Djoković won this match with 2:0. The goal of this paper is to analyze this match from a mathematical point of view.

That recent match gave us the idea to connect tennis and mathematics on next mathematics lesson in Grammar School "Jovan Jovanović-Zmaj, Novi Sad, Serbia in November 2012, with gifted sixteen years old students.

The process of mathematical modelling was applied in order to connect mathematics and tennis, as an introduction to combinatorics and some simple probability teaching contents.

The role of modelling process, as an excellent tool in mathematical education, is increasing due to its good features. Modelling contributes to students' problem solving capabilities, and to their collaboration and critical thinking skills. Despite this widespread interest and concern, the mathematical modelling is not adequately implemented in the mathematical education.

Generally, it refers to using mathematics to solve realistic and open problems (Kaiser & Schwarz, B, 2006). The effectiveness of modelling-based task induced the positive attitude, beliefs and reaction of both students and teachers. Evidence of this can be found in the literature (Boaler, 2001; Mason, 2001; Doerr & English, 2003; Lamon, Parker & Huston, 2003; Kaiser & Shwarz, 2006; Galbraith, Stillman, Brown & Edvards, 2007).

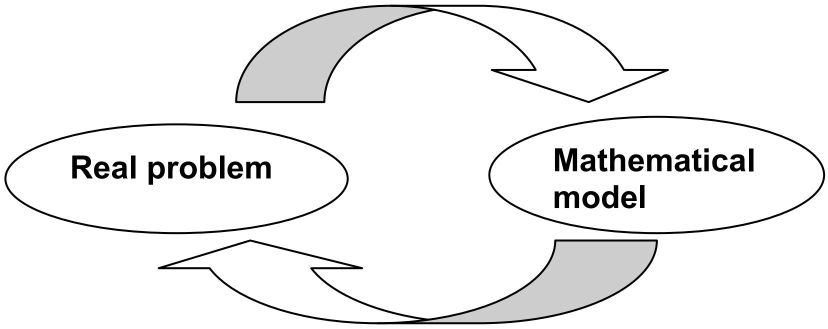

Figure 1

In Figure 1 we present a simplified scheme of mathematical modelling process. The transition from real problem to mathematical model is the most important part in the teaching and learning process. Therefore, we present the transition activities from tennis to mathematics during the lecture, done together with our students.

In the second section we present the results of papers [2], [7], and [11], considering the probability that the tennis player can win the game.

In the third section we describe the process of mathematical modelling done with our high school gifted students.

2. Mathematical model of a tennis game

It is known that in tennis match the scoring system is very unusual and complicated to be followed. It is a hierarchical one “with points required to win games, games required to win sets, and sets required to win the match” ([11]). In order to win the game the player has, first, to collect points from 0, 15, 30, 40, and then if he gets next point he win the game.

In the papers [2], [7] and [11] the authors considered the possibility for a player to win the game in a tennis match. This is the simplest case. But, if both players reach 40 points, then it is called a “deuce”. After deuce the wining point makes “advantage” to the player who wins the game if he wins the next point. If the advantage player looses the next point they return to deuce again and continue the mach. If a player wins six games (with at least 2 games more than his opponent) then he wins the set and if he wins 2 out of 3 sets[1] he wins the game.

The Figure 2 shows the Markov chain of a tennis game ([2], [7]).

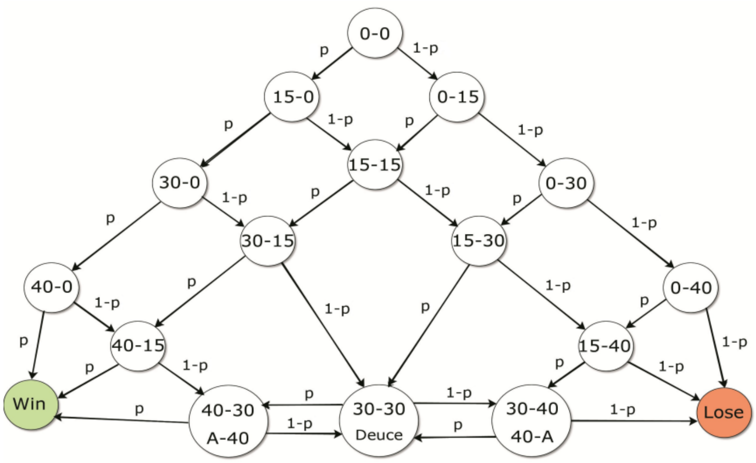

Figure 2. The Markov chain of Tennis game

[1] **Note of the editors** - or 3 out of 5, depending on the rules of the specific tournament

In the paper [11] the author gave the idea for Table 1, and the following explanations (Figure 3.):

PLAYER A

PLAYER B							
	0	**15**	**30**	**40**	Game		
0	1	1	1	1	Win the game		
15	1	2	3	4			
30	1	3	6	10			
40	1	4	10	20	20		
Game	Win the game			20	40	40	
					40	80	80
						80	160
							…

Figure 3. Simplification of the Markov chain rule in tennis

At the start of the game the score is 0-0. There is only one way for player A to reach scores of 15-0, 30-0 and 40-0 in his/her favour, and that is by respectively winning 1, 2 and 3 points in a row. This gives us the first row in Table 1, and due to symmetry also the first column (for player B). If player A or B wins the next point at 40-0 in their favour, s/he wins the game as indicated. So the probability of A winning “game-0 (love)” is p4. However, there are two ways in which the score can become 15-15. For example, player A could lose the first point and win the next point, or win the first point and win the next point. It is left to the reader to check that there are 3 different ways in which the score can become 30-15 in A’s favour, and 4 different ways in which the score can become 40-15 in A’s favour. This gives us the second row in Table 1 (and by symmetry the second column for player B). For A to win game-15, he needs to win the next point….” [11].

3. Mathematical modeling process in the classroom

In this section we present the process of mathematical modelling done together with sixteen gifted grammar school students, already introduced to some probability theory. First, we formulate the real problem as a question:

What is the probability that Djoković wins a game against Federer?

Let us describe the whole process of transition from the real problem stage to the mathematical model that is happening with our students. At the beginning, the students were rather satisfied with the possibility to discuss about the match between Djoković and Federer, since they already knew the result of the match, and they were very proud about it. But, they were also very surprised that we suggested them to apply mathematics to some tennis match, to use a method very different from usual statistical analysis, such as shown in the live commentary in [12]:

We explained that the statistics was not necessary for our work this time. Several students remarked that it was not a simple task, and they suggested to go to the upper mentioned site and analyze more precisely the match under consideration.

Then, they mentioned several components that could influence the results of the mach. Further on, the students were very active in their conversation about chances, the possibilities, the probabilities for Djoković to be the winner. In these discussions they recalled some of their mathematical knowledge about the notion of *probability*, its properties, and so on.

In this class there is a girl, Marija, who is volleyball player. She was the most active student during the modelling session and got excellent mark for her work.

Firstly, she mentioned that mathematical model cannot be done without a lot of unknown propositions that are happening during the match, such as: weather conditions, health and feeling condition of both players, and many others.

We omitted all these factors, because they cannot be expressed mathematically, but the students claimed that there are infinitely many possibilities because of the “deuce”. Then we suggested to analyze the development of the match between Djoković and Federer, before deuce.

Secondly, Marija remarked that the first two games were won by Federer, and as it is known the tennis players alternate in serving. In order to simplify this model the teacher suggested skipping the advantage of servers.

Next, we asked students to draw the graph of the first game and to calculate the probability p of Federer wining this game. They made calculations shown on the blackboard (Figure 4). At the beginning they assumed that $p = 1/2$ for each player winning the point, and they got results $\left(\frac{1}{2}\right)^4$ for Federer. After that we suggested that they denote by p the probability for Djoković to win the point and they calculated it correctly (Figure 4.).

After this first attempt the students were interested in continuing the mathematical modelling process, because they figured out all the possibilities that could have happened before the deuce and almost all of them drew the correct graph in their notebooks (Figures 5 and 6). It is important to note that the students used kind of Markov chain of Tennis game, presented in Figure 2, without knowing it. They started from one side (when Federer won the game after 40:0, and fulfilled the rest of the Markov chain. Of course we did not mentioned the notion of *Markov chains*, we called it a *graph*, but we used it to calculate the number of ways for obtaining a specific score (paths in the graph leading to a specific vertex), for example 40:15, 40:30.

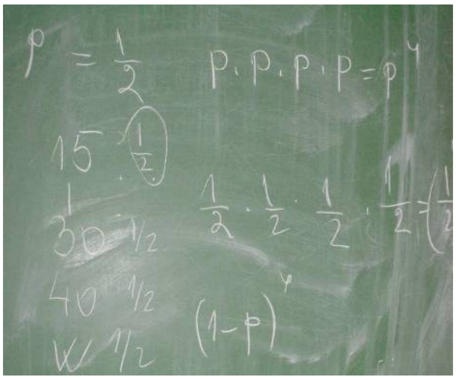

Figure 4. The work of students on the board

Figure 5. The work of students in notebook

Marija, (Figure 7.) was discussing with other students about the graph (in fact a Markov chain) and the possibility and probability for both players to reach specific scores. In that manner, she decided to count the different ways for reaching each score. In order to collect her results, and to visualise them properly, we suggested the form of the table, similar to the table in Figure 3. done in the paper [11]. She fulfilled it successfully (Figure 8. on the right).

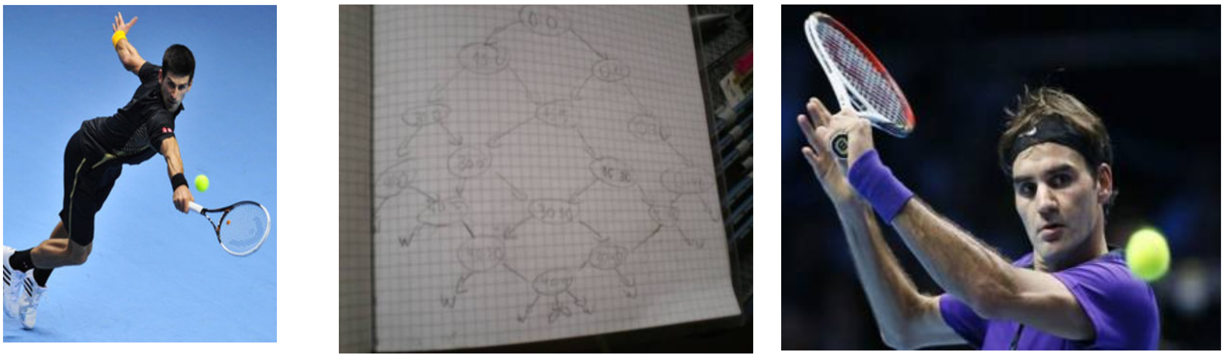

Figure 6. The Graph of the match between Djokovic and Federer

Maria and almost all other students remarked the symmetry in the number of ways to reach some results, for example 15:40 and 40:15. In considering the number of ways (3) to reach 15:30 and 30:15, they recognized that 3=2+1. Further on, they checked this fact with scores 30:30, and all others, and they filled the table rather quickly. When they came to “deuce” score, they wanted to continue with filling the table, but they were confused with the further notation. We suggested them to do it similar as it was presented in Table 3.

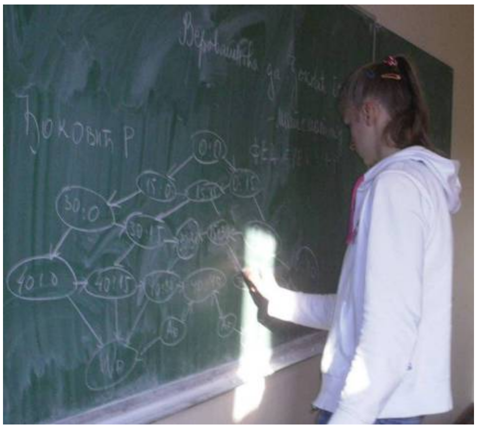

Figure 7. Marija is drawing a graph

Figure 8. Vlada is considering Pascal triangle

When we finished the table similar to Table 3, the student Vlada (Figure 8.) recognized that he already met and worked on a similar table, such that each numbers is the sum of corresponding numbers in previous row. Other students helped him with the name, “Pascal triangle”, and he drew it on the blackboard. At that moment the students were really lucky, to connect the tennis game with the “Pascal triangle”. Further on, we went to the probability. Now the job was rather easy (p is the probability that Djoković wins a point):

- In the first game, at the score 40-0, Federer wins the next point (and thus won the first game). The probability for this score is $(1-p)^4$.
- The probability that Djoković wins the next point after 40-15 is $4p^4(1-p)$.
- The probability that Djoković wins the next point after 40-30 is $10p^4(1-p)^2$.
- The total probability that Djokovic wins the game is $p^4 + 4p^4(1-p) + 10p^4(1-p)^2 = p^4(1+4(1-p)+10(1-p)^2$.
- The probability that Djoković wins the next point after the first “deuce” is $p^4 + 4p^4(1-p) + 10p^4(1-p)^2 + 20p^5(1-p)^3$
- and after the next several “deuces” is $p^4 + 4p^4(1-p) + 10p^4(1-p)^2 + 20p^5(1-p)^3 + 40p^6(1-p)^4 + 80p^7q^5 + ...$

At this moment we stopped the calculations and the students were satisfied, because they got a mathematical formula showing that they were right at the beginning of the lecture, when they proposed that there are infinitely many possibilities for wining, if “deuce” occurs.

Let us note that we finished the whole mathematical modelling process for 45 minutes. Our high school students are gifted students, but they were not introduced to the notion of *limits*, or to the sum of finite numbers of terms of geometric series. We shall work on it next year, and of course we shall repeat this mathematical model of a tennis game.

This year it was an introduction to combinatorics, which is our curriculum for the end of the first semester. The students recognized that it can be connected with combinatorics and were very satisfied that it could be integrated with something they had deep interest in.

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