R E V I E W

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on a competition for occupying the academic position of Associate Professor

area of higher education: *4. Natural Sciences, Mathematics and Informatics*,

professional field *4.5 Mathematics*, scientific speciality *Theory of probability and mathematical statistics*

for the needs of the Institute of Mathematics and Informatics, Bulgarian Academy of Scinecs

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I am a member of the scientific panel for this procedure according to order No. 64/21.03.2024 of the Director of the Institute of Mathematics and Informatics Prof. D.Sc. Peter Boyvalenkov. Documents for participation in the announced competition have been submitted only by Assistant Professor Assen Georgiev Tchorbadjieff. As a member of the scientific panel, I have received from Dr. Tchorbadjieff all the administrative and scientific documents required by the Act on the Development of the Academic Staff in the Republic of Bulgaria (ADASRB), the Rules for its implementationand the Rules on the terms and conditions for awarding of academic degrees andoccupying of academic positions at Bulgarian Academy of Sciences.

Personal data

Dr. Assen Tchorbadjieff was born on November 6, 1978 in the city of Sofia. From 1999 to 2003, he was a bachelor majoring in "Applied Mathematics" at the FMI of Sofia University "St. Kliment Ohridski" (SU), and from 2004 to 2006 he was a master's student at the Faculty of Physics of SU, where he received a master's degree in "Engineering Physics", (specialty "Wireless networks and devices". In January 2008, he was accepted as a doctoral student at the Institute of Nuclear Research and Nuclear Power Engineering at the National Academy of Sciences.He defended his doctoral thesis in 2013 in physical sciences (Physics of elementary particles and high energies).

In the period 2004 – 2007 he worked at Rila Solutions as a programmer with the main activity of programming mobile devices and applications. After that, in the period 2007-2015, he worked as a physicist at INRAE-BAS with the main scientific research activity related to conducting experiments, data processing and statistical analysis. Again in this period (2007-2009) he worked at Reflective Solutions as a programmer. From 2015 until now, he has held the position of chief assistant at the Institute of Mathematics and Informatics at the BAS, Operations Research, Probability and Statistics section.

General characteristics of the candidate's scientific works

Dr. A. Tchorbadjieff has submitted **18** scientific articles for participation in the competition, all published in the period 2015-2023 after acquiring the educational and scientific degree "Doctor" and the competition for chief assistant, among which in **2** he is a sole author, and the rest are co-authored.

These articles satisfy the minimum requirements under the ADASRB. The articles with which A. Tchorbadjieff participated in the competition were cited **18** times by other authors. He has also provided a list of all his publications and citations, which contains **33** articles and **32** citations (excluding self-citations).

As far as I know, A. Tchorbadjieff's articles for the competition were not used to obtain the educational and scientific degree "Doctor" or to occupy the academic position of chief assistant. I would also like to point out that I have not found any cases of plagiarism in Tchorbadjieff's articles. The listed works of A. Tchorbadjieff fall into the field of the competition and more specifically they outline two directions in his scientific activity - the first is in the field of statistical modeling of cosmic rays and atmospheric, geomorphological processes and geostatistics, and the second one is in the theory of branching stochastic processes and their simulation modeling.

In the first direction, taking into account the specificity of observational data, Tchorbadjieff shows in-depth knowledge and skills in statistical data analysis such as classification and cluster analyses, generalized linear regression models, a wide range of methods for checking normality, independence, model adequacy and coherence, Change Point Analysis etc.

Regarding the second direction, let us note that the term *branching processes* was introduced by **A. N. Kolmogorov and N. Dmitriev in 1947** and means an area of mathematics in which the characteristics of populations composed of elements that may have different origins - cells, living organisms, elementary particles, financial instruments, information, etc., but whose development is independent of the others and assumes that they follow the same pattern of development: birth-existence-death accompanied by the birth of new particles (offspring). At its demise, each element gives rise to a random number of the same elements (subject to the same probabilistic laws in their development) and thus constructively describes the evolution of a branching stochastic process (BSP).

The articles presented by Tchorbadjieff address and solve interesting and difficult problems in the indicated areas, where it is evident that the candidate has mastered a wide range of data analysis methods aimed at global climate and water changes and has successfully entered BSP theory and in particular the Markovian ones. Most of them have been published in prestigious mathematical and statistical journals, including **Journal of Applied Statistics, Modern Stochastics: Theory and Applications, Lithuanian Mathematical Journal and Compt. Rend. Acad. Bulg. Sci.,** address and solve interesting and difficult problems in the indicated areas, where it is evident that the candidate has mastered a wide range of data analysis methods aimed at global climate and water changes and has successfully entered BSP theory and in particular the Markovian ones.

Analysis of the candidate's scientific achievements

In the overview of the applicant's results, we will follow the numbering of the articles in his list of publications for the competition, attached to the statement of originality of the submissions.

Articles numbered [1-7] are in the field of statistical modeling of cosmic rays and atmospheric, geomorphological processes and geostatistics.

More specifically, the publications [1-3] are brought together by the collection, processing and statistical analysis of data related to climate observations collected at the Base Environmental Observatory (BEO) on Musala peak of Rila mountain. Interactions between cosmic rays and the Earth's atmosphere, parameters of global change and climate research, natural hazards and technological risks are the goals of research in BEO. The article [1] is an overview and is related to real-time measurements of basic space and atmosphere parameters. The information is transmitted through a high-frequency radio-telecommunication system to the Internet and stored in a database for further analysis within the international networks GAW, EURDEP, EUSAAR (ACTRIS), RECETOX and UNBSS. This year BEO celebrates its 65th anniversary.

Papers [2-3] examine the impact of Saharan dust storms due to long-range mass air transport. The results obtained in [2] were confirmed by satellite data and traced to the origin of the pollution source with back trajectories calculated by the **HYSPLIT** model. The dust source was confirmed by satellite data calculated from several different infrared channels of the **NASA Aqua/AIRS** satellite. In [3], the topic of the transboundary transfer of Saharan sand was continued by creating a software system for automating the tracking of the pollution source. It is based on automated processing and combining of NASA Aqua/AIRS Dust Index satellite data and air transport trajectories using the HYSPLIT model.

Articles [4] and [5] can also be united by a common theme, namely *arsenic contamination of the Ogosta River Valley*. The collected data on heavy metal pollution along the Ogosta valley around town of Chiprovtsi show a strong unevenness of the studied terrain, thta is why the classical methods of multidimensional analysis are not suitable. In order to prevent these features in the data, the samples were grouped according to the geomorphological features of the terrain. For this purpose, a **K-means cluster method** was used, which was subsequently refined with **classification analysis**. Further, applied clustering was combined in [5] with **component analysis** to reduce data inhomogeneity and derive primary geochemical relationships for arsenic concentration with geomorphological and geochemical parameters such as groundwater level, oxidation, electrical conductivity, etc. For this purpose, **generalized linear regression models (Generalized Linear Models)** with *Gaussian and gamma distribution of the dependent variable* were used.

Articles [6] and [7] are devoted to the detection of structural differences as a result of natural changes in the process or a change in the measurement mode, observed during long time periods of measurement of dynamically changing processes. The statistical method for this is known as **Change Point Analysis**. Its main principle is the use of *parametric and non-parametric tests to assess the credibility of the statistical hypothesis* of a systematic change in mathematical expectation and/or variance or other significant statistical assessment. Using a ready statistical package for Change Point Analysis, a model was prepared for automatic detection of the Forbush effect - a physical process of a sharp reduction lasting several days in the intensity of Galactic cosmic rays as a result of modulation by particles from solar coronal eruptions. The model was tested with data from the BEO Musala Muon Telescope. The difficulty in using the standard parameters for the Change Point analysis arises from the presence of a long autocorrelation lag in the atmospheric pressure-corrected muon time series.

The application of the developed model for automatic detection of Forbush events with Change Point Analysis is demonstrated with real data measured at BEO Musala and referenced from NOAA satellite data in work [7]. The research covers a period of 4 years, and more than 10 solar eruptions with different geo-effective intensities have been confirmed and detected.

Articles numbered [8-16] fall into the field of branching process theory. In general, the tasks that are solved in them are motivated and related to the modeling of the cascade processes taking place in the atmosphere, naturally in the framework of BP. This is the reason for the development of one of the directions, namely the study of the influence of the initial conditions on the asymptotic behavior of the MBP, as well as the influence of the individual distribution of the offspring in terms of the BP on their asymptotic behavior.

More specifically, paper [8] presents a brief description, combined with computer simulations, of the modeling by BP of a cascade process caused by the birth of many secondary particles - protons (hydrogen nuclei, 90%), alpha particles (about 9%) with high and ultra-high energies generated by the entry of cosmic rays into the atmosphere.

Article [9] is devoted to the study of the influence of the initial conditions on the development of a Markov branching process (MBP) (BP in continuous time when the lifetime of the particles has an exponential distribution), when the initial number of particles ***is geometric (shifted and unshifted), negative binomial and Pólya-Aeppli distributed.*** The probabilistic generating function (p.g.f.) of the processes in the *supercritical case* with an initial distribution among the listed ones was found in an explicit form. Let us note that the task of finding in an explicit form the p.g.f. of a BP is of particular importance, and as is known in BP theory this has been done in the classical case of the Galton-Watson BP, as well as for its continuous-time Markov counterpart, in the special case where the distribution of the progeny of a particle is with ***two-parameter geometric distribution Ge(a,b), a< 1- b, a, b >0 with*** $p^k = ab^{k-1}$***, k= 1, 2, …*** In the publication [9], however, with the receipt of the p.g.f. invariance of the initial distribution with respect to the BP distribution for arbitrary t > 0 is established, i.e. and the BP distribution is of the initial distribution type, but with other time-dependent parameters. Assen Tchorbadjieff's results are new and in general complement and extend this field and are a step from the study of MBRs with different initial conditions with the aim of subsequent application.

Further studies are completed for critical MBPs with ***negative binomial, Poisson, and Pólya-Aeppli*** as initial particles' number distributions. Degeneracy probabilities and generating functions are found, which are found to depend on the initial conditions. The random initial conditions do not change the critical parameter of the branching process, but they do affect the degeneracy probability of the processes. Subsequently, the obtained results were compared statistically with a specially created simulation code, which is a modification of the program code already created in [8]. This allows the calculation of process distributions with more complex initial conditions, demonstrated in the case of ***Pólya-*** scheme. These results are summarized in a publication [10].

The publication [11] is closely related to [10] and coincides with the onset of the COVID-19 pandemic. The model developed in [10] is applicable in computer modeling of time series data on the number of infected individuals. Despite the measures taken, the intensity of the infection has changed dramatically over the two-year period between 2020 and 2022, causing waves of very high infection rates. To address this problem, the heterogeneity of the data is modeled by change point regime. The developed model is a combination of automatic regime change detection with a linear birth and death process. The results are empirically validated by data for 38 countries and the US for the period February 2020 to April 2022.

The publication [12] is devoted to the application of the *Lagrange Inversion method* for finding the solution of the **backward Kolmogorov equation** for a subcritical and time-homogeneous MBP with a geometric distribution of the offspring of one particle. In the *subcritical case*, the solution of the **backward Kolmogorov equation** for the p.g.f. of the process is expressed as a composition of a **Gaussian hypergeometric distribution and a special Wright function**, and in the *critical case* by a composition of a **special Lambert - W function and a fractional linear function**. The Lambert-W function is considered as the real-valued composite inverse of the function $V(x) = xe^x,\ x \geq -1$. In the subcritical case, the conditional marginal distribution is also obtained in an explicit form. It turns out to be a new kind of discrete integer distribution with support on the positive integers and its dispersion index, defined as the ratio of the variance to the mathematical expectation, turns out to be a solution to a transcendental equation. The moments of the marginal distribution in terms of **Bell polynomials** are also found. *In the critical case,* the probability of degeneracy of the processes and the distribution function expressed in a series containing Bell polynomials, **Stirling and Lach numbers** are obtained.

The publication [13] is a study of the *factorial moments* of a critical MBP with a geometric offspring distribution. They are known to describe the behavior of p.g.f. $F(t, s)$ of the process in a neighborhood of the point $s = 1$. They are used to solve the **forward Kolmogorov equation** for a critical MRP with a geometric distribution of offspring. The solution involves rapidly converging repeated polynomial iterations. The obtained results for the factorial moments make it possible to calculate the skewness and kurtosis of the distribution.

In [14], the study of homogeneous MBPs with geometric offspring distribution is completed by finding the solution of the **backward Kolmogorov equation** in *the supercritical case*. The results include finding the probability distribution of the number of particles present at a given time. It is calculated explicitly by the derivative of a complex function and the **Fáa Di Bruno formula**. Also, the solution of the Kolmogorov equation is expressed by the **special Wright function**. The representation of this function in convergent series is obtained after applying the **Lagrange inversion method**. The asymptotic behavior is described using two different equivalent forms for the Laplace transform. These include the computation of the marginal distribution and its moments. The exact formula for the asymptotic density is expressed by means of the **reduced Wright function**. In particular, when the degeneracy probability q = 1/2, the density of the marginal random variable is expressed by an *incomplete Gamma function*.

This range of problems also includes work [15], in which a *subcritical* MBP *X(t)* is studied, starting with a single particle as an initial condition and the offspring distribution is a mixture of *logarithmic distributions over the non-negative integers*. In this case, the number of existing particles at time *t>0* is found to have a **shifted Extended Sibuya distribution**. This is obtained in terms of p.g.f. of the process as a solution of the **forward and backward Kolmogorov equations**. The distribution of the number of particles present at time *t>0*, the factorial moments, the conditional distribution of the process at time *t>0*, under the condition of non-degeneracy, is also found and a limit theorem is obtained for the conditional probability distribution, which turns out to be a mixture of logarithmic distributions on the non-negative integers or of the type of distributions similar to the extended Shibuya distribution.

The developed software functionalities in R for BP simulation, accompanying Tchorbadjieff 's previous research in the direction of branching processes, are presented in the work [16]. Originally developed to simulate the branching mechanism of a cosmic ray cascade in the atmosphere, starting with an electron-photon cascade, they have subsequently been adapted for applications in epidemiology with an extended range of probability distributions such as **Poisson, negative binomial, shifted geometric and *Pólya-Aeppli*,** used either to model the initial conditions for linear birth-death processes or as a probability distribution of offspring in terms of BP. The simulator is mostly applied when analytical solutions yield convergent infinite series. It uses R's parallel computing capabilities.

Formally, the publications [17] and [18] fall outside the two main directions of A. Tchorbadjieff scientific interests, but essentially they can be related to the two already outlined directions in the candidate's scientific work, respectively.

The work [17] is dedicated to a Levy process (LP) with an infinitely divisible logarithmic distribution (IDLD) and in general can be joined to the more theoretical direction in the scientific activity of A. Tchorbadjieff and more precisely to the study of the characteristics of **Lévy process** subordinated to a Poisson process and its comparison with other similar processes by certain characteristics. For the corresponding **Lévy** measure of LP with IDLD, it is deduced that it can be calculated by means of **Bell polynomials**, and the generating function by Gaussian logarithm and hypergeometric function. The transition probabilities of the process, again involving Bell polynomials, are calculated. Subsequently, two time-replaced random processes are defined and compared – a negative binomial random variable with time replaced by a gamma process and a **Lévy logarithmic process** subordinated to a Poisson process. The **Bernstein functions,** the **Lévy measure** of the subordinated process and the transition probabilities were calculated for them. The motivation for this study is due to the loss of energy of the particles as they penetrate the atmosphere.

The accuracy analysis in [18] is related to the evaluation of a new method such as 3-dimensional laser scanning in the processing of anthropological specimens and more specifically human skulls, until the entry of which a procedure of manual measurements was relied upon. For this purpose, an intercalibration experiment was conducted with comparative measurement of human skulls according to 13 key parameters. The results were analyzed using **Bland-Altman plots**, considering the intra-group variance between the participants and showing compatibility and substitutability between the two methods.

The analysis made so far, in my opinion, is completely sufficient to form a positive attitude towards Assen Tchorbadjieff's candidacy for this competition.

Participation in scientific projects

Assen Tchorbadjieff participated in 3 scientific projects financed by the "Scientific Research Fund" at the Ministry of Education and Science, for one of which he was the package leader.

Personal impressions

I have known A. Tchorbadjieff since he joined IMI-BAN as a Chief Assistant Professor. He worked on the project FNI-KP-06-H22-3/2018 led by me with the Scientific Research Fund of the Ministry of Education and Science. My impressions have been shaped by his work on the project and his presentation at international conferences where he has reported. I believe that in the period that I am reviewing, he carried out modern and substantive research and made serious scientific contributions.

Critical remarks

My critical remarks are of a technical nature and concern the layout of the documents. The numbering of the articles in the reference of scientific contributions and in the list of abstracts is different and this makes it difficult to track the results.

In addition, the Bulgarian translation of the summaries has not been edited, and instead of helping the understanding, it makes it difficult that it is even inaccurate. But this cannot diminish from the serious scientific results.

CONCLUSION

The documents and materials presented by Chief Assistant Professor Assen Tchorbadjieff, meet all the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Rules for the Implementation of ZRASRB and the Rules for the Terms and Conditions for Obtaining an Academic Degree and Holding an Academic Position at the BAS. The results obtained by Assen Tchorbadjieff are an important contribution to the development of the theory of branching stochastic processes and their applications in physics and data analysis related to atmospheric changes and air, soil and water pollution. His scientific achievements exceed the usual requirements for holding the academic position of "Associate Professor".

Based on the comments above, I strongly recommend the scientific jury to propose to the Scientific Council of IMI-BAN to appoint Assistant Professor Dr. Assen Tchorbadjieff as an Associate Professor in the area of higher education 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics, scientific specialty Probability theory and mathematical statistics.

09.06. 2024 Reviewer:

(Prof. Dr. Maroussia Bojkova)