R E P O R T

on the competition with a single applicant
Dr. Danila Dmitrievich Cherkashin
Scientific field: **4. Natural sciences, mathematics and informatics**
Professional field: **4.5. Mathematics (Combinatorics, graph theory)**

The competition is announced in "Durzhaven vestnik" no. 14/10.02.2023 There is one application for the position submitted by:

Dr. Danila Dmitrievich Cherkashin.

1. **Personal Data**

   Dr. Danila Cherkashin graduates from Sankt Petersburg University in 2015. In 2018 he receives the doctor degree from the Sankt Petersburg section of the Steklov Institute of the Russian Academy of Sciences. His PhD thesis is entitled "Extremal problems in hypergraph colorings" under the supervison of Andrey Raigorodsky and Fedor Petrov. From 2015 until 2021 he takes on research positions in the University of Sankt Petersburg and the Moscow Institute for Physics an Technology (MFTI). In the period 2021-2022 Danila Cherkashin is a research in the sankt Petersburg secion of the Institute Steklov. From 2022 on he is a research associate in the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

2. **Research Activity**

   The applicant presents 11 scientific articles. All of them are published in respectable scientific journals with an IF and/or SJR. The papers are published in the following journals:

   - Topological Methods in Non-linear Analysis - 1; (IF 0.87)
   - Doklady Akademii Nauk - 1; (IF 0.534)
   - Algebra and Analysis - 1; (IF 0.533)
   - Discrete Applied Mathematics - 1; (IF 1.139)
   - Journal of Combinatorial Theory, Ser. B - 1; (IF 1.306)
   - European Journal of Combinatorics - 1; (IF 0.89)
   - Zapiski Nauchnih Seminarov POMI - 3; (IF 0.289)
   - Journal of Discrete Mathematics - 1; (IF 0.87)

- Uspehi Matematicheskih Nauk - 1; (IF 1.865)
- Electronic Journal of Combinatorics – 1; (IF 0.69)
- Linear Algebra and Its Applications - 1; (IF 1.307)

The total impact factor of all papers is 10.313. In one of the presented papers the candidate is the only author, seven are with one co-author, and three have two co-authors. I accept that the contribution of the candidate is equal to that of the other co-authors.

All papers are written in the period 2017-2022 and are published after the acquisition of the PhD degree.

The scientific research of the applicant corresponds thematically to the description of the position. They are devoted to problems from the extremal combinatorics. I shall review the papers by the candidate in more detail.

The research of Dr. Cherkashin can be divided thematically in two groups:

(A) papers on the chromatic numbers of real spaces (papers [1–4]);

(B) papers on hypergraph colorings (papers [5–11]).

(A) **The chromatic numbers of real spaces**

Paper [1] considers maps of continuous maps of compact metric spaces. It is proved that each that each pseudotrajectory with sufficiently small errors contains a subsequence of positive density that is pointwise close to a subsequence of an exact trajectory with the same indices. This paper studies also homeomorphisms, such that any trajectory can be shadowed by a finite number of exact orbits.

Paper [2] deals with the classical problem of the coloring of the points in an euclidean space of any dimension for which every two points at distance 1 are differently colored. In particular, the chromatic numbers $\chi(\mathbb{R}^n)$ (resp. $\chi(\mathbb{Q}^n)$) are defined as the minimal number of colors that are enough to color the points of $\mathbb{R}^n$ (resp. $\mathbb{Q}^n$) so that every two points at distance 1 obtain different colors. In this paper the following lower bounds are proved:

$$\chi(\mathbb{R}^9) \geq 22,\ \chi(\mathbb{R}^{10}) \geq 30,\ \chi(\mathbb{R}^{11}) \geq 35,\ \chi(\mathbb{R}^{12}) \geq 37.$$

In paper [3] the authors consider a natural generalization of the problem of the chromatic number of the plane. They investigate the chromatic number of the spaces $R^n \times \{0, \varepsilon\}^k$. The following inequalities are proved

$$5 \leq \chi(\mathbb{R}^2 \times [0, \varepsilon]) \leq 7,\ \ 6 \leq \chi(\mathbb{R}^2 \times [0, \varepsilon]^2) \leq 7,$$

where $\varepsilon > 0$ is sufficiently small.

In paper [4] the sequence of graphs $G_n(V_n, E_n)$ is investigated, where V_n is the set of vectors $\mathbb{R}_n$ with coordinates in $\{-1, 0, 1\}$ and length $\sqrt{3}$, E_n is the set of all pairs of vectors with inner product 1. In this paper, the exact value of the independence number of the graphs G_n is determined. In particular, the following theorem is proved:

Theorem 1. The independence number of the graph G_n is determined by the formula

$$\alpha(G_n) = \max\{6n - 28, 4n - c(n)\},$$

where

$$c_n = \begin{cases} 0 & \text{if } n \equiv 0 \pmod 4, \\ 1 & \text{if } n \equiv 1 \pmod 4, \\ 2 & \text{if } n \equiv 2, 3 \pmod 4. \end{cases}$$

Furthermore, the authors prove new lower bounds on the chromatic numbers $\chi(\mathbb{R}^n)$ and $\chi(\mathbb{Q}^n)$.

Theorem 2. The following inequalities hold

$$\chi(\mathbb{R}^n) \geq \chi(\mathbb{Q}^n) \geq \frac{|V_n|}{\alpha(G_n)} = \frac{8\binom{n}{3}}{\max\{6n - 28, 4n - c(n)\}}.$$

(B) **Hypergraph colorings**

Papers [5] and [6] deal with hypergraphs colored with two colors (red and blue) with an upper bound on the difference of red and blue elements in each edge. This value is denoted by d and is called discrepancy. Let $f(n)$ be the minimal number of edges in an n-uniform hypergraph. The main result in [5] is Theorem 1.2 according to which

$$f(n) \leq c \log \operatorname{snd}(n),$$

where $\operatorname{snd}(x)$ is the minimal positive integer that does not divide x.

In [6] the candidate presents a new method for coloring of generalized Kneser graphs, based on hypergraphs of large discrepancy and a small number of edges. The central result in the paper is contained in Theorem 2 and gives a proper coloring of the graphs $K(n, n/2 - t, s)$ with $(4 + o(1))(s + t)^2$ colors, which is obtained by using Hadamard matrices. It is proved that for colorings by independent set of natural type this result is the best possible up to a multiplicative constant.

In paper [7] the problem of Erdős-Hajnal for 3-uniform hypergraphs is considered. Let $m(n, r)$ denote the minimal number of edges in an n-uniform hypergraph with a chromatic number larger than r. By a result of the candidate contained in [8], it is known

that the sequence $\frac{m(n,r)}{r^2}$ is convergent and the corresponding bound is denoted by L_n. It is well known that $m(2,n) = \binom{n+1}{2}$. In this paper the authors compare the existing methods for finding lower bounds on $m(3,n)$. Furthermore, they present improvements on the existing lower bound. It is proved that $L_3 \geq \frac{4}{e^2} > 0.54$.

Paper [8] contains a proof of a conjecture by Noga Alon on the convergency of the sequence $a_r := m(n,r)/r^n$. Up to this moment it was only known that there exist constants c_n and C_n, for which $c_n r^n \leq m(n,r) \leq C_n r^n$.

Paper [9] is a survey on the problem of finding hypergraphs with a minimal number of edges lying in some special class of hypergraphs. A central problem of this type is the Erdős-Hajnal problem on finding the minimal number of edges in an n-uniform hypergraph of chromatic number at least 3. The survey contains the latest results in this field obtained in the last few years. It also contains a large list of 168 papers containing results in this area of research.

In [10] the authors consider simple graphs with n vertices and weights of the edges ± 1, subject to the condition that the sum of the weights of all edges adjacent to given edge (including the edge itself), is positive. It is proved that the sum of the weights of all edges in G is at least $-n^2/25$. Examples of graphs with the described properties are presented in which the sum of the weights of all edges is $-(1+o(1))\frac{n^2}{8(1+\sqrt{2})}^2$.

In [11] the candidate finds a bound on the independence number by using Lovasz' theta function for the eventown problem and its generalizations on Z_n. A family of subsets of $[n]$ is called an eventown family if every two subsets have an even intersection. A family F of vectors over $\{0,1,\ldots,k-1\}^n$ such that $(f_1,f_2) = 0$ for every two vectors $f_1, f_2 \in F$ is called a k-town family. The main result in this paper is contained in the following theorem:

Theorem 3. Let F be a k-town family. Then $|F| \leq k^{\frac{n}{2}}$. Moreover, suppose that k is a prime, and $(f_1,f_2) \neq 0 \pmod{k}$ for at most $\varepsilon|F|^2$ pairs f_1, f_2. If $\varepsilon < \frac{k-1}{k}$, then

$$|F| \leq \frac{k^{\frac{n}{2}}}{1-\frac{k}{k-1}\varepsilon}.$$

3. **Main results**

In my opinion the most important scientific results of the candidate are the following:

(1) New lower bounds on the chromatic numbers $\chi(\mathbb{R}^n)$ and $\chi(\mathbb{Q}^n)$ are found.

(2) The bound

$$f(n) \leq c \log \mathrm{snd} n$$

is proved, where $f(n)$ is the minimal number of edges in an n-uniform graph, for which there is no coloring with discrepancy 0, and $\mathrm{snd}n$ is the smallest non-divisor of n.

(3) A conjecture by Noga Alon about the convergency of the sequence $a_r := \frac{m(n,r)}{r^n}$ is proved.

(4) A new lower bound on the numbers $m(3, n)$ is proved.

(5) The independence number $\alpha(G_n)$ of a special class of graphs G_n is proved. These graphs have as vertices the vectors with coordiantes $0, \pm 1$, have length $\sqrt{3}$ and edges – the pairs of vectors with inner product equal to 1.

(6) A new method for coloring of generalized Kneser graphs is suggested, which gives a proper coloring of $K(n, n/2 - t, s)$ with $(4 + o(1)(s + t)^2$ colors.

(7) A new bound on the sum of the weights of simple graphs with weights ± 1 and special local properties is proved. Examples of graphs lying close to this bound are constructed.

4. **Projects**

In the period 2016-2020 the cadidate is a participant in three projects sponsored by the Scienece foundation of the Russian Federation. These are the following:

- Combinatorial, discrete, and enumerative combinatorics
- Random graphs and hypergraphs: models and applications
- Group theory, algebraic geometry, representation theory, motive theory, homological algebra

The subjects of the three projects correspond fully to the scientific specification of the position which is combinatorics and graph theory.

5. **Numerical Data**

The presented papers by Dr. Cherkashin can be classified as follows:

- scientific journals with IF: 11

The total impact factor of the presented papers is 10.313. The candidate has presented for the competition 35 citations out of a total of 102 citations of his publications. All of them are in refereed journals. There is no doubt that the scientific output of Dr. Cherkashin is well-known and highly estimated by the mathematicians working in the field of his research interests.

6. **Critical Remarks.**

   I have no notable critical remarks. There are certain difficulties caused by different numeration of the papers in the list of publications, in the file with the abstracts and the names of the files of the publications.

7. **Personal Remarks.**

   I have known the candidate for two years. I have attended several of his talks at various workshops. My impression is most favourable. I am convinced that he is a serious resarcher with deep knowledge of the field of combinatorics and graph theory. It is beyond any doubt that he meets all requirements for the academic position of associate professor at IMI - BAS.

8. **General Assesment of the Applicant**

   In my opinion Dr. Cherkashin has obtained important scientific result that are original and match the level of contemporary mathematics. That is why I assess positively the application of Dr. Cherkashin for the position of Associate Professor in IMI - BAS in the professional field 4.5. Mathematics (Combiantorics, graph theory).

Conclusion

I am deeply convinced that **Dr. Danila Dmitrievich Cherkashin** has all the merits and professional qualifications required for the position of associate professor of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. He fulfills all the legal requirements plus the specific ones of IMI at BAS for the scientific field 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics (Combinatorics, graph theory). I strongly recomend his application for the position of an associate professor at IMI - BAS.

Sofia, 02.06.2023 Member of the Scientific Panel:

(Prof. DSc Ivan Landzhev)