R E V I E W

by Prof. Nikolay Ivanov Yankov, DSc

on the competition for acquiring the academic position of **“associate professor”**
in professional field: 4. Natural sciences, mathematics and informatics,
professional area: **4.5. Mathematics** (Combinatorics, graph theory)
published in State Gazette No 14 from 10.02.2023
for the Institute of Mathematics and Informatics (IMI),
Bulgarian Academy of Sciences (BAS)
with only candidate **Danila Dmitrievich Cherkashin, PhD**

1. General presentation of the received materials in accordance to art. 53 and 54 (1) of RALDASRB

By order No 185/07.04.2023 from IMI-BAS’s Director, I’m appointed as a member of the scientific jury. At the jury’s first meeting, I was voted to write this review. All materials for this competition have been received electronically.

In accordance with art. 57, para. 1 of RALDASRB (Rules on the application of the Law for the development of the academic staff in the Republic of Bulgaria), the assessment of the applicant for the academic position ”associate professor” is done by art. 53 and the results from the reference are in accordance with art. 54, para. 1 of RALDSRB. In connection with the implementation of art. 57a, on the basis of the documents submitted, it was established:

- a copy of diploma 330-NK-15 issued on 12.12.2018 for the educational and scientific doctoral degree PhD (satisfying art. 53, para. 1, item 1) from the St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences. This diploma is certified at BAS as corresponding to the educational and scientific degree PhD in Discrete Mathematics with Certificate No 12/05.01.2023 in accordance with art. 53, para. 1, item 1. After a search in the Bulgarian National Center for Information and Documentation (NACID) database, I can confirm that this degree is registered therein;
- that the applicant has at least two academic years of experience as a researcher in

a scientific organization (St. Petersburg State University and Moscow Institute of Physics and Technology) in accordance with art. 53, para. 1, item 2b;

- 3 publications in specialized scientific journals totaling 100 pts., when 100 pts. account for a monograph. All these works do not repeat the works used for the candidate's doctoral thesis (satisfying art. 53, para. 1, item 3);
- a reference list for meeting the minimal national requirement in accordance with art. 2b, para. 2 and 3 of LDASRB (satisfying art. 53, para. 3);
- a declaration of authorship for the scientific works for this competition.

2. Applicant's general scientific activity and contributions

For his participation in this competition, Dr. Cherkashin has submitted a total of 11 scientific papers. These publications with the exception of the work [5, On small $n-$uniform hypergraphs with positive discrepancy, JCTB, 139:353–359, 2019] do not repeat the ones submitted for the educational and scientific degree of PhD. All presented publications are in specialized scientific journals, with 9 having an impact factor (IF) in WebOfScience (WOS) and one indexed in Scopus having an SJR. About the IF papers: their quartiles are as follows: 2 are in Q1, 3 in Q2, 4 in Q3 and 1 in Q4. All journals are in the field of Mathematics.

The average IF is 0.91 which is a rather high figure for this field. Of the submitted papers, one is by a single author: Dr. Cherkashin; the rest 9 are co-authored, and the candidate is the first author in half of the works.

I partially accept the candidate's list of points for the minimal national requirements, with the exception of paper [5]:

- Group B: 3 scientific articles totaling 100 pts. (the required minimum is 100 pts.);
- Group C: 234 points from 8 scientific articles (the required minimum is 100 pts.). Here, I have to state that the paper [5] from the applicant's list is already used for his PhD thesis (as a preprint in arXiv) (contrary to the requirement in art. 24, par 1.3 from LDASRB) and therefore the number of journal papers should be reduced to 7 with a total of 234 pts.;
- Group D: 35 citations in WOS and Scopus totaling 210 pts. (the required minimum is 50 pts.). All citations cover the period 2017–2021 and the most by year are: 2019 – 12, and 2020 – 9;

- Group E: 3 participation in scientific projects totaling 30 pts.

The total sum of all papers under review here is 170 pages. The major scientific contributions in the candidate's works can be classified in the following groups:

- Topological dynamics (paper [1]);
- Chromatic numbers of spaces (papers [2]–[4]);
- Hypergraph colorings (papers [5]–[9]);
- Graphs and hypergraphs applications (papers [10], and [11]);

A wide variety of instruments and ideas from combinatorics and graph theory, as well as analytical and linear algebra methods, are used.

The paper [1] is dedicated to continuous maps of compact metric spaces. The newly obtained necessary and sufficient conditions for multitracking lead to new necessary conditions for tracking. An important achievement of the candidate is the improvement of Aoki and Hirade's result for the condition $CR(X,T) = \Omega(X,T)$ i.e. to match the set of recurrent points in the $\varepsilon-$networks with the set of non-wandering points. It's proven that it's sufficient to have multitracking of $CR(X,T)$ in order to achieve equality. Moreover, it's shown that the ω-limit point of any pseudotrajectory of a homeomorphism with multishadowing is close to a minimal point of the modeled system.

The work [2] is a short note that presents an improvement in the results of a classical combinatorial geometry problem: the finding of the chromatic number of the Euclidean space $\mathbb{R}^n$. Using a sequence of distance graphs, their independence number $\alpha(G_n)$ is found, and as a result, new lower bounds for the chromatic number $\chi(\mathbb{R}^n)$ for $9 \leq n \leq 12$ are established. New asymptotic results for the chromatic numbers and the independence numbers of the random subgraphs $G_{\frac{1}{2}}(n,3,1)$ and $G_{n,\frac{1}{2}}$ are also obtained.

The study [3] advances knowledge of the chromatic number of Euclidean spaces. A natural generalization to Euclidean metric spaces $\mathbb{K}^n \times \{0,\varepsilon\}^k, n,k \geq 1$ for $\mathbb{K} = \mathbb{R}$ or $\mathbb{Q}$ is shown. The case $n = 2$ is mainly considered, and it is proved that when $k = 1$ the chromatic number is in the closed interval [5,7] whenever $\varepsilon < \sqrt{\frac{3}{7}}$; when $k = 2$ the only possible values are 6 and 7. In the rational case, the value 3 is given for the chromatic number $\chi(\mathbb{Q} \times [0,\varepsilon]^3_{\mathbb{Q}})$.

The work [4] considers the graph sequence $G_n = (V_n, E_n)$ with vertices V_n the vectors v with coordinates $0, \pm 1$, $|v| = \sqrt{3}$ and edges having scalar product 1. For the chromatic numbers $\chi(\mathbb{R}^n)$ and $\chi(\mathbb{Q}^n)$ of the Euclidean spaces, a new bound is found. This bound

is linked with the ratio $\frac{|V_n|}{\alpha(G_n)}$ and in the case of $9 \leq n \leq 12$ it is the best bound known sofar.

The paper [5] is dedicated to a very important problem of Erdös and Hajnal: finding a bound for $f(n)$ – the fewest number of edges in a $n-$uniform hypergraph with positive discrepancy. The Alon et. al's 1987 upper bound for $f(n)$ is improved to $O(\log \mathrm{snd} n)$, where snd denotes the lowest nondivisor. Alas, the results published here are the main results of Chapter 4 of the candidate's PhD thesis and cannot be considered for the associate professorship.

In the work [6] it's proven that there exist a coloring for the Kneser graph $K(n, \frac{n}{2} - t, s)$ with a number of colors bounded by $(4 + o(1))(s + t)^2$. This results improves previous known bounds. This is achieved by using Hadamard matrices, and the method is extended to study Kneser hypergraphs as well. It is shown that the chromatic number of such a hypergraph with parameters $(n, r, \frac{n}{r} - t, s)$ is less than or equal to $2|E|$.

A continuation of the study of Erdös and Hajnal's problem of finding $m(n, r)$: the fewest number of edges in a $n-$uniform hypergraph with a chromatic number of at least r is shown in [7]. In the case of 3-graphs, when n is much smaller than r, a new and improved bound for L_3 : the limit of $\frac{m(n,r)}{r^n}$ is pushed to $\frac{4}{e^2}\cdot$ In the proof, a modified version of the Pluhár's idea of coloring using a greedy algorithm is applied. Akolzin-Shabanov's proof about the probability of finding the bad edges is improved when $n = 3$.

The N. Alon's hypothesis that for fixed n there exists the limit $\frac{m(n,r)}{r^n}$ is proved in [8]. In order to achieve this, an inequality – an upper bound for the maximal chromatic number in a $n-$uniform hypergraph with N-edges, is used. For the first time in the literature, a proposition by B. Sidakov is proved. With it, for the minimal number of edges in an $n-$uniform hypergraph with chromatic number $\chi > r$, the same result is shown: in the case of a list coloring, the limit $\frac{m_c(n,r)}{r^n}$ also exists. In the proofs, mainly analytic and combinatorial arguments are used.

The candidate's submitted work [9] is an expose on the extremal problems of coloring hypergraphs. It's 168 referenced papers cover the main results on these problems. The fact that more than a quarter of the cited titles are from the last 5 years, as well as that classic results are cited (for example D. Hilbert (1892), F. Bernstein (1907), and B. L. van der Waerden (1927)) shows the contemporality and importance of the

candidate's research. Written in eight chapters, spanning 60 pages, this work is a detailed and systematic analysis of the following topics:

- Erdös-Hajnal's problem: classical bounds are shown as well as new and improved results by greedy and hybrid approaches. The case when a lot of colors are used is also considered. In §2.4.3, the candidate's contribution: the proof of N. Alon's hypothesis, is exposed. The next section covers the case of 3−graphs and Dr. Cherkashin's results on $\frac{m(3,r)}{r^3}$ are used.

- Classical hypergraphs: simple linear and b-simple hypergraphs; some families of hypergraphs that are cliques. The candidate's contributions are in Theorems 3.2.5, and 3.2.10, as well as in the sharpening the upper bounds of Erdös-Lovász on the maximal number of edges in a non-trivial hypergraph. Cross-intersecting families and non-uniform hypergraphs are also considered.

- List coloring of graphs and hypergraphs: for fixed and random hypergraphs. Here, also the graph container method (D. Kleitman, K. Winston) is described and its applications to hypergraph is also discussed.

- Panchromatic coloring: upper and lower bounds are shown in the case when n/r is small. Dr. Cherkashin's contributions in this chapter are Theorems 5.1.1, 5.2.1, and 5.2.2.

- Discrepancy. The candidate's work here is connected to uniform hypergraphs with positive discrepancy (Theorem 7.3.2.) with results mainly already covered in [5].

- Explicit constructions and small values of variables. The main result present here is Theorem 8.3.2 which is also included in Dr. Cherkashin's PhD thesis; therefore, I do not review it.

- Applications of hypergraph coloring: the Hilbert's monochromatic cubes; van der Waerden's function; explicit estimates in Folkman's theorem (a generalization of Schur's 1916 theorem). The main results are for Kneser's graphs and are already covered in [6]. The main novelties here are: Theorems 9.5.2 and 9.5.3 which are presented in the context of other modern results on the coloring of generalized Kneser graphs.

The work [10] is focused on the minimal sum of the weights on the edges in a signed edge-dominated graph (a SED graph). The known bounds for the minimal sum are confirmed, and for the bound $-(1+o(1))\frac{n^2}{54}$: its optimality is proved whenever the SED pair (G, f) satisfies the following condition: for every $e \in E_-$ it is connecting a vertex from V_+ with a vertex from V_- and $e \in E_+$ connects some vertices from V_+.

In the paper [11, Lovász's θ-function is used in eventown problem. It's proved that when the cardinality of the family is $|F| = 2^{\frac{n}{2}} + s$, then the number of pairs $f_1, f_2 \in F$ with odd intersection is bounded by $op(F) \geq s2^{\lfloor \frac{n}{2} \rfloor - 2}$. Applying these new results it's showed that the cardinality of the k-town family is upper-bounded by $k^{\frac{n}{2}}$. This bound is then improved in two different scenarios: for a prime k and $(f_1, f_2) \not\equiv 0 \pmod{k}$; when $(f_1, f_2) \equiv t \pmod{k}$ for every pair (f_1, f_2).

I accept as achieved all of the scientific contributions proposed by the candidate. I am not aware of any plagiarism in the works presented.

3. Assessment of candidate's personal contributions

Of the 10 papers submitted for this assessment, three have three authors, six have two authors, and one is candidate's sole work. I consider the contributions of Dr. Cherkashin to the articles to be equal to those of all other authors.

4. Assessment of expert and project activities

Dr. Danila Cherkashin has declared his participation in three scientific projects in Russia:

- 2016-2020: Project No 16-11-10039 "Combinatorial, discrete, and enumerative geometry" financed by the Russian Scientific Fund with Project Lead Prof. P. G. Zograf. The candidate has three co-authored papers registered within this project;

- 2016-2020: Project No 16-11-10014 "Random graphs and hypergraphs: models and applications" financed by the Russian Scientific Fund with Project Lead Prof. A. M. Raygorodskiy. The candidate has three papers registered within this project; two of the papers are single-authored;

- 2018-2020: Mega-Project at St. Petersburg state university (financed by the Russian Federation's government) "Group theory, algebraic geometry, representation theory motive theory, homological algebra" with Project Lead Prof. Dipendra Prasad.

5. Candidate's scientometric indicators

Dr. Danila Cherkashin has Erdös number of three (A. Raygorodskiy MR3700970, Bèla Bollobás MR3403515, Paul Erdös MR1670561). At the Scopus database I found 17 indexed papers and $h-$index (without self-citing) 7 with a total of 174 citations. Web Of Science search shows 16 publications by the candidate and $h-$index also 7 coming from 103 citations. This shows the importance of the research as well as the wide area of application of Dr. Cherkashin's work.

6. Critical notes and recommendations

There is a discrepancy in the various lists of publications that are submitted for this competition. The journal paper [5] has been used in the candidate's PhD thesis and cannot be submitted for the academic position of associate professor. In the submitted documentation, I didn't find any information about the candidate's talks at scientific conferences, which I consider very important for disseminating the achievements of a candidate for the associate professor's position at a scientific institute.

7. Conclusion

All mentioned above, and the fact that the candidate satisfies all requirements of LDASRB, RALDASRB and IMI-BAN's rules, form the basis for me to propose that Dr. Danila Dmitrievich Cherkashin be awarded the academic position of **"associate professor"** in area: **4.5. Mathematics**, scientific speciality: "Combinatorics, graph theory".

Shumen, May 28, 2023 **Scientific jury member**:

/prof. Nikolay Yankov, DSc/