
Zbl 1198.34049

Iliev, Iliya D.; Li, Chengzhi; Yu, Jiang

Bifurcations of limit cycles in a reversible quadratic system with a center, a saddle and two nodes. (English)

Commun. Pure Appl. Anal. 9, No. 3, 583-610 (2010). ISSN 1534-0392; ISSN 1553-5258
http://dx.doi.org/10.3934/cpaa.2010.9.583
http://www.aimsciences.org/journals/cpaa/current_CPAA.jsp

Consider the system

$$\dot{x} = -y - 3x^2 + by^2, \quad \dot{y} = x(1 - 2y),$$

where b is a real parameter. The exact upper bound for the number of limit cycles produced by the period annulus of the system under quadratic perturbations is called the cyclicity of the period annulus. By using the properties of related complete elliptic integrals and the geometry of some plane curves, the authors prove that, if $b \geq 2$, $b \neq 3$, then the cyclicity of the period annulus is two. This bound is exact.

M. A. M. Alwash (Culver City)

Keywords : bifurcation; period annulus; complete elliptic integrals; limit cycles

Classification :

∗34C07 Theory of limit cycles of polynomial and analytic vector fields
34C08 Connections of ODE with real algebraic geometry
37G15 Bifurcations of limit cycles and periodic orbits
34C23 Bifurcation (periodic solutions)
34C14 Symmetries, invariants

Zbl 1187.35211

Hakkaev, Sevdzhan; Iliev, Iliya D.; Kirchev, Kiril

Stability of periodic traveling waves for complex modified Korteweg-de Vries equation. (English)

J. Differ. Equations 248, No. 10, 2608-2627 (2010). ISSN 0022-0396
http://dx.doi.org/10.1016/j.jde.2010.02.001
http://www.sciencedirect.com/science/journal/00220396

Summary: We study the existence and stability of periodic traveling-wave solutions for complex modified Korteweg-de Vries equation. We also discuss the problem of uniform continuity of the data-solution mapping.

Keywords : Korteweg-de Vries equation; periodic waves; stability

Classification :

∗35Q53 KdV-like equations
35C07
35B10 Periodic solutions of PDE

35B35 Stability of solutions of PDE
35B60 Continuation of solutions of PDE

Zbl 1178.34037

Gautier, Sébastien; Gavrilov, Lubomir; Iliev, Iliya D.

Perturbations of quadratic centers of genus one. (English)

Discrete Contin. Dyn. Syst. 25, No. 2, 511-535 (2009). ISSN 1078-0947; ISSN 1553-5231
http://dx.doi.org/10.3934/dcds.2009.25.511
http://www.aimsciences.org/journals/dcdsA/online.jsp

Suppose the first integral of a given planar system of ordinary differential equations with a center defines elliptic curves. This paper presents a program for answering the question of the maximum number of limit cycles that can bifurcate from the open period annulus under small perturbation of the system. In the quadratic case under consideration it is enough to consider suitable one-parameter (rather than multi-parameter) quadratic perturbations, here called essential perturbations, together with the corresponding generating functions, expressed as complete elliptic integrals. These were found in previous work of the third author and are reproduced here. Then the zeros of the generating functions, which determine the limit cycles that appear, are studied using tools from algebraic geometry. The ideas are applied to obtain the exact cyclicity of two families of reversible systems.

Douglas S. Shafer (Charlotte)

Keywords : quadratic centers; elliptic integrals; limit cycles

Classification :

∗34C07 Theory of limit cycles of polynomial and analytic vector fields
34C08 Connections of ODE with real algebraic geometry
37G15 Bifurcations of limit cycles and periodic orbits

Zbl 1177.34046

Gavrilov, Lubomir; Iliev, Iliya D.

Quadratic perturbations of quadratic codimension-four centers. (English)

J. Math. Anal. Appl. 357, No. 1, 69-76 (2009). ISSN 0022-247X
http://dx.doi.org/10.1016/j.jmaa.2009.04.004
http://www.sciencedirect.com/science/journal/0022247X

Authors' abstract: We study the stratum in the set of all quadratic differential systems

$$\dot{x} = P_2(x, y),\ \dot{y} = Q_2(x, y)$$

with a center, known as the codimension-four case Q_4. It has a center and a node and a rational first integral. The limit cycles under small quadratic perturbations in the system are determined by the zeros of the first Poincaré-Pontryagin-Melnikov integral I. We show that the orbits of the unperturbed system are elliptic curves, and I is a

complete elliptic integral. Then using Picard-Fuchs equations and the Petrov's method (based on the argument principle), we set an upper bound of eight for the number of limit cycles produced from the period annulus around the center.

Alexey Remizov (Trieste)

Keywords : quadratic differential systems; center; limit cycles; first integral; zeros of Abelian integrals; Picard–Fuchs equations

Classification :

∗34C07 Theory of limit cycles of polynomial and analytic vector fields
34C05 Qualitative theory of some special solutions of ODE
34C08 Connections of ODE with real algebraic geometry
34C23 Bifurcation (periodic solutions)

Zbl 1136.35317

Hakkaev, Sevdzhan; Iliev, Iliya D.; Kirchev, Kiril

Stability of periodic travelling shallow-water waves determined by Newton's equation. (English)

J. Phys. A, Math. Theor. 41, No. 8, Article ID 085203, 31 p. (2008). ISSN 1751-8113; ISSN 1751-8121
http://dx.doi.org/10.1088/1751-8113/41/8/085203
http://iopscience.iop.org/1751-8121/

Summary: We study the existence and stability of periodic travelling-wave solutions for generalized Benjamin-Bona-Mahony and Camassa-Holm equations. To prove orbital stability, we use the abstract results of Grillakis-Shatah-Strauss and the Floquet theory for periodic eigenvalue problems.

Keywords : Benjamin-Bona-Mahony equation; Camassa-Holm equation; Floquet theory

Classification :

∗35B35 Stability of solutions of PDE
35B10 Periodic solutions of PDE
35Q35 Other equations arising in fluid mechanics
35Q53 KdV-like equations
35B25 Singular perturbations (PDE)
34C08 Connections of ODE with real algebraic geometry
34L40 Particular ordinary differential operators

Zbl 1093.34015

Gavrilov, Lubomir; Iliev, Iliya D.

The displacement map associated to polynomial unfoldings of planar Hamiltonian vector fields. (English)

Am. J. Math. 127, No. 6, 1153-1190 (2005). ISSN 0002-9327; ISSN 1080-6377
http://dx.doi.org/10.1353/ajm.2005.0039
http://muse.jhu.edu/journals/american_journal_of_mathematics/v127/127.6gavrilov.pdf

http://muse.jhu.edu/journals/american_journal_of_mathematics
http://www.jstor.org/action/showPublication?journalCode=amerjmath

In this paper, the authors study the displacement map associated to small one-parameter polynomial unfoldings of polynomial Hamiltonian vector fields on the plane. The leading term of the displacement map, the generating function $M(t)$, has an analytic continuation in the complex plane and the real zeros of $M(t)$ correspond to the limit cycles bifurcating from the periodic orbits of the Hamiltonian flow. The authors give a geometric description of the monodromy group of $M(t)$, use it to formulate sufficient conditions for $M(t)$ to satisfy a differential equation of Fuchs or Picard-Fuchs type, and consider some special examples.

Valery A. Gaiko (Minsk)

Keywords : planar Hamiltonian vector field; polynomial unfolding; displacement map; monodromy group; periodic orbit; limit cycle
Classification :
∗34C07 Theory of limit cycles of polynomial and analytic vector fields
37C10 Vector fields, flows, ordinary differential equations
37C27 Periodic orbits of vector fields and flows
58K05 Critical points of functions and mappings
58K10 Monodromy
34C08 Connections of ODE with real algebraic geometry

Zbl 1077.34035

Iliev, Iliya D.; Li, Chengzhi; Yu, Jiang

Bifurcations of limit cycles from quadratic non-Hamiltonian systems with two centres and two unbounded heteroclinic loops. (English)

Nonlinearity 18, No. 1, 305-330 (2005). ISSN 0951-7715
http://dx.doi.org/10.1088/0951-7715/18/1/016
http://www.iop.org/Journals/no

The authors investigate bifurcations of limit cycles in a class of planar quadratic integrable (non-Hamiltonian) systems with two centres, both surrounded by unbounded heteroclinic loops, under small quadratic perturbations. By a careful study of the number of zeros of Abelian integrals based on the geometric properties of some planar curves, defined by the ratios of such integrals, they obtain complete results on the number and distribution of limit cycles bifurcating from the two period annuli.

Valery A. Gaiko (Minsk)

Keywords : planar quadratic non-Hamiltonian system; Abelian integral; bifurcation; limit cycle; heteroclinic loop
Classification :
∗34C07 Theory of limit cycles of polynomial and analytic vector fields
37G15 Bifurcations of limit cycles and periodic orbits

34C08 Connections of ODE with real algebraic geometry
34C23 Bifurcation (periodic solutions)

Zbl 1043.34031

Gavrilov, Lubomir; Iliev, Iliya D.

Complete hyperelliptic integrals of the first kind and their non-oscillation. (English)

Trans. Am. Math. Soc. 356, No. 3, 1185-1207 (2004). ISSN 0002-9947; ISSN 1088-6850
http://dx.doi.org/10.1090/S0002-9947-03-03432-9
http://www.ams.org/tran/2004-356-03/S0002-9947-03-03432-9/home.html
http://www.ams.org/tran/

Let $O(h)$, $h \in \Sigma$, be a continuous family of ovals which are contained in the family $\{H(x,y) = h,\ (x,y) \in \mathbb{R} \times \mathbb{R},\ h \in \Sigma\}$, where $H(x,y)$ is a real polynomial. For ε sufficiently small and generic polynomials $f(x,y)$ and $g(x,y)$, the limit cycles of the perturbed Hamiltonian system $dH + \varepsilon(f\,dx + g\,dy) = 0$ which tend to certain ovals from the continuous family when $\varepsilon \to 0$, are in one-to-one correspondence with the zeros of the complete Abelian integral

$$I(h) = \int_{O(h)} f\,dx + g\,dy, \qquad h \in \Sigma.$$

For this reason, V. I. Arnold called the problem to find the exact number of the zeros of $I(h)$ a “weakened 16th Hilbert problem”. In the so-called “elliptic case” (the complex algebraic curve is of genus at most one), it was proved that the vector space $A(H,d)$ of Abelian integrals of dth-degree polynomials, along the ovals of H, is Chebyshev (the number of the zeros of each integral is smaller than the dimension of the vector space $A(H,d)$). In the so-called “hyperelliptic case” ($H(x,y) = y^2 + P(x)$, where $P(x)$ is a real polynomial of degree $2p+1$), Arnold asked wether the p-dimensional vector space of Abelian integrals

$$I(h) = \int_{O(h)} \{(\alpha_0 + \alpha_1 x + \cdots + \alpha_{p-1} x)/y\}\,dx, \quad h \in \Sigma,\ p > 1,$$

is Chebyshev.
Here, the authors show that the answer to the above question is negative in general (there are hyperelliptic Hamiltonians H and continuous families of ovals $O(h)$ belonging to $\{H = h\}$ such that $I(h)$ can have at least $[(3/2)p] - 1$ zeros in Σ). Further, they show that if $p = 2$ (then $\deg P(x) = 5$), exceptional families of ovals $\{O(h)\}$ exist so that each Abelian integral of the form

$$I(h) = \int_{O(h)} \{(\alpha_0 + \alpha_1 x)/y]\,dx$$

is not oscillatory on the interval Σ.

Jinghuang Tian (Phoenix)

Keywords : weakened 16th Hilbert problem; complete Abelian integral; Chebyshev property

Classification :
- ∗34C08 Connections of ODE with real algebraic geometry
- 34C07 Theory of limit cycles of polynomial and analytic vector fields
- 14K20 Analytic theory; abelian integrals and differentials

Zbl 1046.34056

Gavrilov, Lubomir; Iliev, Iliya D.

Two-dimensional Fuchsian systems and the Chebyshev property. (English)

J. Differ. Equations 191, No. 1, 105-120 (2003). ISSN 0022-0396
http://dx.doi.org/10.1016/S0022-0396(02)00116-X
http://www.sciencedirect.com/science/journal/00220396

Summary: Let $(x(t), y(t))^T$ be a solution of a Fuchsian system of order two with three singular points. The vector space of functions of the form $P(t)x(t) + Q(t)y(t)$, where P, Q are real polynomials, has a natural filtration of vector spaces, according to the asymptotic behavior of the functions at infinity. We describe a two-parameter class of Fuchsian systems, for which the corresponding vector spaces obey the Chebyshev property (the maximal number of isolated zeros of each function is less than the dimension of the vector space). Up to now, only a few particular systems were known to possess such a nonoscillation property. It is remarkable that most of these systems are of the type studied in the present paper. We apply our results in estimating on the number of limit cycles that appear after small polynomial perturbations of several quadratic or cubic Hamiltonian systems in the plane.

Classification :
- ∗34C08 Connections of ODE with real algebraic geometry
- 34C05 Qualitative theory of some special solutions of ODE
- 34C07 Theory of limit cycles of polynomial and analytic vector fields
- 37C25 Fixed points, periodic points, fixed-point index theory

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Zbl 1026.34037

Gavrilov, Lubomir; Iliev, Iliya D.

Bifurcations of limit cycles from infinity in quadratic systems. (English)

Can. J. Math. 54, No.5, 1038-1064 (2002). ISSN 0008-414X; ISSN 1496-4279
http://cms.math.ca/cjm/

The authors study the bifurcation of limit cycles in one-parameter unfoldings of quadratic systems of differential equations on the plane. For the zero parameter value, the system is supposed to have an elliptic critical point at the origin and a degenerate critical point at infinity. It is proved that there are three types of quadratic systems possessing an elliptic critical point which bifurcates from infinity together with eventual limit cycle around it. These limit cycles are studied by performing a degenerate transformation which brings the system to a small perturbation of well-known reversible systems with a center. The corresponding displacement function is than expanded in a Puiseux series with respect to the small parameter and its coefficients are expressed in terms of Abelian integrals. The authors prove a theorem on the asymptotic behavior of limit cycles. Finally, they consider four important examples (namely, the Bogdanov-Takens system, the isochronous center $\mathcal{S}_3$, the reversible Lotka-Volterra center, and the Hamiltonian system with a center and two saddles) and estimate the number of uniformly large limit cycles around the critical point coming from infinity. It is proved that in each case there are at most two such cycles. The proof is by construction of the bifurcation diagram of zeros for certain Abelian integrals in a complex domain.

Dimitrii Rachinskii (Moskva)

Keywords : planar quadratic system; bifurcation of limit cycle from infinity; reversible system; estimates on the number of cycles; center; Puiseux series; Abelian integrals

Classification :

∗34C07 Theory of limit cycles of polynomial and analytic vector fields
34C10 Qualitative theory of oscillations of ODE: Zeros, etc.
34C05 Qualitative theory of some special solutions of ODE
37G15 Bifurcations of limit cycles and periodic orbits

Zbl 0992.37054

Gavrilov, Lubomir; Iliev, Iliya D.

Second-order analysis in polynomially perturbed reversible quadratic Hamiltonian systems. (English)

Ergodic Theory Dyn. Syst. 20, No.6, 1671-1686 (2000). ISSN 0143-3857; ISSN 1469-4417

http://dx.doi.org/10.1017/S0143385700000936

http://journals.cambridge.org/action/displayJournal?jid=ETSbVolume=y

This paper is devoted to the exact upper bound for the number of zeros of the second-order Poincaré-Pontryagin integral $M_2(h)$ related to small nth-degree polynomial perturbations

(1) $$\dot{x} = Hy + \varepsilon f(x, y, \varepsilon), \quad \dot{y} = -H_x + \varepsilon g(x, y, \varepsilon)$$

of a Hamiltonian vector field X_H

(2) $$\dot{x} = Hy, \quad \dot{y} = -H_x$$

corresponding to a reversible cubic Hamiltonian H with just one saddle point and one center. The authors show that when the perturbation is quadratic they obtain a complete result, there is a neighborhood of the initial Hamiltonian vector field in the space of all quadratic vector fields, in which any vector field has at most two limit cycles.

Messoud Efendiev (Berlin)

Keywords : exact upper bound; number of zeros; second-order Poincaré-Pontryagin integral; polynomial perturbations; Hamiltonian vector field; reversible cubic Hamiltonian; saddle point

Classification :

∗37J40 Perturbations, etc.
70H09 Perturbation theories
34C07 Theory of limit cycles of polynomial and analytic vector fields

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Zbl 0945.34020

Iliev, Iliya D.

On the limit cycles available from polynomial perturbations of the Bogdanov-Takens Hamiltonian. (English)

Isr. J. Math. 115, 269-284 (2000). ISSN 0021-2172; ISSN 1565-8511
http://dx.doi.org/10.1007/BF02810590
http://www.springerlink.com/content/0021-2172/
http://www.ma.huji.ac.il/ ijmath/

Summary: The displacement map related to small polynomial perturbations of the planar Hamiltonian system $dH = 0$ is studied in the elliptic case $H = \frac{1}{2}y^2 + \frac{1}{2}x^2 - \frac{1}{3}x^3$. An estimate of the number of isolated zeros for each of the successive Melnikov functions $M_k(h)$, $k = 1, 2, \ldots$ is given in terms of the order k and the maximal degree n of the perturbation. This sets up an upper bound to the number of limit cycles emerging from the periodic orbits of the Hamiltonian system under polynomial perturbations.

Keywords : limit cycles; polynomial perturbations; Bogdanov-Takens Hamiltonian; Melnikov functions; periodic orbits

Classification :

∗34C07 Theory of limit cycles of polynomial and analytic vector fields
34C05 Qualitative theory of some special solutions of ODE
37J05 Relations with symplectic geometry and topology
37J40 Perturbations, etc.

Zbl 0967.34026

Iliev, Iliya D.

The number of limit cycles due to polynomial perturbations of the harmonic oscillator. (English)

Math. Proc. Camb. Philos. Soc. 127, No.2, 317-322 (1999). ISSN 0305-0041; ISSN 1469-8064
http://dx.doi.org/10.1017/S0305004199003795
http://www.journals.cambridge.org/journal_MathematicalProceedingsoftheCambridgePhilosophic

Consider the two-dimensional autonomous differential system

$$(*) \qquad dx/dt = y + \varepsilon f(x, y, \varepsilon), \quad dy/dt = -x + \varepsilon g(x, y, \varepsilon),$$

where f and g are polynomials in x and y with coefficients depending analytically on the small parameter ε. The author derives an upper estimate for the number of limit cycles of $(*)$ for small ε. The estimate depends on the degree of f and g and on the order k of the first nonvanishing Melnikov function M_k in the expansions of the first return map with respect to ε.

Klaus R.Schneider (Berlin)

Keywords : number of limit cycles; polynomial perturbations; harmonic oscillator
Classification :
∗34C07 Theory of limit cycles of polynomial and analytic vector fields
34C05 Qualitative theory of some special solutions of ODE
34C15 Nonlinear oscillations of solutions of ODE

Zbl 0926.34033

Iliev, I.D.; Perko, L.M.
Higher order bifurcations of limit cycles. (English)
J. Differ. Equations 154, No.2, 339-363 (1999). ISSN 0022-0396
http://dx.doi.org/10.1006/jdeq.1998.3549
http://www.sciencedirect.com/science/journal/00220396

Summary: The authors show that asymmetrically perturbed, symmetric Hamiltonian systems of the form

$$\dot{x} = y, \quad \dot{y} = \pm(x \pm x^3) + \lambda_1 y + \lambda_2 x^2 + \lambda_3 xy + \lambda_4 x^2 y,$$

with analytic $\lambda_j(\varepsilon) = O(\varepsilon)$, have at most two limit cycles that bifurcate for small $\varepsilon \neq 0$ from any period annulus of the unperturbed system. This fact agrees with previous results of Petrov, Dangelmayr and Guckenheimer,and Chicone and Iliev, but shows that the result of three limit cycles for the asymmetrically perturbed, exterior Duffing oscillator, recently obtained by Jebrane and Żołądek, is incorrect.
The proofs follow by deriving an explicit formula for the kth-order Melnikov function, $M_k(h)$, and using a Picard-Fuchs analysis to show that, in each case, $M_k(h)$ has at most two zeros. Moreover, the method developed for determining higher-order Melnikov functions applies to more general perturbations of these systems. © Academic Press.

Keywords : bifurcation; limit cycles; kth-order Melnikov function
Classification :
∗34C23 Bifurcation (periodic solutions)
34C05 Qualitative theory of some special solutions of ODE
37G15 Bifurcations of limit cycles and periodic orbits
37J20 Bifurcation problems

Zbl 0922.34037

Iliev, I.D.
On second order bifurcations of limit cycles. (English)
J. Lond. Math. Soc., II. Ser. 58, No.2, 353-366 (1998). ISSN 0024-6107; ISSN 1469-7750

http://jlms.oxfordjournals.org/

A formula for the second variation of the displacement function is derived in case of polynomial perturbations of Hamiltonian vector fields with elliptic or hyperelliptic Hamiltonians $H(x,y)=\frac{1}{2}y^2-U(x)$, $\deg U\geq 2$, in terms of the coefficients of the perturbation. The result is applied to prove the conjecture stated by *C. Chicone* [Lect. Notes Math. 1455, 20-43 (1990; Zbl 0731.34020)] and *T. R. Blows* and *L. M. Perko* [SIAM Rev. 36, 341-376 (1994; Zbl 0807.34051)] that a specific cubic system appearing in a deformation of singularity with two zero eigenvalues has at most two limit cycles.

I.D.Iliev (Sofia)

Keywords : bifurcation; planar vector field; second variation; polynomial perturbations; hyperelliptic Hamiltonians; limit cycles

Classification :

∗34C23 Bifurcation (periodic solutions)
37G15 Bifurcations of limit cycles and periodic orbits
37G99 Bifurcation theory
37J40 Perturbations, etc.
34C05 Qualitative theory of some special solutions of ODE

Zbl 0921.58044

Horozov, Emil; Iliev, Iliya D.

Linear estimate for the number of zeros of Abelian integrals with cubic Hamiltonians. (English)

Nonlinearity 11, No.6, 1521-1537 (1998). ISSN 0951-7715
http://dx.doi.org/10.1088/0951-7715/11/6/006
http://stacks.iop.org/Non/11/1521
http://www.iop.org/Journals/no

Let $H(x,y)$, $(x,y)\in\mathbb{R}^2$, be a polynomial of degree m (called Hamiltonian) and let $f(x,y)$, $g(x,y)$ be polynomials of degrees not exceeding n. Let $\Sigma=\{h:h_1<h<h_2\}\subset\mathbb{R}$ be a maximal interval of existence of a continuous family of closed connected components $\delta(h)$ of the algebraic curve $H(x,y)=h$, $h\in\Sigma$, free of critical points.

The infinitesimal 16th Hilbert problem [*V. I. Arnol'd*, Funct. Anal. Appl. 11, 85-92 (1977; Zbl 0411.58013)] is to give an estimate for the number $Z(m,n)$ of zeros of the Abelian integral

$$I(h)=\int_{\delta(h)}[g(x,y)dx-f(x,y)dy],\quad h\in\Sigma,$$

in terms of the degrees of the polynomials H, f, g. The infinitesimal 16th Hilbert problem is known to be closely connected with the number of limit cycles which is the main concern of Hilbert's 16th problem.

In the paper, for cubic Hamiltonians H $(m=3)$ a linear estimate $Z(3,n)\leq 5n+15$ for the number of zeros of $I(h)$ is obtained. The proof is based on the properties of the Picard-Fuchs system satisfied by the four basic integrals $\iint_{H<h}x^iy^jdxdy$, $i,j=0,1$,

generating the module of complete Abelian integrals $I(h)$ over the ring of polynomials in h.

E.Ershov (St.Peterburg)

Keywords : infinitesimal 16th Hilbert problem; Hamiltonian system; limit cycle; Abelian integral; Picard-Fuchs equation; Melnikov function

Classification :

∗37G15 Bifurcations of limit cycles and periodic orbits
37J20 Bifurcation problems
37K50 Bifurcation problems
34C05 Qualitative theory of some special solutions of ODE

Zbl 0920.34037

Iliev, Iliya D.

Perturbations of quadratic centers. (English)

Bull. Sci. Math. 122, No.2, 107-161 (1998). ISSN 0007-4497
http://dx.doi.org/10.1016/S0007-4497(98)80080-8
http://www.sciencedirect.com/science/journal/00074497

This paper addresses the bifurcation of limit cycles from a center of a quadratic system. More precisely, consider a quadratic system

$$\dot{x} = f(x,y), \qquad \dot{y} = g(x,y),$$

with a rest point at the origin that is surrounded by an annulus of periodic solutions and the perturbed system

$$\dot{x} = f(x,y) + \varepsilon P(x,y,\varepsilon), \qquad \dot{y} = g(x,y) + \varepsilon Q(x,y,\varepsilon),$$

where P and Q are quadratic polynomials in the space variables. The question is: How many of the periodic orbits surrounding the center of the original quadratic system persist as limit cycles for $\varepsilon \neq 0$? This is a difficult unsolved problem that has received a great deal of attention. The usual idea used to study this problem is to notice that the periodic orbits near the origin correspond to the zeros of a displacement function defined on a Poincaré section for the original period annulus. Indeed, if ξ is a real coordinate along such a Poincaré section, then the displacement function has the form

$$d(\xi,\varepsilon) = \varepsilon M(\xi) + O(\varepsilon^2)$$

where M is the Andronov-Poincaré-Melnikov function. Thus, if M is not identically zero, then the simple zeros of M correspond to the continuable periodic orbits. Of course, to carry out this program, the coefficient M must be identified and then its simple zeros must be computed. Moreover, if M happens to be the zero function, then a higher-order leading coefficient must be computed and analyzed. It turns out that M, or the leading order coefficient of the series representation for the displacement, can be represented by a line integral over the unperturbed periodic orbits where the integrand involves the perturbation, a first integral of the unperturbed system, and an integrating factor.

Here, one of the main contributions is a complete characterization of appropriate forms for the integral representation of these leading order coefficients for all cases of quadratic centers together with a characterization of the "essential" perturbation terms that are required to obtain the maximum possible number of continuable periodic orbits for a general quadratic perturbation.
An analysis of the number of continuable periodic orbits is carried out for two important cases of quadratic centers. In addition to the main results of the paper, that provides a framework for future research, the author gives a useful introduction to the literature on the subject of bifurcations of limit cycles from period annuli of quadratic systems.

C.Chicone (Columbia)
Keywords : limit cycles; quadratic systems; Melnikov function; center; Andronov-Poincaré-Melnikov function
Classification :
∗34C05 Qualitative theory of some special solutions of ODE
37G15 Bifurcations of limit cycles and periodic orbits

Zbl 0896.34019

Iliev, Iliya D.
Inhomogeneous Fuchs equations and the limit cycles in a class of near-integrable quadratic systems. (English)
Proc. R. Soc. Edinb., Sect. A, Math. 127, No.6, 1207-1217 (1997). ISSN 0308-2105; ISSN 1473-7124
http://www.ingentaconnect.com/content/rse/proca
http://journals.cambridge.org/action/displayJournal?jid=PRM

The author considers quadratic perturbations of a special quadratic system in the plane. This particular system is related to the problem of finding the cyclicity of the period annulus in the component Q_4^- in the intersection $Q_3^R \cap Q_4$ (in the notation of H. Zoladek). The main theorem of the paper states that the cyclicity of a period annulus of the reversible codimension four centre Q_4^- under quadratic perturbations is three.

J.Knobloch (Ilmenau)
Keywords : planar vector fields; limit cycles; cyclicity; quadratic perturbations
Classification :
∗34C05 Qualitative theory of some special solutions of ODE
37G15 Bifurcations of limit cycles and periodic orbits
34C23 Bifurcation (periodic solutions)

Zbl 0854.34035

Horozov, Emil Ivanov; Iliev, Iliya Dimov
Perturbations of quadratic Hamiltonian systems with symmetry. (English)
Ann. Inst. Henri Poincaré, Anal. Non Linéaire 13, No.1, 17-56 (1996). ISSN 0294-1449
numdam:AIHPC_1996__13_1_17_0
http://www.sciencedirect.com/science/journal/02941449

The main result of the paper is the following Theorem 1. Let H be a generic cubic Hamiltonian with a centroid symmetry: $H(-x,-y)=-H(x,y)$. Then any quadratic perturbation of the corresponding Hamiltonian system $\dot{x}=H_y+\varepsilon f(x,y)$, $\dot{y}=-H_x+\varepsilon g(x,y)$ has at most two limit cycles for ε small enough.
The technique used is based essentially on the notion and on the properties of the centroid curve formed by the mass centres of a continuous family of ovals within the level curves of H.

Yu. V.Rogovchenko (Kiev)
Keywords : cubic Hamiltonian; centroid symmetry; quadratic perturbation; Hamiltonian system; two limit cycles
Classification :
∗34C05 Qualitative theory of some special solutions of ODE
37J99 Finite-dimensional Hamiltonian etc. systems
34C23 Bifurcation (periodic solutions)
34D10 Stability perturbations of ODE

Zbl 0853.58084

Iliev, Iliya D.
The cyclicity of the period annulus of the quadratic Hamiltonian triangle. (English)
J. Differ. Equations 128, No.1, 309-326 (1996). ISSN 0022-0396
http://dx.doi.org/10.1006/jdeq.1996.0097
http://www.sciencedirect.com/science/journal/00220396

The paper investigates the number of limit cycles for small quadratic perturbations of quadratic Hamiltonian systems $x'=H_y+\varepsilon f(x,y,\varepsilon)$, $y'=-H_x+\varepsilon g(x,y,\varepsilon)$, where H has the form $H=l_1l_2l_3$ with $l_j=a_j+b_j+c_j$. This case is known as the Hamiltonian triangle. It is well known that the center surrounded by the triangle is the unique one among the quadratic Hamiltonian centers having cyclicity three. The main result of the paper is the following theorem. For small ε, the maximal number of limit cycles which emerge from the center and the period annulus altogether is equal to three. Concerning the cyclicity under quadratic perturbation of the triangle polycycle itself, recent results were obtained by *H. Żołądek* in J. Differ. Equations 122, No. 1, 137-159 (1995; Zbl 0840.34031).

G.Osipenko (St.Peterburg)
Keywords : center; focus; separatrix; limit cycles; quadratic Hamiltonian systems; Hamiltonian triangle
Classification :
∗37G15 Bifurcations of limit cycles and periodic orbits
34C05 Qualitative theory of some special solutions of ODE
37E99 Low-dimensional dynamical systems

37G99 Bifurcation theory

Zbl 0851.34042

Iliev, I.D.

Higher-order Melnikov functions for degenerate cubic Hamiltonians. (English)

Adv. Differ. Equ. 1, No.4, 689-708 (1996). ISSN 1079-9389
http://www.aftabi.com/ADE/ade.html

The author considers quadratic perturbation problems of Hamiltonian systems in the plane with degenerate cubic Hamiltonians. He first uses the scheme of J. P. Francoise to compute explicitly the first four Melnikov functions $M_1(h),\dots,M_4(h)$, then he proves the following five interesting theorems.
Theorem 1. Assume H is a nongeneric cubic Hamiltonian with a center. Then the perturbed system (1) $\dot{x}=H_y+\varepsilon f(x,y)$, $\dot{y}=-H_x+\varepsilon g(x,y)$, where (2) $H=x[y^2+Ax^2-3(A-1)x+3(A-2)]$, $A\in\mathbb{R}$, is integrable and belongs to the union $Q_3^H\cup Q_3^R$ (in the notation of H. Zoladek) provided: (i) $M_1(h)=M_2(h)\equiv 0$, for H corresponding to $A=0$ in (2); (ii) $M_1(h)+\cdots+M_4(h)\equiv 0$, for H corresponding to $A\not\equiv -1$ in (2).
Theorem 2. In any quadratic Hamiltonian system the cyclicity of the saddle loop (or the total cyclicity of two such loops, if they exist) under quadratic perturbation is two.
Theorem 3. Suppose in (1), H is the standard elliptic Hamiltonian. Then for ε small and for arbitrary quadratic perturbations (f,g) the system has no more than two limit cycles in the finite part of the plane.
Theorem 4. Suppose in (2) we have $A=0$, then (1) has at most two limit cycles simultaneously born from the center and the set of periodic orbits of $(1)_{\varepsilon=0}$.
Theorem 5. Assume the formula for the higher-order Melnikov functions is $I(h)=k_1J_1+k_0J_0+k_{-1}J_{-1}=c_1+c_2(h-h_s)|n|h-h_s|+c_3(h-h_s)+\cdots$, where $J_i(h)=\int_{H=h}x^iydx$ $(i=0,\pm1)$, then $c_1=c_2=c_3=0$ implies $k_0=k_1=k_{-1}=0$.

Ye Yanqian (Nanjing)

Keywords : quadratic perturbation problems; Hamiltonian systems in the plane; degenerate cubic Hamiltonians; Melnikov functions; cyclicity; saddle loop

Classification :

∗34C23 Bifurcation (periodic solutions)
34C05 Qualitative theory of some special solutions of ODE
37G15 Bifurcations of limit cycles and periodic orbits

Zbl 0874.35104

Iliev, I.D.; **Khristov, E.Kh.**; **Kirchev, K.P.**

Spectral methods in soliton equations. (English)

Pitman Monographs and Surveys in Pure and Applied Mathematics. 73. Harlow: Longman Scientific & Technical. New York, NY: John Wiley & Sons, Inc. x, 384 p. £ 64.00 (1994). ISBN 0-582-23963-X/hbk; ISBN 0-470-23477-6

The authors analyze some methods based on the spectral theory of certain ordinary differential equations (such as the Schrödinger equation and the Dirac system) that are associated with nonlinear evolution equations and solitons. The emphasis is on the spectral theory of recursion operators (Λ-operators). The Λ-operators [*F. Calogero* and *A. Degasperis*, Spectral transform and solitons, Vol. I, North-Holland, Amsterdam (1982; Zbl 0501.35072)] are certain integro-differential operators related to Bäcklund transformations. The book consists of a 24-page introduction and four chapters. In the introduction, the main ideas and some historical notes are given.

In Chapters 1, 2, and 3, the spectral theory of the Λ-operators is studied on a finite interval, on the half line, and on the full line, respectively, and its application on solving various evolution equations is considered. In Chapter 4 the stability of traveling-wave solutions is analyzed for the Cauchy problem associated with some evolution equations such as the Korteweg-de Vries equation, nonlinear Schrödinger equation, Benjamin-Bona-Mahoney equation, and their generalizations. There are 251 references listed in the bibliography; the English translations are indicated for some but not all the original Russian references. Each chapter is followed by some appendices and bibliographical notes. An index is missing in the book although it would have been useful.

The book should be helpful for researchers interested in inverse spectral problems and applications to nonlinear evolution equations.

T.Aktosun (Fargo)

Keywords : inverse spectral transform; stability of solitary waves; spectral theory of recursion operators; Korteweg-de Vries equation; nonlinear Schrödinger equation; Benjamin-Bona-Mahoney equation

Classification :

∗35Q51 Solitons

35Q53 KdV-like equations

35B35 Stability of solutions of PDE

35R30 Inverse problems for PDE

35-02 Research monographs (partial differential equations)

Zbl 0808.34041

Horozov, E.; Iliev, I.D.

On saddle-loop bifurcations of limit cycles in perturbations of quadratic Hamiltonian systems. (English)

J. Differ. Equations 113, No.1, 84-105 (1994). ISSN 0022-0396

http://dx.doi.org/10.1006/jdeq.1994.1115

http://www.sciencedirect.com/science/journal/00220396

This paper studies the bifurcations (of saddle-loop connections) that arise from a perturbation of quadratic Hamiltonian vector fields on the plane. The problem is closely related to the second part of Hilbert's 16th problem concerning the number of limit cycles of polynomial vector fields. Guckenheimer et al. have conjectured that the number of limit cycles of vector fields of the form $X_\varepsilon(x, y) = (H_y + \varepsilon f, -H_x + \varepsilon g)$ (where ε is small and X_0 is a quadratic Hamiltonian vector field) does not exceed two and gave numerical evidence for that. In this present paper this conjecture is proved.

M.A.Teixeira (Campinas)

Keywords : bifurcations; quadratic Hamiltonian vector fields on the plane; Hilbert's 16th problem; number of limit cycles; polynomial vector fields

Classification :

∗34C23 Bifurcation (periodic solutions)
34C05 Qualitative theory of some special solutions of ODE
37J99 Finite-dimensional Hamiltonian etc. systems
37G15 Bifurcations of limit cycles and periodic orbits

Zbl 0802.58046

Horozov, E.; Iliev, I.D.

On the number of limit cycles in perturbations of quadratic Hamiltonian systems. (English)

Proc. Lond. Math. Soc., III. Ser. 69, No.1, 198-224 (1994). ISSN 0024-6115; ISSN 1460-244X

http://dx.doi.org/10.1112/plms/s3-69.1.198
http://plms.oxfordjournals.org/
http://www.journals.cambridge.org/journal_ProceedingsoftheLondonMathematicalSociety

The authors determine the exact upper bound for the number of limit cycles which appear in quadratic perturbations of Hamiltonian vector fields in the plane for generic cubic Hamiltonians with three saddles and one center.

E.Horozov (Sofia)

Keywords : plane Hamiltonian systems; upper bound; limit cycles; quadratic perturbations

Classification :

∗37G15 Bifurcations of limit cycles and periodic orbits
34C05 Qualitative theory of some special solutions of ODE

Zbl 0783.35071

Iliev, Ilya D.; Kirchev, Kiril P.

Stability and instability of solitary waves for one-dimensional singular Schrödinger equations. (English)

Differ. Integral Equ. 6, No.3, 685-703 (1993). ISSN 0893-4983

http://www.aftabi.com/DIE/die.html

Stability of soliton (solitary wave) solutions to the generalized nonlinear Schrödinger equation,

$$iu_t = -u_{xx} + u\Big(\big[f(|u|^2) + 2kh'(|u|^2)h(|u|^2)\big)_{xx}\Big],$$

is investigated by means of the spectral decomposition technique. A soliton solution has a free parameter ω, $u(x,t) = v(x)\exp(i\omega t)$, where the function $v(x)$ must vanish at infinity. To analyze the soliton's stability, the following Lyapunov function is introduced: $d(\omega) = E + \omega Q$, where the energy

$$E(\omega) = \int_{-\infty}^{+\infty}\Big(|u_x|^2 + g(|u|^2) - k\big(h(|u|^2)_x\big)^2\Big)dx,$$

and the "number of particles" is $Q = \int_{-\infty}^{+\infty}|u|^2 dx$.
The main theorem proved in the work is that the soliton is stable provided $d''(\omega) > 0$, and unstable in the opposite case.

B.A.Malomed (Ramat Aviv)

Keywords : solitary wave; stability; generalized nonlinear Schrödinger equation; spectral decomposition; soliton solution; Lyapunov function

Classification :

∗35Q55 NLS-like (nonlinear Schroedinger) equations
35B35 Stability of solutions of PDE
35Q51 Solitons