Abstracts of the papers

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The abstract of [1]. We consider continuous maps of compact metric spaces. It is proved that every pseudotrajectory with sufficiently small errors contains a subsequence of positive density that is point-wise close to a subsequence of an exact trajectory with the same indices. Also, we study homeomorphisms such that any pseudotrajectory can be shadowed by a finite number of exact orbits. In terms of numerical methods this property (we call it multishadowing) implies possibility to calculate minimal points of the dynamical system. We prove that for the non-wandering case multishadowing is equivalent to density of minimal points. Moreover, it is equivalent to existence of a family of ε-networks ($\varepsilon > 0$) whose iterations are also ε-networks. Relations between multishadowing and some ergodic and topological properties of dynamical systems are discussed.

The abstract of [2]. Let $m(n,r)$ denote the minimal number of edges in an n-uniform hypergraph which is not r-colorable. It is known that for a fixed n one has $c_n r^n < m(n,r) < C_n r^n$. We prove that for any fixed n the sequence $a_r := m(n,r)/r^n$ has a limit, which was conjectured by Alon. We also prove the list colorings analogue of this statement.

The abstract of [3]. We apply the bound on independence number via Lovász theta function to eventown problem and its generalizations over $\mathbb{Z}_n$.

The abstract of [4]. Let $m(n,r)$ denote the minimal number of edges in an n-uniform hypergraph which is not r-colorable. For the broad history of the problem see [5]. It is known [2] that for a fixed n the sequence

$$\frac{m(n,r)}{r^n}$$

has a limit. The only trivial case is $n = 2$ in which $m(2,r) = \binom{r+1}{2}$. In this note we focus on the case $n = 3$. First, we compare the existing methods in this case and then improve the lower bound.

The abstract of [5]. Extremal problems in hypergraph colouring originate implicitly from Hilbert's theorem on monochromatic affine cubes (1892) and van der Waerden's theorem on monochromatic arithmetic progressions (1927). Later, with the advent and elaboration of Ramsey theory, the variety of problems related to colouring of explicitly specified hypergraphs widened rapidly. However, a systematic study of extremal problems on hypergraph colouring was initiated only in the works of Erdős and Hajnal in the 1960s. This paper is devoted to problems of finding edge-minimum hypergraphs belonging to particular classes of hypergraphs, variations of these problems, and their applications. The central problem of this kind is the Erdős–Hajnal problem of finding the minimum number of edges in an n-uniform hypergraph with chromatic number at least three. The main purpose of this survey is to spotlight the progress in this area over the last several years.

The abstract of [6]. A two-coloring of the vertices V of the hypergraph $H = (V,E)$ by red and blue has discrepancy d if d is the largest difference between the number of red and blue points in any edge. Let $f(n)$ be the fewest number of edges in an n-uniform hypergraph without a coloring with discrepancy 0. Erdős and Sós asked: is $f(n)$ unbounded? N. Alon, D. J. Kleitman, C. Pomerance, M. Saks and P. Seymour proved upper and lower bounds in terms of the smallest non-divisor (snd) of n. We refine the upper bound as follows:

$$f(n) \leq c \log \operatorname{snd} n.$$

The abstract of [7]. We suggest a new method for coloring generalized Kneser graphs based on hypergraphs with high discrepancy and a small number of edges. The main result provides a proper coloring of $K(n, n/2 - t, s)$ in $(4 + o(1))(s + t)^2$ colors, which is produced by Hadamard matrices. Also, we show that for colorings by independent set of a natural type, this result is the best possible up to a multiplicative constant. Our method extends to Kneser hypergraphs as well.

The abstract of [8]. Let G be a simple graph with n vertices and ± 1-weights on edges. Suppose that for every edge e the sum of edges adjacent to e (including e itself) is positive. Then the sum of weights over edges of G is at least $-\frac{n^2}{25}$. Also we provide an example of a weighted graph with described properties and the sum of weights $-(1+o(1))\frac{n^2}{8(1+\sqrt{2})^2}$.

The previous best known bounds were $-\frac{n^2}{16}$ and $-(1+o(1))\frac{n^2}{54}$. respectively. We show that the constant $-1/54$ is optimal under some additional conditions.

The abstract of [9]. New lower bounds are found for the minimum number of colors needed to color all points of a Euclidean space in such a way that any two points at a distance of 1 have different colors.

The abstract of [10]. This paper is devoted to the study of the graph sequence $G_n = (V_n, E_n)$, where V_n is the set of all vectors $v \in \mathbb{R}^n$ with coordinates in $-1, 0, 1$ such that $|v| = \sqrt{3}$ and E_n consists of all pairs of vertices with scalar product 1. We find the exact value of the independence number of G_n. As a corollary we get new lower bounds on $\chi(\mathbb{R}^n)$ and $\chi(\mathbb{Q}^n)$ for small values of n.

The abstract of [11]. This paper is devoted to a natural generalization of the problem on the chromatic number of the plane. The chromatic number of the spaces $\mathbb{R}^n \times [0, \varepsilon]^k$ is considered.

It is proved that $5 \leq \chi(\mathbb{R}^2 \times [0, \varepsilon]) \leq 7$ and $6 \leq \chi(\mathbb{R}^2 \times [0, \varepsilon]^2) \leq 7$ for $\varepsilon > 0$ sufficiently small.

Also, some natural questions arising from these considerations are posed.

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