S T A N D P O I N T

on the contest for the Acquisition of the Academic Position
" Associate Professor"
in Professional Direction 4.5 Mathematics (Combinatorics, Graph theory)
at the Institute of Mathematics and Informatics (IMI)
of Bulgarian Academy of Sciences (BAS)
announced in St. Gaz. 14/10.02.2023 and at the Website of IMI-BAS

The standpoint is written by Prof. Azniv Kirkor Kasparian, Section of Algebra, Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", Professional direction 4.5 Mathematics, as a member of the scientific juri for the contest, according to Order 185/7.04.2023 of the Head of the Institute of Mathematics and Informatics at Bulgarian Academy of Sciences.

The only applicant for the announced contest is Ph.D. Danila Dmitrievich Cherkashin.

1 General description of the presented materials

1.1 Brief biography of the applicant

Ph.D. Danila Dmitrievich Cherkashin obtains his Master's Degree in Mathematics in 2015 at Saint-Petersburg University. In 2018 he defends his Ph.D. Thesis in Discrete Mathematics at Saint-Petersburg's Section of Steklov Mathematical Institute of Russian Academy of Sciences. Ph.D. Danila Cherkashin was a researcher at Saint-Petersburg University for more than three and a half years. He has practiced for more than three and a half years at Moscow Institute of Physics and Technology. For part of the time, he hold an Associate Professorship there. Ph.D. Danila Cherkashin has worked two years at Saint-Petersburg's Economics University and three years at a lyceum in Saint-Petersburg. From 2022 he is a researcher at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences.

Ph.D. Danila Cherkashin has 19 publications in highly prestigious specialized scientific journals. Four of them are independent and the other 15 are joint. The research of Ph.D. Danila Cherkashin has gained 102 citations, from which 67 refer to articles, presented for the contest.

The aforementioned data illustrates the extensive experience of Ph.D. Danila Cherkashin in performing theoretical research and teaching mathematics at a high professional level.

1.2 General characterization of the scientific works and the contributions, presented for the contest

Ph.D. Danila Cherkashin works on vertex coloring problems for hypergraphs, Euclidean spaces and their infinitesimal layers. His articles combine elaborate probabilistic techniques with highly sophisticated combinatorial constructions. This makes him a prominent specialist in the extremal graph theory. I do not know personally Ph.D. Danila

Cherkashin but the documents, presented for the competition have convinced me that he complies completely with the requirements of the Law on the Development of the Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences. The scientific contributions of Ph.D. Danila Cherkashin exceed considerably the minimal national requirements of Decree 26/13.02.2019 on the Amendments of the Rules for Implementation of the Law on Development of the Academic Staff of Republic Bulgaria, as well as the specific requirements of the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Bulgarian Academy of Sciences. More precisely, he competes with 11 articles, one of which is independent and the other 10 are joint. These earn him 384 points, instead of the obligatory 320. Two of the aforementioned articles are published in specialized journals with IF from the first quartile. Three of them appear in journals with IF from the second quartile, four are with IF from the third quartile, one is with IF from the fourth quartile and one is with SJR. Instead of the requires 10 citations, Ph.D. Danila Cherkashin competes with 35 citations of his works. From them 18 are on two articles, presented for the competition, and 17 citations refer to three articles outside the contest.

Ph.D. Danila Cherkashin has participated in three research projects of Russian Science Foundation.

The scientific works of Ph.D. Danila Cherkashin, presented for the competition do not include ones, used in previous procedures for acquisition of academic degrees and occupation of academic positions. I am strongly convinced that there is no plagiarism in the aforementioned scientific works of Ph.D. Danila Cherkashin

1.3 Contential analysis of the scientific contributions of the materials, presented for the contest

One of the articles of Ph.D. Danila Cherkashin, presented for the competition, pertains to the topological dynamics and covers his Master's Thesis. A dynamical system is a pair (X, T) of a compact metric space (X, ρ) and a continuous map $T : X \to X$. For any $y \in X$, let us consider the trajectory $O^+(y) := \{T^k(y) \,|\, k \in \mathbb{Z}^{\geq 0}\}$ and its closure $\overline{O^+(y)} \subseteq X$. If for any $z \in \overline{O^+(y)}$ the set $O^+(z)$ is dense in $\overline{O^+(y)}$, the point y is said to be minimal. A sequence $\{x_k\}_{k=1}^{\infty} \subset X$ is a δ-pseudotrajectory for some $\delta \in \mathbb{R}^{>0}$, if $\rho(x_{k+1}, T(x_k)) \leq \delta$ for $\forall k \in \mathbb{N}$. The δ-pseudotrajectories can be viewed as T-trajectories with "errors", obtained by some numerical method. A continuous map $T : X \to X$ satisfies the shadowing property if for any $\varepsilon > 0$ there exists such $\delta > 0$ that any δ-pseudotrajectory $\{x_k\}_{k=1}^{\infty} \subset X$ can be approximated by an exact trajectory $\{T^k(y_0)\}_{k=0}^{\infty} \subset X$, satisfying $\rho(x_k, T^k(y_0)) < \varepsilon$ for $\forall k \in \mathbb{N}$. A joint article of Danila Cherkashin and S. Kryzhevich from 2017 shows that any pseudotrajectory has a subsequence, which can be shadowed by a subsequence of an exact trajectory. Moreover, the probability for the existence of a minimal point in a neighborhood of a random point of a pseudotrajectory is positive. The article generalizes the notion of shadowing by introducing the multishadowing. A dynamical system (X, T) satisfies the multishadowing property if for any $\varepsilon > 0$ there exists such $\delta > 0$ that for any δ-pseudotrajectory $\{x_k\}_{k=1}^{\infty} \subset X$ there are finitely many points $y_1, \ldots, y_N \in X$ with

$\min\{\rho(x_k, T^k(y_i)) \mid 1 \le i \le N\} < \varepsilon$ for $\forall k \in \mathbb{N}$. The article of Cherkashin and Kryzhevich from 2017 establishes that a dynamical system (X, T) with a homeomorphism $T : X \to X$ satisfies the multishadowing property if and only if for any $\varepsilon > 0$ there exists such $\delta > 0$ that for any δ-pseudotrajectory $\{x_k\}_{k=1}^{\infty} \subset X$ and any limit point x_o of $\{T^n(x_k)\}_{k,n=1}^{\infty}$ the closed ball of radius ε, centered at x_o, contains a minimal point. A dynamical system (X, T) is non-wandering if for any $x \in X$ and any open neighborhood $U \subset X$ of x there exists $k \in \mathbb{N}$ with $T^k(U) \cap U \neq \emptyset$. A subset $Y \subset X$ is an ε-network for some $\varepsilon > 0$ if for any $x \in X$ there is $y \in Y$ with $\rho(x, y) \le \varepsilon$. The article proves that a non-wandering dynamical system (X, T) with a homeomorphism $T : X \to X$ satisfies the multishadowing property if and only if the minimal points of (X, T) are dense in X. This is shown to hold exactly when for any $\varepsilon > 0$ there is a finite subset $Y = \{y_1, \dots, y_{N(\varepsilon)}\} \subset X$, providing ε-networks $T^n(Y)$ of X for all $n \in \mathbb{Z}$.

The remaining scientific works of Ph.D. Danila Cherkashin, presented for the competition, are on vertex colorings of hypergraphs, as well as on the chromatic numbers of Euclidean spaces and their infinitesimal layers. A hypergraph $H = (V, E)$ is a pair of a finite set of vertices V and a family E of subsets $e \subset V$. If all edges $e \in E$ are of one and a same cardinality $|e| = n \in \mathbb{N}$, then H is said to be n-uniform. The maps $f : V \to \{1, \dots, r\}$ are referred to as r-colorings of V. An r-coloring f is proper if any edge $e \in E$ contains at least two vertices $v_1, v_2 \in e$ of different color $f(v_1) \neq f(v_2)$. The chromatic number $\chi(H)$ of a hypergraph $H = (V, E)$ is defined as the minimal natural number, for which H admits a proper $\chi(H)$-coloring. Let $m(n, r) \in \mathbb{N}$ be the minimal number of edges of an n-uniform hypergraph $H = (V, E)$ with $\chi(H) > r$. Previous works of other authors derive lower and upper bounds on $\frac{m(n,r)}{r^n}$, depending on n and appropriate positive real constants. In a joint article with F. Petrov from 2020, D. Cherkashin proves the existence of a positive limit L_n of $\frac{m(n,r)}{r^n}$ for any fixed $n \in \mathbb{N}$. Let $H = (V, E)$ be a hypergraph and $\{L(v)\}_{v \in V}$ be a family of finite sets $L(v)$, associated with the vertices $v \in V$. The maps $f : V \to \cup_{v \in V} L(v)$ with $f(v) \in L(v)$ for $\forall v \in V$ are called list coverings of V. Let $m_c(n, r)$ be the minimal number of edges of an n-uniform hypergraph $H = (V, E)$ with list chromatic number $\chi_c(H) > r$. The aforementioned article with F. Petrov from 2020 establishes also the existence of a positive limit of $\frac{m_c(n,r)}{r^n}$ for any fixed $n \in \mathbb{N}$. An independent article of Cherkashin from 2019 improves the known lower bounds on L_3. A finite sequence of edges $e_1, \dots, e_r \in E$ is an r-chain if $|e_i \cap e_j| = 1$ for $|i - j| = 1$ and $e_i \cap e_j = \emptyset$ for $|i - j| \neq 1$. The aforementioned article of Cherkashin from 2019 obtains the upper bound $|E| \left(\frac{|E|-1}{r-1} \right)^{r-1}$ on the number of the r-chains of an arbitrary hypergraph $H = (V, E)$.

Let $f : V \to \{1, 2\}$ be a 2-coloring of an n-uniform hypergraph $H = (V, E)$. The discrepancy $\mathrm{disc}(e)$ of an edge $e \in E$ is the absolute value of $\left| f^{-1}(1) \cap e \right| - \left| f^{-1}(2) \cap e \right|$. The discrepancy of f is defined as $\mathrm{disc}(f) := \max\{\mathrm{disc}(e) \mid e \in E\}$ and the discrepancy of H is $\mathrm{disc}(H) := \min\{\mathrm{disc}(f) \mid f : V \to \{1, 2\}\}$. Denote by $f(n)$ the minimal number of edges in an n-uniform hypergraph with $\mathrm{disc}(H) > 0$. In 1987 Alon, Kleitman, Pomerance, Saks and Seymour obtain a lower and an upper bound on $f(n)$. A paper of D. Cherkashin and F. Petrov from 2019 refines the upper bound on $f(n)$ by an explicit construction of an n-uniform hypergraph H of positive discrepancy $\mathrm{disc}(H) > 0$.

Let $K(n, k, s)$ be the generalized Kneser graph, whose vertices are the subsets $v \subset$

$\{1, \ldots, n\}$ of cardinality $|v| = k$ and whose edges are $\{v_1, v_2\}$ with $|v_1 \cap v_2| < s$. A joint article of J. Balogh, D. Cherkashin and S. Kiselev from 2019 describes the asymptotic of the chromatic number $\chi\left(K\left(n, \frac{n}{2}, s\right)\right)$ for sufficiently large s. For any $A \subset \{1, \ldots, n\}$ of cardinality $|A| \geq s$, Frankl's set I_A consists of the vertices v of $K(n, k, s)$ with $|v \cap A| \geq \frac{|A|+s}{2}$. The F-chromatic number $\chi_F\left(K(n, k, s)\right)$ is the minimal natural number, for which $K(n, k, s)$ admits a coloring by Frankl's sets I_A. The aforementioned work of Balogh, Cherkashin and Kiselev from 2019 obtains lower and upper bounds on $\chi_F\left(K\left(n, \frac{n}{2} - t, s\right)\right)$, depending on $s + t$ and n. The upper bound on χ_F is obtained by explicit $(4+o(1))(s+t)^2$-colorings of hypergraphs with large discrepancy, associated with Hadamard matrices. Under specific assumptions on the parameters and the existence of an Hadamard matrix of appropriate size, the article of Balogh, Cherkashin and Kiselev from 2019 derives an upper bound on the chromatic number of Kneser's hypergraph $KH\left(n, r, \frac{n}{2} - t, s\right)$.

A joint survey of A. Raigorodskii and D. Cherkashin from 2020 discusses the recent progress on extremal problems for vertex colorings of hypergraphs.

Let us consider the Euclidean space $\mathbb{R}^n$ as a graph with an infinite set of vertices $\mathbb{R}^n$, whose edges connect the points $v_1, v_2 \in \mathbb{R}^n$ at Euclidean distance 1. The chromatic number $\chi(\mathbb{R}^n)$ of this graph is referred to as the chromatic number of $\mathbb{R}^n$. A work of Larman and Rogers from 1972 provides an asymptotic upper bound on $\chi(\mathbb{R}^n)$, while an asymptotic lower bound is found by an article of Raigorodskii from 2000. In a paper from 2018, D. Cherkashin, A. Kulikov and A. Raigorodskii improve the available lower bound on $\chi(\mathbb{R}^n)$, making use of a sequence of graphs $G_n = (V_n, E_n)$. The vertex set V_n consists of the points $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ with $x_i \in \{-1, 0, 1\}$, $||x|| = \sqrt{3}$, while the edges $\{x, y\} \subset V_n$ connect the vertices with scalar product $\langle x, y\rangle = 1$. An independent set I in a graph $G = (V, E)$ is a subset $I \subset V$, which does not contain an edge $e = \{v_1, v_2\} \in E$. The independence number $\alpha(G)$ of G is the maximal cardinality of an independent set $I \subset V$. Cherkashin, Kulikov and Raigorodskii compute the independence number $\alpha(G_n) = 6n - 28$ for sufficiently large $n \in \mathbb{N}$. That enables them to conclude that $\chi(\mathbb{R}^n) \geq \chi(\mathbb{Q}^n) \geq \chi(G_n) \geq \frac{|V_n|}{\alpha(G_n)} = \frac{8}{6n-28}\binom{n}{3}$ for sufficiently large $n \in \mathbb{N}$. The developed techniques yield also improved lower bounds on $\chi(\mathbb{R}^n) \geq \chi(\mathbb{Q}^n)$ for $9 \leq n \leq 12$. As a generalization of the chromatic number $\chi(\mathbb{R}^n)$ of $\mathbb{R}^n$, one can consider the chromatic numbers $\chi(\mathbb{R}^n \times [0, \varepsilon]^k)$ of the infinitesimal layers $\mathbb{R}^n \times [0, \varepsilon]^k$, $k \in \mathbb{N}$ of $\mathbb{R}^n$. In a joint article from 2017, A. Kanel-Belov, V. Voronov and D. Cherkashin obtain lower and upper bounds on $\varepsilon > 0$, for which $\chi(\mathbb{R} \times [0, \varepsilon]^k) \in \{3, 4\}$. They show that $\chi(\mathbb{Q} \times [0, \varepsilon]^3_{\mathbb{Q}}) = 3$ for a sufficiently small $\varepsilon > 0$, $5 \leq \chi(\mathbb{R}^2 \times [0, \varepsilon]) \leq 7$ for $0 < \varepsilon < \sqrt{\frac{3}{7}}$ and $\chi(\mathbb{R}^2 \times [0, \varepsilon]^2) \geq 6$ for all $\varepsilon > 0$. The article establishes that for any $k \in \mathbb{N}$ there exists such $\varepsilon_o(k) \in \mathbb{R}^{>0}$ that $\chi(\mathbb{R}^2 \times [0, \varepsilon]^k) \leq 7$ for all $0 < \varepsilon < \varepsilon_o(k)$.

Let F be a family of $x = (x_1, \ldots, x_n)$ with $x_i \in \{0, 1, \ldots, k-1\}$, $\forall 1 \leq i \leq n$ for some natural number $k > 1$. An article of M. Antipov and D. Cherkashin from 2022 establishes that if the scalar products $\langle x, y\rangle$ of all $x, y \in F$ satisfy the congruence $\langle x, y\rangle \equiv 0 (\operatorname{mod} k)$ then $|F| \leq k^{\frac{n}{2}}$. The work derives upper bounds on $|F|$ for families F with a fixed residue $\langle x, y\rangle \equiv t (\operatorname{mod} k)$, $0 \leq t \leq k-1$ for all $x, y \in F$, as well as for F with negligible number of $x, y \in F$ with $\langle x, y\rangle \not\equiv t (\operatorname{mod} k)$.

For any edge $e \in E$ of a graph $G = (V, E)$, let us denote by $N[e]$ the union of the adjacent edges $e' \in E$ of e with e. A simple graph $G = (V, E)$ and a weight function $f : E \to \{1, -1\}$ form a signed edge-dominated pair (G, f) if $\sum_{e' \in N[e]} f(e') \geq 0$ for all edges $e \in E$. Let $s(G, f)$ be the sum of the weights of all edges of G and $g(n)$ be the minimal sum $s(G_n, f_n)$ for all signed edge-dominated pairs (G_n, f_n) with $|V(G_n)| = n$ vertices. In 2000 Akbari, Bolouki, Hatami and Siami obtain a lower bound on $g(n)$ and show the existence of a sequence (G'_n, f'_n) of signed edge-dominated pair, whose sums $s(G'_n, f'_n)$ are subject to a negative asymptotic upper bound. An article of D. Cherkashin and P. Prozorov from 2022 improves the aforementioned lower bound on $g(n)$ and constructs a sequence (G_n, f_n) of signed edge-dominared pairs with improved negative asymptotic upper bound on $s(G_n, f_n)$.

1.4 Conclusion on the application

After getting acquainted with the materials and the scientific works, presented for the competition, and based upon the aforementioned analysis of their scientific significance and applicability, I confirm that the scientific contributions comply with the Law on Development of the Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences, for occupation by the applicant of the academic position " Associate Professor" in the scientific field and the professional direction of the contest. In particular, the applicant satisfies the minimal national requirements in the professional direction and no plagiarism was found in the presented scientific works. That is why,
I evaluate positively the application of Ph.D. Danila Dmitrievich Cherkashin.

2 General conclusion

Based upon the aforementioned, **I strongly recommend** the Scientific Juri to propose the appropriate election authority of the Institute of Mathematics and Informatics at Bulgarian Academy of Sciences to elect

Ph.D. Danila Dmitrievich Cherkashin
as an " Associate Professor" in Professional Direction 4.5 Mathematics (Combinatorics, Graph Theory) at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences.

May 26, 2023

Standpoint written by:
Prof. Ph.D. Azniv Kasparian
Section of Algebra
Department of Mathematics and Informatics
Sofia University "St. Kliment Ohridski"