I mainly work in graph theory and combinatorics and the contribution is a theoretical research. Let us describe its main branches.

Hypergraph colorings. Some of the papers continue the research from my PhD thesis. Define the value $m(n, r)$ as the smallest number of edges in an n-uniform hypergraph (that is, a system of n-element sets) which cannot be properly colored in r colors (the coloring of the vertices of a hypergraph is called *proper* if there are no monochromatic edges).

In [1] it is proved that for a fixed n the value $m(n, r)/r^n$ has a limit — this was previously stated as a conjecture by Noga Alon. A similar regularity theorem is proved for the case of list colorings.

Paper [2] consider a particular case $n = 3$ and refines the lower bound in this case so that now $0.54 < L_3 \leq 4/3$, where L_3 is a limit from the previous paragraph.

Let us denote by $f(n)$ the smallest number of edges in an n-uniform hypergraph, which cannot colored in two colors with zero discrepancy — that is, so that each edge contains $n/2$ vertices of each of the two colors. Alon, Kleitman, Pomerance, Saks, and Seymour proved that

$$c_1 \frac{\log \operatorname{snd} n}{\log \log \operatorname{snd} n} \leqslant f(n) \leqslant c_2 \frac{\log^3 \operatorname{snd} n}{\log \log \operatorname{snd} n}$$

where $\operatorname{snd}(n)$ is the smallest natural non-divisor of n. In [3] the upper bound is improved to $O(\log \operatorname{snd} n)$.

The work [4] applies the notion hypergraph discrepancy to the colorings of the generalized Kneser graph $K(n, k, s)$ whose vertices are k-subsets of the n-set, and the edges correspond to pairs of sets which have less than s elements in common. Using the construction, based on hypergraphs of large discrepancy, a proper coloring of $K(n, n/2-t, s)$ in $(4+o(1))(s+t)^2$ colors is constructed. Namely, we use a classical example of a hypergraph with large discrepancy based on Hadamard matrices.

The survey [5] is devoted to the results in extremal prolems in hypergraph colorings obtained during the last decade.

Chromatic numbers of spaces. Let M be a metric space and let G be the corresponding graph, whose vertices are points of M and edges connect vertices at distance 1 apart. Then the chromatic number $\chi(M)$ is defined as the chromatic number of G. Usually M is a subset of Euclidean space.

Paper [6] shows that for every positive ε one has $\mathbb{R}^2 \times [0, \varepsilon]^2 \geq 6$.

Papers [7, 8] refine lower bounds on $\chi(\mathbb{R}^n)$ for n in the range $\{9, 10, 11, 12\}$. For this purpose we find the independence numbers of the series of graphs $F_n \subset G(\mathbb{R}^n)$, where the vertex set of F_n is the set of all vectors $v \in \mathbb{R}^n$ with coordinates in $-1, 0, 1$ such that $|v| = \sqrt{3}$ and the edge set consists of all pairs of vertices with scalar product 1. Surprisingly, for a large n the maximal size of an independent set is $6n - 28$, and an example is quite complicated.

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