

REPORT

By Prof. Jordanka Paneva-Konovska, DSc.
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
Member of the Scientific Jury, appointed by order №: 206/16.07.2024
of the Director of IMI – BAS

on the documents and publications presented by
Assoc. Prof. DSc Emilia Grigorova Bazhlekova
as an application to a competition for academic post of (Full) Professor
for the needs of IMI-BAS, in the domain of Higher education
4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5. Mathematics,
Scientific specialty: 'Mathematical analysis'
(Applications of fractional calculus)

The competition, with a term of 2 months, has been announced in the State Gazette No. 43/17.05.2024 for the needs of the Institute of Mathematics and Informatics (IMI), BAS. Associate Prof. DSc. Emilia Grigorova Bazhlekova has submitted documents for participation in it.

In accordance with Order No: 206/16.07.2024 of the Director of IMI – BAS I was appointed as a member of the Scientific Jury for this procedure. According to the decision of the first meeting of the Jury, held on 29.07.2024, I present a report. IMI - BAS is accredited by NAOA for the doctoral program 'Mathematical Analysis'.

After examining the materials presented by the applicant E. Bazhlekova, I am convinced that both they and the applicant satisfy the requirements of Chapter 2 of the Act on Development of the Academic Staff in the Republic of Bulgaria (ZRASRB). E. Bazhlekova was awarded a PhD degree at Eindhoven University of Technology, the Netherlands, on 01.10.2001 (the diploma is legalized in Bulgaria: diploma 000001 / 16.12.2011) and scientific degree 'Doctor of Science' at Institute of Mathematics and Informatics, Bulgarian Academy of Sciences (2022). All the recommended scientometric criteria of Chapter 2 of the Regulations on the Conditions and Order for Acquiring Scientific Degrees and Occupying Academic Positions at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences are satisfied and significantly exceeded.

1. Brief biographical data

Emilia Bazhlekova graduated from the 'Geo Milev' Mathematics High School in the city of Pleven, then - higher education (master's) in mathematics in 1986 at the Faculty of Mathematics and Informatics of the Sofia University 'St. Kliment Ohridski' with a specialization in Complex Analysis and a subsequent one-year specialization in the period 1986-1987. In the period December 1997 - December 2001, Bazhlekova was on a scientific specialization abroad (the Netherlands), Applied Analysis group, Eindhoven University of Technology, Eindhoven, where in 2001 she defended her PhD thesis. The topic of her PhD thesis is 'Fractional evolution equations in Banach spaces' and is under the scientific supervision of prof.dr.ir. J.de Graaf and

prof.dr. Ph. Clément. The topic of her DSc thesis is 'Subordination principle for generalized fractional evolution equations'. She speaks English (very well), Dutch (very well) and Russian.

The candidate worked as a teacher and research scientist in the specialty of mathematics. She began her scientific and teaching career as a part-time assistant in Mathematical analysis at the Faculty of Physics of Sofia University 'St. Kliment Ohridski' and teacher of Mathematics and English in the town of Septemvri. From February 1995 to October 2004 and then from March 2011 to October 2011, she worked as a mathematician in the 'Complex Analysis' section (now AGT) of IMI. From 2011 to 2014, she was an assistant professor in the same section, and from 2014 - an associate professor. Her scientific interests are mainly in the field of Mathematical analysis, namely: fractional calculus, fractional evolutionary equations (including an operator-theoretic approach for abstract problems in Banach spaces, spectral expansions of solutions for boundary value problems, estimates, asymptotics, generalized Mittag-Leffler functions), applications of fractional calculus in mathematical modelling, integral transformations and special functions, convolutional calculus and its application to finding Duhamel representations of solutions of nonlocal linear boundary value problems.

2. General description of the documents and presented scientific publications:

For her participation in the competition Assoc. Prof. Bazhlekova presented the following materials:

- Author reference for the original scientific contributions
- Abstracts of scientific publications (in Bulgarian and English)
- Complete list of scientific publications
- List of scientific publications related to the procedure
- Complete list of citations
- List of citations for participation in the competition
- Reference for participation in research projects
- Sample reference for fulfillment of the minimum requirements
- Other required documents (CV, diplomas, internship references, declarations, State Gazette).

The information contained in these materials and related to the applicant's scientific activity is analyzed in details below.

• Scientific works related to the procedure - 22 scientific articles.

For participation in the competition, Assoc. Prof. Bazhlekova submitted 22 scientific articles, all published in English. The articles are published in peer-reviewed and indexed journals in Scopus/WoS/Zentralblatt and are distributed as follows:

- Scientific articles in journals with IF/SJR – 18 items
- Scientific articles in journals without IF/SJR – 4 items

Four of them are single-authored and 18 co-authored. All the works presented in the competition are from the period 2014–2024 after her habilitation in January 2014. She has

not participated with any of them in previous procedures. Six of these works (1 with SJR and 5 with IF) were published after the defense of her doctoral dissertation for the scientific degree 'Doctor of Sciences' (which is the requirement of IMI - BAS, Art. 3. (1) of the regulations of IMI-BAS for the implementation of the ZRASRB). As required in this regulation, at least 12 publications with IF or SJR are required to participate in the competition, of which at least 6 are with IF. In addition, at least half of the publications required for the competition must be published or accepted for publication after the date of the last procedure.

- **Scientometric indicators of articles:**

11 of the presented articles are in journals with an impact factor (Web of Science, Thomson Reuters), with a cumulative IF = 24.918. Of these, 9 publications are in the prestigious Q1 category (450 p.), in Q4 – 2 works (48 p.), with SJR – 7 works (140 p.), and without IF/ SJR – 4 works (2 of them are in journals indexed in Scopus/WoS and 2 in Zentralblatt – 48 т.).

Regarding the fulfillment of the recommended criteria by Assoc. Prof. Bazhlekova for occupying the academic position of 'professor', the following comparative table can be composed:

Group	Min. number of points	Scientometric indicator	Candidate value
A	50 p.	Dissertation work for the award of the educational and scientific degree 'Doctor'	50 p.
B	100 p.	Habilitation work - scientific publications	134 p.
Г	220 p.	Scientific publications in Journals that are refereed and indexed in world-renowned databases of scientific information (Web of Science, Scopus, Zentralblatt, MathSciNet, ACM Digital Library, IEEE Xplore and AIS eLibrary), outside of the habilitation thesis	552 p.
Д	140 p.	Citations in scientific publications, monographs, collective volumes and patents, referenced and indexed in world-renowned databases of scientific information (Web of Science, Scopus, Zentralblatt, MathSciNet, ACM Digital Library, IEEE Xplore and AIS eLibrary)	1320 p.

E	150 p.	Obtained scientific degree 'Doctor of Sciences' (75 p.) Participation in a national scientific or educational project (3x10 p.=30 p.) Participation in an international scientific or educational project (5x20 p.=100 p.)	205 p.
Total	660 p.		2261 p.

3. Analysis of scientific results and contributions in the presented scientific publications

The presented works mainly fall into 5 thematic directions on the theme of the competition. Each of the 22 scientific publications submitted for participation in the competition contains original scientific contributions in the field of fractional calculus and its applications. The only exception is publication [2], which is not related to fractional calculus, but is part of a cycle of articles related to a group of indicators "B", which are dedicated to Dimovski's convolutional calculus. To describe the results in these works hereafter we follow their numbering used in List 7 of the materials.

1) The first of the directions is related to the study of the solutions of some fractional evolutionary equations and obtaining Duhamel representations through Dimovski's convolutional calculus. This includes the articles [1], [2], [3], [9], [12] and [13], which practically replace the monographic work and represent a habilitation work (these are the publications from group of indicators "B" (item 4)).

In the publications [1], [2], and [9], Duhamel representations of the solutions of boundary value problems of mathematical physics are obtained, and in [1] and [9] problems with fractional derivatives are considered. The method used for these representations was proposed by Ivan Dimovski and consists in the development of a convolutional calculus for the specific problem, which leads to its algebraization in the corresponding ring of multiplier quotients. The formal algebraic solution is then found, which is interpreted as a function. An interesting application of Dimovski's convolutional calculus is given in [2], where the classical inverse problem for the heat conduction equation is considered. In [2], Dimovski's approach is applied to obtain Duhamel representations for four types of regularization problems. The resulting Duhamel representations of the solution in publications [1], [2], and [9] are used for numerical experiments that confirm the potential of these representations in applied problems. In the publications [3], [12] and [13] the solutions of various equations with fractional derivatives are studied analytically.

2) The second direction is 'Analysis of linear viscoelastic models with fractional derivatives'. Publications [7], [8], [10], [19] and [20] belong to it. In these works, various constitutive laws are considered, which are fractional generalizations of some classical models, such as Maxwell and Jeffrey's fractional laws and Zener's generalized law. In [8], [10], [19] and [20] graphs are presented, which are obtained on the basis of the derived integral

representations for the corresponding functions. They serve to visualize the presented analytical results.

3) The third group of articles is devoted to uniqueness/existence for inverse problems for fractional differential equations. It includes articles [15], [16] and [22]. In the articles [15] and [16], a non-local boundary value problem is solved, and different convolutional derivatives are considered in them. Moreover, in [15] the problem is considered in a classical sense (in spaces of continuous functions), and in [16] the authors work in Sobolev spaces. The solution to the problem in [15] is represented by the so-called multinomial Prabhakar functions, which were first defined there. In [22], uniqueness is proved for the inverse problem of finding the potential function in the considered equation.

4) The articles [4], [5] and [17], which discuss numerical methods for some equations with fractional derivatives, belong to the direction 'Analysis of numerical methods for fractional evolutionary equations'. In publication [4], various numerical methods are developed and investigated for a special case of the considered equation with Dirichlet boundary conditions. The main contribution of the candidate is in Section 2 of this publication, in which a detailed analysis of the solution is made, estimates in Sobolev spaces are obtained, properties of the time components in the eigenfunction expansion of the solution are investigated. The derived estimates are essentially used in the further analysis of the considered numerical methods. In [5], a numerical algorithm for solving an equation that uses finite differences is developed. The work [17] summarizes the Adams-type method for numerical solution of fractional differential equations containing fractional convolutional derivatives.

5) The fifth direction is dedicated to other applications of fractional calculus for modeling of complex processes (publications [6], [11], [14], [18] and [21]). For example, in [6] a peristaltic flow of a viscoelastic fluid with an Oldroyd-B fractional constitutive law is considered. A detailed analytical and numerical study of the time evolution of the pressure gradient is presented. The influence of various material parameters is discussed, as well as necessary constraints on the parameters at which the model makes physical sense. In [11], diffusion-controlled adsorption of a surfactant at the air/liquid interface is considered, which is described by the classical Ward-Tordai equation generalized there. It can be written in the form of an equation of fractional order $1/2$. The publication [14] analyzes the application of fractional time derivatives for the mathematical modeling of bioreaction-diffusion process. Various approaches to incorporating fractional order derivatives into classical models are discussed. On the basis of numerical results, hypotheses for dependencies between model parameters are derived, in which important characteristics of the modeled process (for example, monotonicity) are guaranteed. In publication [18], a fractional order generalization of the classical Maxwell model for axisymmetric flow of a viscoelastic thin liquid film between two interacting drops is considered. In publication [21], a class of equations that generalize the fractional Jeffrey model is studied. Additional constraints on the parameters are derived to ensure that the model makes physical sense. The considered equation describing the model is recast as an integral Volterra equation, whose kernel is represented by Mittag-Leffler multinomial functions. Applying an operator-theoretic approach, a subordination is established with respect to a suitable evolution equation of integer order depending on the considered range of parameters. The analyticity of the corresponding solution operator is also

discussed. The main tools in the proofs are the Laplace transform and the technique of Bernstein functions, especially some properties of sets of real powers of complete Bernstein functions.

4. General characteristics of the research activity of the candidate:

In addition to the given characteristics of scientific publications and their international impact, I will also note some other data:

- Assoc. Prof. Bazhlekova is a member of the editorial board of the international scientific journal *Fractional Calculus and Applied Analysis* (indexed in *Web of Science*, Q1 category) since 2014 and Associate Editor therein.

- She has 8 participations in research projects (RP), as follows: 1 national project under OP "Science and education for intelligent growth", 2 national ones under the Scientific Research Fund (SRF)- Ministry of Education and Science (MES); 4 on bilateral international cooperation at BAS and 1 international at SRP.

- Reviewer of a number of international mathematical journals.

5. Reflection of the candidate's scientific publications in the literature:

Dr. Bazhlekova presented a list of 220 citations she noticed (and verified) in scientific publications indexed in the SCOPUS database (only 56 of them are included in the minimum requirements table). The citations presented for the competition were not used in previous procedures and give a total of 1,320 points (according to Appendix 1 of the BAS Regulations for the implementation of the RSARB on 6 points per citation). The author's personal Hirsch index is $h=13$ (according to Thomson Reuters' Web of Science & Scopus), and $h=17$ (according to Google Scholar Profile). Most of the citations are from well-known foreign authors in the field of special functions and fractional calculus, and are in reputable international publications.

6. Approbation of the results:

The results of E. Bazhlekova from the works proposed in the competition have been published in prestigious international journals and proceedings of international conferences, indexed in the primary sources of the world bibliographic network. In addition, the main part of them were reported at the general seminar "Analysis, Geometry and Topology" at IMI-BAS, at the annual scientific sessions of the section "Analysis, Geometry and Topology" at IMI-BAS, as well as at the seminar in Mathematical Modeling of FMI-SU. Moreover, as noted above, they have been noticed and cited many times. They have been presented at a number of international scientific forums in Bulgaria and abroad (Serbia, Greece), and at national ones and have received very positive reviews. These results are not a self-serving development, but represent an essential part of the work programs of the already mentioned scientific research

projects (of SRF, IMI-BAS, bilateral cooperation), part of the teams of which the candidate is. In view of all this, I consider the approbation of the results to be very good.

7. Evaluation of the candidate's personal contribution:

For her participation in the competition, Assoc. Prof. Bazhlekova submitted 4 single-authored articles, which are undoubtedly her personal work. The remaining 18 are co-authored (16 with one more author, 1 with two more and 1 with three more authors) - I consider that the candidate has an equal contribution in them.

8. Critical Notes:

From a technical point of view, the materials for the competition are carefully prepared and are a pleasure to look at. I have no critical notes.

9. Personal impressions:

I have known Assoc. Prof. Bazhlekova since her postgraduate studies in the 'Complex Analysis' section of IMI (1990). She works actively in IMI collectives on SRP at SRF - MES, on budget projects of IMI-BAS, on bilateral cooperation of BAS with the Academy of Serbia. Because of this, I am closely familiar with her scientific and scientific-organizational activities on topics close to my scientific interests and pursuits. I also have impressions of her presentations at international conferences and national forums and how they are received by the specialists in the field. They are that the candidate is an established and recognized author, in our country and abroad, in the field of fractional calculus and its numerous applications.

10. Conclusion:

The review of the presented documents and scientific works shows that Assoc. Prof. DSc. Bazhlekova is an internationally recognized specialist in mathematical analysis, and in particular in the field of fractional calculus and its applications, with significant personal contributions. I appreciate the contributions of the papers as theoretical generalization and development of theory and applications in both an old classical and a rapidly developing area of mathematical analysis. According to the Regulations of IMI-BAS for the application of ZRASRB, Ch. 2, the quantitative criteria for occupying the academic position 'professor' at IMI and their implementation in this case are **significantly exceeded**:

1) at least **12** publications with IF or SJR, of which at least **6** are with IF - **18** items presented, **11** of which with IF;

2) at least half of the publications required for the competition must have been published or accepted for publication after the date of the last procedure - **6** articles were presented (**5** - with an impact factor and **1** - with an impact rank);

3) Sum of indicators from 5 to 10 for publications (distributed by categories) – at least **220** points. The candidate has **552** points.

4) Citations (without self-citations) of the publications on the competition: the candidate must have at least **140** points. The candidate's score is **1320** points (from the Scopus/WoS database).

I confirm that no plagiarism has been found in the submitted works of the candidate, which are close to my narrow scientific specialty.

Based on the above, I affirm that Assoc. Prof. DSc Emilia Grigorova Bazhlekova satisfies all the requirements of the ZRASRB and the Regulations of the IMI and the BAS for its application under this procedure. I give a positive conclusion for her election to the academic position of 'professor' at IMI in the field of higher education: 4. Natural sciences, mathematics and informatics, in professional direction 4.5 'Mathematics', scientific specialty 'Mathematical analysis' (Applications of fractional calculus). I vote 'For' the Scientific Jury according to the procedure to propose to the Scientific Council of IMI-BAS to choose the candidate Assoc. Prof. DSc Emilia Grigorova Bazhlekova at the academic position of 'professor' at IMI-BAS.

16.08.2024
Sofia

The report is prepared by:
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