MR1373471 (97a:34075) 34C05 (58F21)

Horozov, Emil Ivanov (BG-SOFIM); **Iliev, Iliya Dimov** (BG-AOS)

Perturbations of quadratic Hamiltonian systems with symmetry. (English, French summaries)

Ann. Inst. H. Poincaré Anal. Non Linéaire **13** (1996), *no. 1,* 17–56.

The authors use the method of Abelian integrals to study limit cycles appearing after general quadratic perturbation of a Hamiltonian system generated by the Hamiltonian $H = xy^2 + x^3/3 + \mu y - x$. The unperturbed system is invariant with respect to the central symmetry and has two centers. It is shown that the total number of limit cycles bifurcating from the level curves of H (around both centers) does not exceed 2.

Reviewed by *Henryk Żołądek*

MR1400888 (97e:35129) 35Qxx (35-02 35B35 35P30)

Iliev, I. D. (BG-AOS); **Khristov, E. Kh.** (BG-SOFIM); **Kirchev, K. P.** (BG-AOS)

★**Spectral methods in soliton equations. (English summary)**

Appendix 3.B by A. Yanovski [A. B. Yanovskiĭ].

Pitman Monographs and Surveys in Pure and Applied Mathematics, 73.

Longman Scientific &Technical, Harlow; copublished in the United States with John Wiley &Sons, Inc., New York, 1994. x+384 *pp.* £70.00. *ISBN* 0-582-23963-X

Among all the books on the method of the inverse scattering transform (IST) for solving integrable nonlinear evolution equations, only a very few address the questions of stability, existence and uniqueness of the solution to the Cauchy problem. Indeed, answering these questions requires the use of the spectral theory of differential operators. This constitutes precisely the subject of the book under review, whose chief aim is to cover this matter of great technical difficulty and general interest.

After a very complete and instructive introduction, the book is organized into four chapters which can almost be read separately. Chapter 1 is mainly devoted to the spectral theory for the Sturm-Liouville operator on the finite interval; in particular, the continuous analogue of Newton's method is used to generate explicit formulae for the potentials. The tools are illustrated by application to the Korteweg-de Vries equation with periodic boundary conditions. Chapter 2 proposes a detailed presentation of the spectral theory for the Schrödinger operator on the half-line, and also for the

Dirac operator.

Then Chapter 3 deals with the Dirac operator on the infinite line with the aid of which many interesting nonlinear integrable evolutions can be studied (nonlinear Schrödinger, sine-Gordon, modified Korteweg-de Vries, etc.) and in particular can be given an action-angle Hamiltonian formulation.

Lastly, Chapter 4 treats the question of the stability of a solitary wave, with particular attention given to the (integrable) case of Korteweg-de Vries compared with the (non-integrable) case of Benjamin-Bona-Mahony. This study is performed on the basis of a precise set of definitions which provide a rigorous and efficient tool for such useful studies.

Although the deliberate choice of presentation in mathematically rigorous fashion sometimes makes for difficult reading, the richness of the content and results makes this book very useful for any researcher in the field, as indeed it fills a gap in the mathematical part of the subject.

Reviewed by *Jérôme Leon*

MR1296162 (95h:34046) 34C05 (34C23 58F21)

Horozov, E. (BG-SOFIM); **Iliev, I. D.** (BG-AOS)

On saddle-loop bifurcations of limit cycles in perturbations of quadratic Hamiltonian systems.

J. Differential Equations **113** (1994), *no. 1,* 84–105.

Here, the authors study the system $\dot{x} = \partial H/\partial y + \varepsilon f(x, y)$, $\dot{y} = -\partial H/\partial x + \varepsilon g(x, y)$, where ε is a small parameter, f, g are quadratic functions and $H(x, y)$ is a generic cubic function. By genericity we mean that the level curves $H(x, y) = h$ are either smooth curves different from straight lines or contain a double point forming a vertex of a separatrix loop. There can be one or two such loops. The authors study limit cycles appearing near the loops. They prove that the cyclicity of any loop is 2. Moreover, the total number of limit cycles appearing near two loops does not exceed 2. If near one loop there are two limit cycles then the other focus is hyperbolic and is not surrounded by any limit cycle. Some analysis is devoted to the case with a saddle loop and a straight line. Here the unperturbed system belongs to the intersection of the space of Hamiltonian systems with the space of reversible systems with a center. It is natural to use the second-order Poincaré-Mel′nikov integral in such situations. Because the authors do not use it their result is incomplete.

Reviewed by *Henryk Żołądek*

MR1272426 (95e:58146) 58F21 (34C05 58F14)
Horozov, E. (BG-SOFIM); **Iliev, I. D.** (BG-AOS)
On the number of limit cycles in perturbations of quadratic Hamiltonian systems. (English summary)
Proc. London Math. Soc. (3) **69** (1994), *no. 1,* 198–224.

The main result is as follows: Let H be a generic cubic Hamiltonian with three saddles and one centre, and X_H be the Hamiltonian vector field. Then there exists a neighbourhood $\mathcal{U}$ of X_H in the space of all quadratic vector fields such that each $V \in \mathcal{U}$ has at most two limit cycles. The bound is exact, as one can construct perturbations with two limit cycles.

Under the generic assumption, limit cycles bifurcate from Hamiltonian cycles which fill a disk bounded by a saddle connection. Then the proof splits into two parts: (1) Solve the weak Hilbert problem for H, which is to find an upper bound for the number of zeros of the integral $I(h)$ associated to the perturbation. This gives the number of limit cycles which bifurcate from regular Hamiltonian cycles. (2) Determine the number of limit cycles which bifurcate from the saddle connection.

The authors prove that these two numbers are equal to two. The first problem is replaced by an equivalent one: prove that the centroid curve is convex. This curve is defined by taking two ratios of integrals as a function of the value of the Hamiltonian.

Reviewed by *Robert Roussarie*

MR1202566 (94a:35103) 35Q55 (34A99 35B35)
Iliev, Iliya D. (BG-AOS); **Kirchev, Kiril P.** (BG-SOFIM)
Stability and instability of solitary waves for one-dimensional singular Schrödinger equations. (English summary)
Differential Integral Equations **6** (1993), *no. 3,* 685–703.

The authors study the stability of the standing waves $u_\omega(t,x) = e^{i\omega t}\varphi_\omega(x)$ of the nonlinear Schrödinger equation (1) $iu_t + u_{xx} = u[f(|u|^2) + 2kh'(|u|^2)(h(|u|^2))_{xx}]$, for $t \geq 0$ and $x \in \mathbf{R}$. Under appropriate assumptions on f and h, equation (1) has a family of standing waves corresponding to functions φ_ω that are positive and even, for ω in an interval of $\mathbf{R}$. Setting $d(\omega) = E(\varphi_\omega) + \omega Q(\varphi_\omega)$, where Q and E are the charge and energy associated with equation (1), the authors show that u_ω is stable if $d''(\omega) > 0$ and that u_ω is unstable if $d''(\omega) < 0$. Since equation (1) is phase and translation invariant, stability and instability are understood modulo phase and translation. The method of proof is adapted from the method of Grillakis, Shatah and Strauss.

Reviewed by *Thierry Cazenave*