

REVIEW

by Prof. D.Sc. Virginia Kiryakova,

Institute of Mathematics and Informatics (IMI) – Bulgarian Academy of Sciences (BAS)

on the documents and publications, presented by

Asso. Prof. D.Sc. Emilia Bazhlekova

as application to a competition for academic post of (Full) Professor
for the needs of IMI-BAS, in domain Higher education 4. Natural sciences,
mathematics and informatics, professional field 4.5. Mathematics, scientific speciality
“Mathematical analysis (Applications of fractional calculus)”,
announced in SG, issue 43/ 17.05.2024

I am a member of the Academic Board on this procedure according to Order № 206 / 16.07.2024 of Director of IMI-BAS. At the first meeting of this Academic Board I was appointed as a Reviewer (Minutes № 1 / 29.07.2024).

I have been acquainted with the required documents and presented works of the only candidate on this competition – Asso. Prof. D. Sc. E. Bazhlekova. Checking them, I am convinced that the presented stuff and the candidate satisfy all the requirements of Ch. 3, Sect. IV of the Law Act for Development of Academic Staff in R. Bulgaria (abbrev. further as “Law”, of 26.02.2019). She has obtained the degree of PhD at Technical University Eindhoven / on 1.10.2001, with legalized Diploma № 1 / 16.12.2011 – at Bulgarian Academy of Sciences, and has also the next academic degree “Doctor of Science” in IMI-BAS (15.11.2022), with work experience as Docent (Asso. Prof.) for more than 10 years in IMI-BAS.

All minimal science-metric indicators from Ch. 4 of the Regulations of IMI-BAS (updated 25.03.2022) and Regulations of BAS for application of the Law (update 20.05.2019) are satisfied and essentially exceeded.

1. Short CV data for applicant

Asso. Prof. Bazhlekova graduated from the High Mathematical School in the town of Pleven, then had higher education in Mathematics as M.Sc. from the Fac. Mathematics and Informatics (FMI) – Sofia University (1986), with a 1 year next specialization in FMI and PhD studies in IMI-BAS. During her 4 years specialization in the Netherlands, she defended there a PhD thesis: „*Fractional Evolution equations in Banach*“ (1.10.2001) in Technical University Eindhoven (legalized by Bulgarian Academy of Sciences). In 2022 she defended in IMI-BAS a dissertation „*Subordination principle for generalized fractional differential equations*“ for the academic degree Doctor of Science (scientific speciality Mathematical Analysis).

The applicant has working experience as a school lecturer, assistant professor on mathematical analysis at Faculty of Physics – Sofia University, and then in IMI-BAS – subsequently as a mathematician, assistant professor, and since 2014 – as Associate Professor.

Concerning the requirements of the Law, she has working experience as Asso. Prof. (Docent) for more than 10 years in IMI-BAS.

E. Bazhlekova has very good commands in English, Dutch, Russian and possesses a good computer literacy.

The research interests and contributions of D.Sc. Emilia Bazhlekova are in a well established and actual scientific area, in which she definitely is a leading scholar, recognized since many years by the foreign colleagues on fractional calculus and its applications. Her contributions are in the field of Mathematical Analysis and its applications – fractional calculus, applications in mathematical modelling, subordination principle, integral transforms and special functions (generalized Mittag-Leffler functions, Bernstein functions), spectral expansions of the solutions of boundary value problems, convolutional calculus and applications to nonlocal BVPs, etc.

2. Description of the presented publications and other materials:

Along with the works presented for application in the competition for academic post Professor and the rest required documents, the candidate E. Bazhlekova has provided also:

- **Full list of all (57) scientific publications**, with clear description with respect to science metrics (indexation) and resp. numbers: for obtaining PhD (2001), for the academic position of Asso. Prof. (2014, 14 items), for the procedure for Dr.Sc. degree (2022, 11 items), and these presented as application on the present competition (22 items).

- **From this full list of 57 scientific publications:** indexed in WoS and / or Scopus are 44; with impact factor and / or impact rang – 41, of which in journals with IF – 23.

- **List of publications presented for application at this procedure for professor:** 22 items, with their science-metric indicators, as from academic system Sonix. Of them:

- with **IF/ SJR** - total 18; which are with **IF** (total sum IF = 24.918) are 11 items;
- other items indexed in WoS and/ or Scopus – 2; in ZBLMath – other 2.

Their distribution according to WoS quartiles and Scopus rangs, with the corresponding scores, is precisely given by the applicant in document № 7.

- **Author's summary for the contributions in the presented 22 works**
- **Abstracts** of the presented 22 works
- **List of citations (872)** of all her publications, and separately – citations only of the works on this procedure (**220**) as available also in academic system Sonix, as well as data for these according to the data bases of Scopus, Web of Science, Google Scholar.

The scientific works presented for application in the procedure for Professor, are:

Total – 22 published articles, of which:

- 18 items with IF / SJR:
 - of which 11 with IF: (in Q1 – 9, in Q4 – 2), and 7 with SJR only
- 2 items – indexed in Scopus / Web of Science, but with no IF / SJR
- 2 items – in „other“ editions indexed in ZBLMath

These works have not been used in other procedures on the Law. All are published already, and are in English. Of them, 4 are single authored (on the basic theoretical achievements by Bazhlekova), 16 – with 1 co-author, 2 – with 2 and 3 co-authors (which are related to applications of her results to mathematical models in mechanics, physics, biology, and to numerical results).

The presented publications, their citations and Bazhlekova's activities (Criterion E) are accurately distributed and scored in a Table (Document № 19) to reflect the minimal requirements of BAS / IMI-BAS (note that these are higher than the national adopted ones), and these data are also available in the Sonix system. Especially, for the groups of indicators B (4.) the publications [1], [2], [3], [9], [12], [13] are concerned, and for the group of indicators Γ (7.) are the rest 16 publications.

This table shows that all necessary science metric indicators are fulfilled, and for the groups of criteria Г, Д, E these are even essentially exceeded:

Groups of indicators	Content	Minimal requirements for Prof.	Applicant's values
A	Ind. 1: PhD	50	50
Б	Ind. 2: for other procedure	-	-
B	Ind. 3-4: Habil. thesis or equivalent publications	100	134
Г	Ind. 5-10: Other publications	220	552
Д	Ind. 11: Citations of works on the procedure (as presented for NACID: 56 x 6, but in fact from the List № 12 and Sonix system: 220 cits in Scopus, 220 x 6 = 1320)	140	336 (according to minimal data necessary for NACID, but in fact are: 1320 points)
E	Ind. 12-20: Others	150	205
Total:		660 points	1277 points (but in fact: 2261 points)

3. Analysis of the scientific results and contributions in the works presented for the competition for academic post of Professor

The presented works are in the area of mathematical analysis / applications of fractional calculus, evolution differential equations of fractional order, special functions, as the basic subject of applicant's scientific interests and contributions. The employed apparatus is mainly from analysis (real, functional), differential equations, special functions, integral transforms, spectral theory and their applications, some methods for numerical analysis.

The so-called Fractional Calculus (Fractional Analysis), as part of the classical Calculus, is a theory where the operators of integration and differentiation can be of arbitrary (fractional) order, not obligatory of integer order. These are nonlocal integro-differential / differential-integral operators with singular kernels, and with their help many processes with memory, in physics, mechanics etc. natural and social sciences and the fractal properties of the nature, are much more adequately described. From an exotic, purely theoretical idea arising in 17th century, in the end of 20th and nowadays in 21st century, it became an extremely fast developing and "fashion" area with numerous applications, with hundreds books and thousands of articles devoted to the topic, which results are used and cited repeatedly.

In her works, Asso. Prof. Bazhlekova employs basically the classical operators of fractional calculus, related to the names of Riemann, Liouville and Caputo, because these are the most frequently encountered in real mathematical models, most of which are described by means of evolution differential equations of fractional order. An essential contribution for these studies is due to the "subordination principle", introduced by herself in fractional calculus and recognized as her own theory, and let us note also her contributions to special functions like these of Mittag-Leffler type with many indices and variables, to completely monotone functions and these of Bernstein and Stieltjes type.

The author's summary reflects clearly and adequately the applicant's contributions.

The contributed results are in the following interrelated topics on the scientific speciality of this competitions:

1. *Studies on the solutions of fractional evolution equations and Duhamel type representations*, [1], [2], [3], [4], [9], [12],[13]

In the publications [1], [2], [9] the convolutional method of Ivan Dimovski is applied, for finding Duhamel type representations of solutions of boundary value problems of mathematical physics (incl. also of fractional order), as well as their expansions in eigen functions. By means of the Laplace transform, estimations are derived for the functions depending on the time, and these are necessary to establish the regularity of solutions and analysis of the numerical methods in [4]. Such examples of the treated problems are the Oldroyd-B model for visco-elastic streams, the one-dimensional boundary value problem for time-diffusion equation of distributed order, the classical inverse problem for heat conduction, while the problems for its incorrectness in Hadamard sense are overcome via an correctly posed regularization problem.

In [3],[12],[13] there are investigated analytically the solutions of different fractional order equations, in a more general abstract form (where the operator of solution is a bounded analytical operator). The subordination principle is used, imposed in fractional calculus by the candidate herself.

The obtained integral representations of the solutions are employed for numerical experiments and visualizations, that confirm the analytical results.

2. *Analysis of linear visco-elastic models with fractional derivatives*, [7], [8], [19], [20]

Various constitutive laws involving fractional derivatives are considered as generalizations of classical models. For the treated generalized laws of Maxwell, Jeffrey and Zener of fractional order, it is proved that these have physical meanings if and only if the corresponding relaxation module is completely monotone function. The employed tools include the subordination principle, the Laplace transform and its inverse, and integral representations are derived for the relaxation modules with different kernels.

A novelty is the example with the multinomial Mittag-Leffler type function. On the base of the integral representations, are presented graphs to illustrate the analytical results ([8],[10],[19],[20]). Three classes of functions – of Bernstein, Stieltjes and of completely Bernstein type, are used.

3. *Uniqueness and existence of solutions of inverse problems with fractional derivatives*, [15], [16], [22]

A nonlocal boundary value problem for equation with convolutional fractional derivative, generalizing this of Caputo type, is considered in spaces of continuous functions, as well as in Sobolev spaces. The candidate uses expansions in biorthogonal basis, corresponding to the nonlocal operator in x -variable, for which the eigen spaces are two-dimensional. Having in mind the obtained bilateral estimations, one observes decreasing of the smoothness – a result, generalizing known ones and corresponding to the expectation that the inverse problem is moderately ill-posed.

Among the considered examples of convolutional fractional derivatives, the most interesting is the one with multi-term generalization of the Caputo derivative. An important contribution of Bazhlekova is the introduction of a generalization of the multinomial Mittag-Leffler functions of Prabhakar type (first appearing in her publication [15]).

The uniqueness of the inverse problem is proven, for finding the potential function in multi-term fractional differential equation. It is essentially used that under the posed conditions the subordination principle is satisfied.

4. *Analysis of some numerical methods for fractional differential equations*, [4], [5], [17]

In [4] there are proposed and considered different numerical methods for a particular case of equation describing the distribution of visco-elastic stream with fractional order constitutive model of Oldroyds-B, with Dirichlet boundary conditions. Bazhlekova proposes analysis of the solution, estimations in Sobolev spaces, properties of the expansion in eigen functions. In [5] it is developed a numerical algorithm to solve a similar equation with the finite differences method. The paper [17] generalizes a method of Adams type for fractional (linear and nonlinear) differential equations with convolutional derivative. The results of the numerical solutions are compared with the exact solution.

5. *Applications of fractional calculus for complex processes*, [6], [11], [14], [18], [21]

In [6] there are considered some models of streams of visco-elastic fluid, with analytical and numerical studies of the evolution in time, and the influence of different parameters of the material. In [11], the Ward-Tordai equation is generalized in two ways. The Laplace transform is applied, and to calculate the solution two numerical techniques are compared. There is analysis of the application of fractional derivatives in time, for modeling of bioreaction in diffusion, [14]. Generalizations of fractional order of the classical Maxwell model are considered, with two interactive drops ([18]), and in [21] – generalizations of the fractional Jeffrey model are considered, as the equation is reworked as an integral one of Volterra type with kernel – multinomial Mittag-Leffler functions. The used tools include the Laplace transform and the properties of sets of powers of Bernstein functions.

4. General description of the research and organizational activities of the candidate:

Additionally to the mentioned characterization of the presented publications and their international impact, let me mention also some other data for E. Bazhlekova's activities in IMI-BAS:

- Member of organizing committees and co-chair for 6 international mathematical conferences, organized by IMI-BAS;
- Member of the Editorial Board and currently Associate Editor in the international scientific journal "Fract. Calc. App. Anal." (Springer, with support by IMI), having Top 10 positions by rank in Web of Science (Q1);
- Participation in scientific projects: at National level (2, NSF-Min. ES), under bilateral collaboration between academies (3, Russia, Serbia), project on OP "Science and education for intelligent development" (1), project on NNP „Information and communication technologies for united digital market ...", (1);
- Long years secretary of the Dept. „Analysis, Geometry and Topology" in IMI-BAS
- Currently, leader for the budget project „Mathematical analysis and applications" at Dept. „Analysis, Geometry and Topology" in IMI-BAS;
- Member of the attestation commission at Scientific Council of IMI-BAS;
- Reviewer for several international mathematical journals.

5. Impact of candidate's results in works of other authors:

E. Bazhlekova has provided a detailed list (extracted from academy system Sonix) of the observed and checked 872 citations (without self-citations) of all her publications (of them: indexed in WoS/ Scopus are 474; in other international editions – 329, in dissertations – 11, etc.). Her data for the Hirsh index are respectively:

h = 13 (Scopus, Author ID 6507427736), **h = 13** (Web of Science), **h = 17** (Google Scholar), as available at her profiles in international data bases:

ORCID # [0000-0001-7755-4093](https://orcid.org/0000-0001-7755-4093)

<https://www.scopus.com/authid/detail.uri?authorId=6507427736>

<https://scholar.google.com/citations?user=SIV5wscAAAAJ&hl=en>

Only for the 22 works presented for this competition for professor, there are 220 citations, found in editions indexed in WoS / Scopus. That is, for Criterium D 11, instead of the minimally required **140 points**, and presented in her own table as with **336 points** (according to minimal data necessary for NACID), Bazhlekova **has in fact much more points: $220 \times 6 = 1320$ points.**

Most of these citations are by foreign authors and in prestigious editions, evidence for which is the total sum of their IFs (**25**) + SJR.

6. Approbation of the results:

The results of E. Bazhlekova in the presented works, as well as in her previous works, are published in prestigious journals and proceedings of international conferences, most of them indexed in the world bibliographic networks and cited many times. They are also reported at several international mathematical forums both in Bulgaria and abroad, and on national seminars. These results are also essential part of the working programs of several scientific projects (on national programs, in IMI-BAS, bilateral with Russian and Serbian academies).

7. Evaluation of the personal contributions of the candidate:

Of all (22) publications presented for this procedure: 4 items are *single-authored* (containing the main theoretical results of the candidate), 16 – are with 1 co-author and 2 – with 2 or 3 co-authors (related to applications of her results to suitable mathematical models in mechanics, physics, biology, to numerical evaluations and their visualizations).

The entire ideological approaches and the main analytical results in the area of fractional calculus and its applications, evolution equations of fractional order and subordination principle, are evidently personal contributions of the candidate, as result of her long years professional development as a world recognized expert in the domain.

There is no plagiarism detected.

8. Critical notes:

None. I would recommend to candidate to publish her D.Sc. thesis as a monograph.

9. Personal impressions:

I know personally Emilia since 1989, when she entered in our Department “Complex Analysis” at IMI as a PhD student and later, as a mathematician. Then, and also after her return to Bulgaria and to Institute, she always made impression with her modesty, diligence, good mathematical culture, and taste for precise investigation of significant problems of Analysis. As a member of the international academic jury, I have attended personally the defense of her PhD thesis in 2001 in the Eindhoven University of Technology, could witness the excellent presentation that led to unanimously taken decision to award the PhD diploma.

Most of the results and publications of Bazhlekova are devoted to analytical and numerical methods for solving differential equations of fractional order and present essential contributions to the applications of fractional calculus. She is a member of the international editorial board and Associate editor of the leading journal in this domain, “Fractional Calculus and Applied Analysis”, and part of its success is due also to her professional expert activities. I have been personally convinced in the highly positive opinions for her results and personality by the world leading figures in our field.

My personal opinion – based on our joint work in IMI-BAS team on the topic, on the presented publications in leading specialized journals and on her prestige among the colleagues in fractional calculus – is that Asso. Prof. Dr.Sc. Emilia Bazhlekova, since long time ago, is a well established and world recognized scholar in the field.

10. Conclusion:

The review of the presented publications and documents related to this procedure shows that Asso. Prof. D.Sc. Emilia Bazhlekova is an internationally established expert in the field of mathematical analysis, and in particular - in fractional calculus and its applications.

I can evaluate the contributions in her works as theoretical generalizations and development of a theory that is currently actual and very fast developing, as part of the area of mathematical analysis and applications to mathematical modelling.

According to the Regulations of IMI-BAS (Ch. 2, 25.03.2022) and Regulations of BAS (20.05.2019) for application of the Law, all quantitative indicators for achieving the academic post Professor in IMI-BAS are fulfilled and these are as follows:

1. Of all presented 22 works on the procedure, to have at least 12 items in publications with IF or SJR: she has presented 18 such papers (of them: with IF - 11 items, with SJR - 7 items); at least half of these required 12 publications should be published after the date of last previous procedure (in her case, for Dr.Sc. 2022) – Yes, such are 6 of the works (Nos 17 to 22).

2. All minimal requirements according to the Regulations on the Law of BAS an IMI-BAS are fulfilled, and for the indicators of the groups Γ , \mathcal{D} , E are essentially exceeded: see the table presented by the candidate (as updated here on page 3). These are 1277 (but in fact 2261 points), in times more than the required total 660 points for Professor.

It is then evident that all requirements for occupation the academic post of (Full) Professor in BAS and in IMI-BAS are satisfied.

I confirm that in the presented works of candidate, which are very close as subject to my own research, there is **no detected plagiarism**.

Having all above said in mind, I can esteem that Asso. Prof D.Sc. Emilia Bazhlekova satisfies all requirements of the Law (Law Act for Development of Academic Staff in R. Bulgaria) and of the Regulations of IMI and of BAS on this Law, as applicant for the academic post on this procedure.

I give my positive conclusion for the appointment of the candidate on the academic post of Professor in IMI-BAS and vote “For” (positively) the Academic Board on this procedure to propose to the Scientific Council of IMI-BAS to elect the candidate Asso. Prof. D.Sc. Emilia Bazhlekova for the academic post of (Full) Professor in IMI-BAS.

Reviewer and member of the Academic Board:

15 August 2024

(Prof. D.Sc. V. Kiryakova)