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Evaluation of Passenger Waiting Time in Public Transport by Using the Monte Carlo Method

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Abstract. The paper proposes the use of a simulation based on the Monte Carlo method to determine the waiting time of a passenger vehicles in urban passenger transport. The method is based on real data on the operation of urban passenger transport in the conditions of Ruse, Bulgaria. The given test results were obtained with the software product Matlab R2017b. Adjustments to the vehicle schedules may be made on the basis of the results obtained from the test. These corrections lead to the development of more accurate timetables, which has a positive impact on the quality of the transport service.

Key words: urban passenger transport, travel time, simulation, Monte-Carlo method

1. INTRODUCTION

Transport has an impact on the viability of life in the cities, on their economic, social and ecological characteristics. The growing demand for transport is creating more and more problems related to mobility. In recent years, there has been a tendency of growth in the number of private vehicles used for daily transportation in the cities, which is the reason for considerable slowing of the speed of movement, increasing the noise level and pollution, as well as creating problems with parking [1, 2, 3, 4]. It has also led to problems with the organization of mass passenger transport such as non-compliance with timetables, irregularity of movement, and poor quality of the transport service. Offering a quality transport service by the mass urban passenger transport is one of the conditions for stimulating the passengers to use it.

We can define two main stages in the development of mass urban passenger transport, which differ considerably. The priorities of the first stage (until the 1990ies) are related to building infrastructure and traffic equipment, construction and high budget expenditure. The second stage (the last 20-30 years) is characterized by development in the concept of attracting passengers to use mass urban transport, which has increased the attention paid to the service considerably. Cities are striving to create a sustainable mass urban transport, convenient for the passengers.

Designing the mass urban transport is related not only to evaluating the demand for transport, but also to evaluating the time spent on travelling. The latter involves the regularity of public transport, the reduced number of transfers and looking for shorter routes. Often these models include environmental expenses, as well as have an impact on greenhouse gasses emissions.

The time for transportation T_{tr} at regular intervals of vehicle movement can be expressed by the following mathematical model

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$$T_{tr} = t_{af1} + t_{af2} + \sum_{i=1}^{n} t_{wi} + \sum_{i=1}^{n} t_{ri}\ , \quad (1)$$

where n is the number of stops until the final destination of the passenger;

The walking times t_{af1} and t_{af2}, are the time the passenger walks to the stop where he gets on the vehicle and the time he/she walks from the stop he gets off at to his destination, respectively. They depend on the speed of walking, the density of the transport network, the average distance between stops, as well as on random factors. This time is different for the different routes and sections of these routes, but for a defined transport network and for the purposes of this paper we can assume that it is a constant value for the passenger and he/she can estimate it;

$t_{ri}\ is$ time spent in the vehicle;

t_w – waiting time at bus stop i. This parameter depends on the regularity of movement and the extent of filling the vehicle capacity for passengers. Waiting time for vehicle – this is the time from the moment of passenger arrival at the bus stop to his/her getting on the vehicle. Obviously, this time will be a function of the interval of movement, changing within the following limits

$$T_{MIN}=0\ ;\ T_{MAX}=I. \quad (2)$$

Usually, the average waiting time is determined as half of the planned interval of movement of vehicles on the route

$$T= I/2\ \text{min}, \quad (3)$$

where I is the planned interval of movement of vehicles on the given route, min.

The waiting time for a vehicle in the mass urban passenger transport is generally determined by three factors:

- interval of movement of vehicles on the route;

-driver's keeping the timetable;

-vehicle's passenger carrying capacity.

Taking into account these three factors, we can write down the following dependence for the waiting time:

$$T = \frac{I}{2} + \frac{\sigma^2}{2I} + P_{opt-out} I_{ef} = (0{,}5 + P_{opt-out}) I_{ef}, \quad (4)$$

where σ is the mean quadratic deviation from the interval of movement;

$P_{opt-out}$ - probability of passenger giving up due to limited passenger carrying capacity of the vehicle;

I_{ef} - real interval of movement of vehicles on the route.

The widely used traditional model for determining the average waiting time of passengers is based on the following assumptions [5]:

-passengers arrive at random;

-passengers get on the first arriving vehicles;

-vehicles arrive regularly.

This model is frequently mentioned by other researchers, who investigate the passenger waiting time. Most of them consider the first two of the above-mentioned assumptions acceptable. The regularity of arrival of vehicles is considered a problem as passengers are expected to wait longer than the average waiting time.

In [6,7,8] following theoretical research, a model has been developed for the bus waiting time, defining it as a function of mean headways between buses μ and variances of headways between buses:

$$t_w = \mu\left(1 + \frac{\mu^2}{s^2}\right)/2, \quad (5)$$

where μ is mean headways between buses;

s^2 – variances of headways between buses.

Some empirical research done subsequently proves that the assumption is reasonable.

In [9] there is a model based on empirical data collected in Australia. In [10] is using hazard-based duration model. In 1999 the model T=2,0+0,3μ, based on a large quantity of empirical data accumulated is published, [11].

In [12] a method is suggested which takes into account passengers' behaviors. They can be divided into three categories: the first one is passengers, arriving when the bus arrives, because they see it, thus, they have 0 waiting time. The second category are passengers who optimize their waiting time, and the third category are passengers, arriving at random.

To improve the quality of the transport service, in [13] a smart urban transit systems is given. In this work for improving the transit system and service, evaluation models on service reliability, service accessibility, timetable robustness, and energy consuming are proposed.

In [14] a waiting time strategy is suggested. The results, presented in the document, indicate how the models could be developed, in order to manage to reduce waiting times better.

Researchers' efforts have provided grounds for studying the passenger waiting times, but no one of the models could provide a systematically and quantitatively satisfying picture of passenger waiting time.

Based on experimental research, this paper proposes a method for correction of timetables of vehicles on a given route, based on estimation of real passenger waiting time.

2. A MASS URBAN PASSENGER TRANSPORT SYSTEM AND PASSENGER TRACK TIME

In terms of the theory of systems, mass urban passenger transport is a large system with all its inherent features, which is sufficient for using mathematical modelling.

The mass urban passenger transport has three main characteristics (Fig. 1.)

The state of these characteristics depends on the behavior of their subsystems, which is a function of the state of their elements and the connection between them.

The operation and management of the mass urban passenger transport consists of n routes, served by m types of transport and k bus stops.

Each route is characterized by the matrix of corresponding instances $R_i = \lfloor R_{ij} \rfloor$, where $R_{ij}$is the passenger flow between points i and j (i,j=1,2,....,n); number of vehicles W_z per route z; length S_z of route z; interval of movement I_z per route z.

The aim of the management of transport service system is satisfying the demand for quality transport service and defining the limits of sustainability of this system.

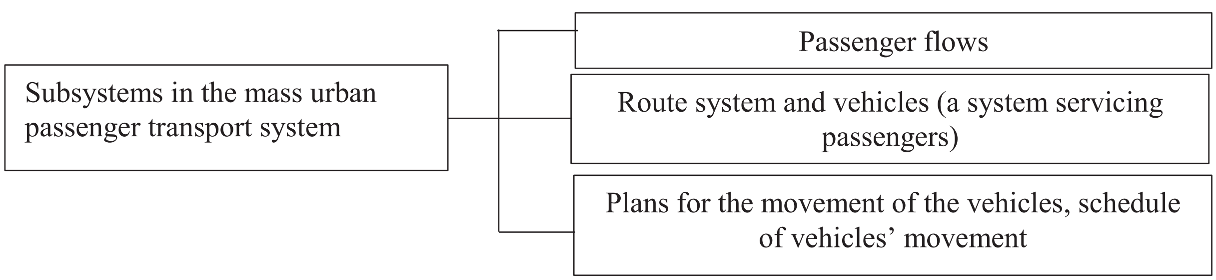

FIGURE 1. *Characteristics of the mass urban passenger transport system*

The quality of system performance is determined as a result of an assessment for the level of needs satisfaction, the main needs being:

- the routes of the mass urban passenger transport coinciding with the numbers of corresponding passenger flows;
- time for moving of passengers to/from the bus stop;
- ensuring regularity of movement on the route;
- ensuring minimum waiting time for vehicle;
- ensuring minimum time for changing from one vehicle to another;
- ensuring minimum travel time on the vehicle;
- ensuring travel comfort, safety and a certain level of filling capacity of the vehicle, etc.

If the above-mentioned indexes are expressed by the total passenger movement time, his/her interests are down to minimization of the values for these indexes

$$minH(\rho), \tag{6}$$

where H is a function of movement times for population.

Thus the task for effective operation and management of urban passenger transport is down to finding solutions, satisfying the interaction of passenger flows, and the number of routes and vehicles that satisfy the condition min H(ρ) for given values of R_{ij}.

Generally, the movement time is a random value, which is a sum of mathematical expectations of the times for:

-walking t_{af1}, t_{af2};

-waiting at bus stops (when changing buses) t_{wi};

-travelling t_{ri}.

Therefore

$$\mathrm{T}_{tr} = E[t_{af1}] + E[t_{af2}] + \sum_{i=1}^{n} E[t_{wi}] + \sum_{i=1}^{n} E[t_{ri}] \tag{7}$$

The mathematical estimation of a random variable ξ is expressed by

$$E[\xi] = \int_{-\infty}^{\infty} x f_{\xi}(x)dx, \tag{8}$$

While dispersion is expressed by $D[\xi] = \int_{-\infty}^{\infty}(x - E[\xi])^2 f_{\xi}(x)dx,$ (9)

where the distribution density of the random value ξ is $f_{\xi}(x)$.

2.1. Determining Mean Passenger Waiting Times at the First Bus Stop

Since the exact law for distribution of random values $t_{чi}$, in (7) is unknown, it is convenient to use the Monte Carlo method, [15]. It is based on the Law of Large Numbers (LLN), fully proved by Chebishev and Hinchin. With the Monte Carlo method, a large number of random experiments (situations), which are similar to the one of interest to us, have been simulated. The idea is that the mathematical expectation of the unknown value is the mean value at the realisation (simulation) of a large number of experiments.

The Monte Carlo menthods vary, but as a rule, they follow a given model (Fig. 2.):

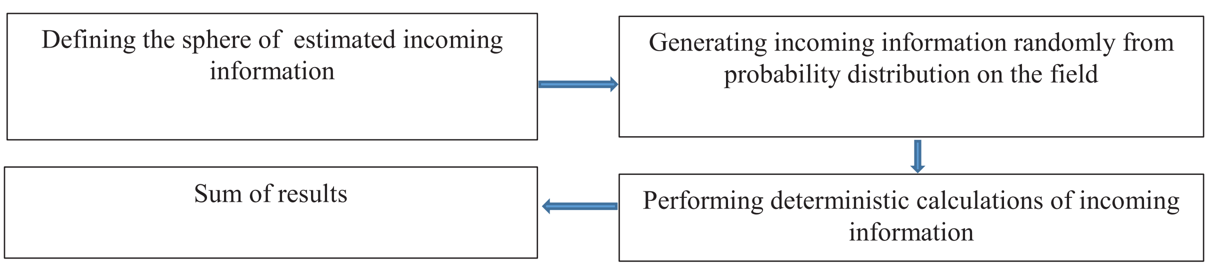

FIGURE 2. *Summarized sequencing model of the Monte Carlo methods*

The Monte Carlo method has two important features:

-simple structure of calculating algorithm;

- Calculation error proportional to $\sqrt{\frac{\sigma^2}{N}}$, where σ^2 is the variance, and N is number of trials.

Thus it can be seen that to reduce the error by one order (10 times) the number of trials should be increased N – 100 times. Therefore, to receive a good enough result, a large number of trials is necessary.

A variant for applying the Monte Carlo method for determining the average waiting times t_{wi} between $i^{-\text{та}}$ and $(i+1)^{-\text{ва}}$ bus stop is shown on Fig. 3 and realized in ten steps.

As incoming data we need the actual bus timetable of public transport for the bus stops under survey and the real times of arrival of vehicles.

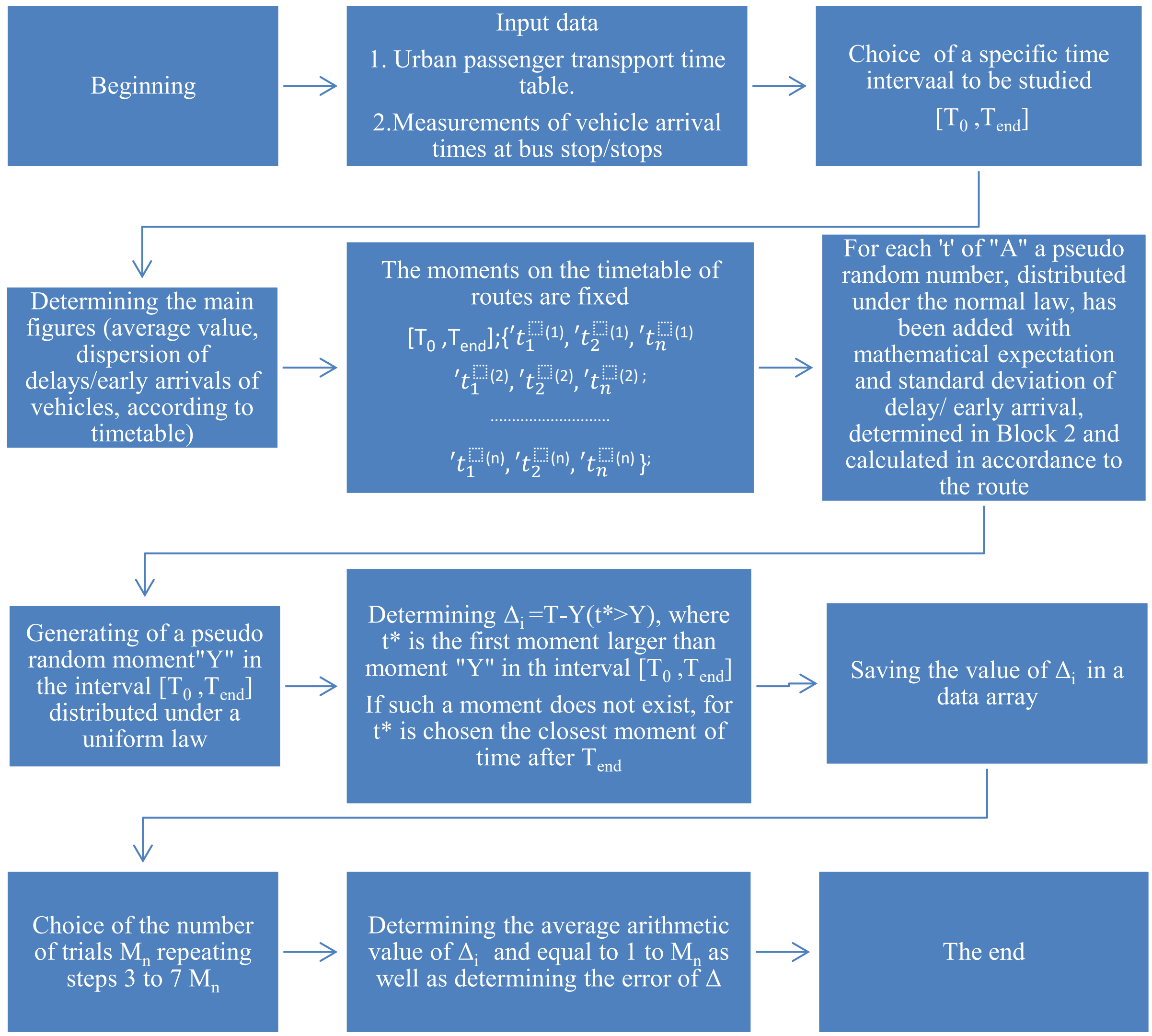

FIGURE 3. *Algorithm for determining the passenger waiting time by applying the Monte Carlo method*

The beginning time (T_0) and the time of ending (T_{end}) between which a random passenger arrives at the bus stop, which is initial for him/her is also needed.

In this time interval [T_0;T_{end}] all times of arrival T1,T2,T3.........Tn at the desired bus stop on routes by the passenger are shown.

To these moments T1,T2..... Tn, pseudo random numbers, distributed under the normal law, have been added

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp((x-\mu)^2/(2\sigma^2)). \tag{10}$$

With mathematical expectation μ and variance σ^2.

The mathematical expectation μ and variance σ^2 of a given moment $T_к$ are approximately known from the statistical processing of delays along the given routes in the starting bus stop.

An evenly distributed random number W is generated with distribution density $f(x) = 1/(T_{end} - T_0)$, in the interval [T_0; T_{end}].

It is checked which is the first moment Tк larger than W, and if there isn't such a moment in the interval [T_0; T_{end}], the next closest moment (larger than T_{end}) of any route is taken.

The difference between $T_к$ and W is the passenger waiting time at one simulation.

A large number of simulations (N) is made and the results are saved in a data array.

The average value of the data from the array is the average waiting time value.

The line of error in the Monte Carlo method is given with $\frac{\sigma}{\sqrt{N}}$,

where N is the number of simulations;

σ is the standard deviation of the pseudo randomly generated numbers.

$$\left|\overline{\vartheta_N} - I\right| \le x_\beta \sqrt{\frac{\sigma^2\theta}{N}} = x_\beta \frac{\sigma\theta}{\sqrt{N}} \quad , \tag{11}$$

where $\overline{\vartheta_N}$ is the value calculated under the Monte Carlo method;

I is the exact value;

$\sigma^2\theta$ – is the largest variance of simulated values;

N is the number of trials;

x_β is quantum of the standard normal distribution, guaranteed by probability β.

Warranty probability β at the value of 0,95 is accepted as standard. In this case the quantum is $x_\beta = 1{,}96$ i.e, the error in the average waiting times is guaranteed with probability $\beta = 0{,}95$.

2.2. Algorithm for Dynamic Correction of a Given Route Timetable

When developing timetables for work on a given route the following scheme is usually used (Fig. 4).

The estimated driving times and downtime along the route serve as starting information for creating timetables.

Driving time estimation is related to the estimation of:

-intervals of movement between bus stops (speeds of movement);

-downtime at the bus stops and the end stops along the route.

The correct time also determines the minimum travel time for the passengers. The unjustified time leads to unjustifiably low speeds, downtime at the interim or end bus stops, due to spare time, violation of traffic and safety rules, or the rules for getting on and off the vehicle, due to lack of time, etc.

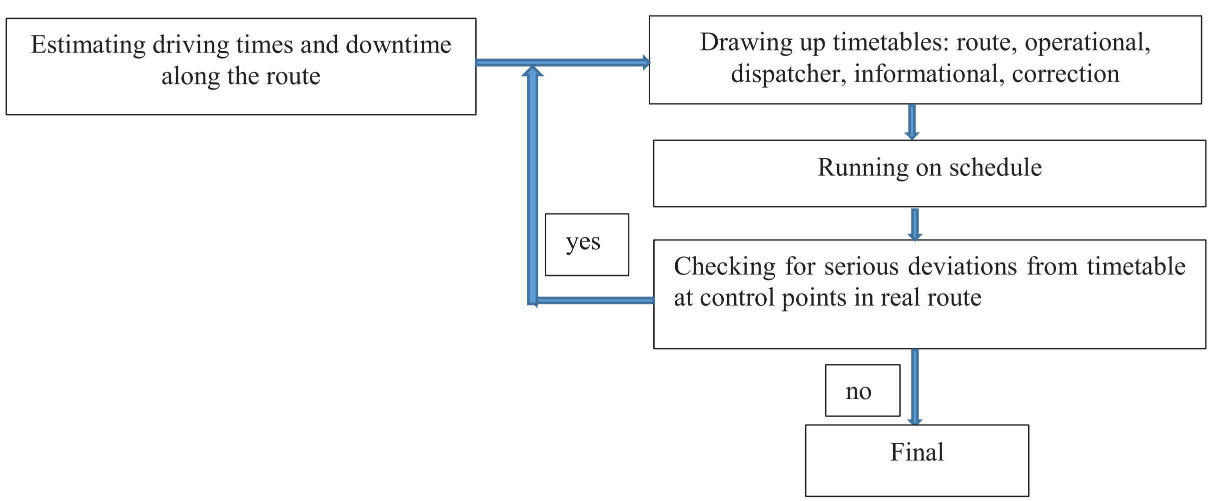

FIGURE 4. *Block scheme of existing practice for developing timetables for the urban public transport*

Various factors influence timing:

- When stops are too close to each other, the driver is not able to accelerate the vehicle properly and, as a consequence, low technical speed is achieved;

- Traction and dynamic properties of vehicles affecting acceleration after the bus stop;
- Construction characteristics of vehicles (doors, height of floor, holders, etc.). Narrow doors, height of floor, lack of holders for passengers slow down their getting on and off the bus and reduce the exchange of passengers at bus stops and, vice versa, the better the construction of the vehicle is the less time is used for getting on and off the bus;
- The volume of passenger flow along the route. Overcrowded vehicles have lower speed;
- The number of passengers passing through one door of the vehicle. The average time for getting on or off the bus in summer is about 2s while in winter it increases by 8-10%.
- The intensity of traffic flow along the route; the state of the transport infrastructure (road cover, etc.); Climatic conditions of traffic. These factors influence the technical speed of movement. During the dark part of the day/night, when there is no street lighting, the technical speed is reduced to 10-15%.
- Restricting speed of movement, due to traffic regulation;
- Experience and psycho-physiological condition of the driver, etc.

To determine the normative travel time of vehicles along the route and the time for covering the whole route, the chronometric method is used once again. With this method we measure the physical time for one lapse from beginning to end and its elements (movement, downtime, delays for different reasons). When chronometric measurement is conducted, a number of conditions are met:

-only the planned vehicles move along the route;

-the road lane must be dry;

-when different types of vehicles are used, the measurement is conducted on the least dynamic one;

-measurements are conducted all work day with subsequent separation of characteristic periods and differentiation of the travel time from beginning to end stop for each characteristic period;

-during the measurements traffic control is cancelled and drivers choose their driving speed, taking into account road conditions and ensuring safety of transportation.

The measurement results are entered into chronometric maps, prepared in advance, taking into account the minimum times measured (of higher weight) and maximum times measured.

Norms are differentiated according to the period of day, the different traffic conditions in the days of the week, the seasons, etc.

In Fig. 5 we present a flow chart for developing timetables of the public transport in cities with corrections depending on the statistical evaluation of waiting times.

2.3. Empirical Evaluation of Passenger Waiting Time by Using the Monte Carlo Method in Ruse

It is of particular importance for the quality of transport service to determine the average passenger downtime, for a passenger arriving at a random moment of the rush hour. The busiest intervals (peak intervals), are between 07:00 – 09:00, and from 17:00 to 19:00, April-May 2020. In our case we investigate the interval between 07:00 and 09:00, from „Petar Karaminchev“ to „Yalta “ bus stops in Ruse.

We review the data for a week day and for a holiday.

The data needed to apply the Monte Carlo method for calculating the average passenger waiting time for a passenger, who wished to move from „Petar Karaminchev“ to „Yalta “ bus stop on a work day, between 07:00 and 09:00 h, are given in Tab. 1.

The error with a standard Monte Carlo method is expressed by $|\overline{\vartheta_N} - I| \leq x_\beta\sqrt{\frac{\sigma^2\theta}{N}}$, [16], also additional study has in [17], where $\overline{\vartheta_N}$ is the value calculated using this method, I is the exact value, $\sigma^2\theta$ – is the largest variance of simulated values, and N is the number of trials. χ_β is the quantum of the standard normal distribution, guaranteed with probability β.

In the specific case, 0.95 is taken for the guaranteed probability β. Then the quantum is χ_β=1.96 i.e, the error in the mean waiting times is guaranteed with probability β=0.95.

When evaluating the error, the highest standard deviation value is taken. Thus the error is increased. With the simulation made in N=100000 trials, for the mean waiting time T_mean1 and an error Error1 for work days in seconds we have obtained:

T_mean1=270.97s;

Error1=0.291s;

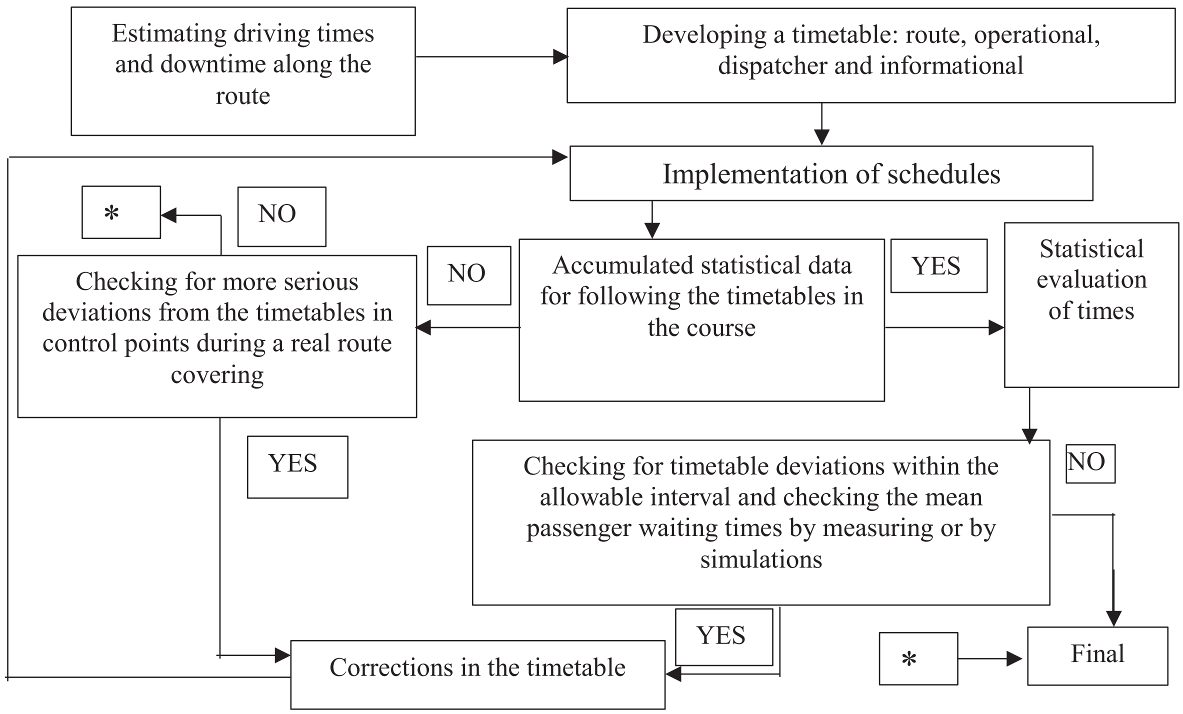

FIGURE 5. *Flow chart for developing timetables for public transport in cities with correction, depending on the statistical evaluation of times*

Error is the error in the mean waiting times on work days and holidays, guaranteed with probability β=0.95.

It can be seen that the waiting time on a work day is comparable to that on a holiday. This could be explained by the fact that during a work day the passenger flow is more intensive, but travel routes are more frequent, contrary to holidays, when intensity is weaker but the travel routes are at longer intervals.

On a holiday, the mean passenger waiting time from Petar Karaminchev to Yalta bus stop is 4.46 min.

TABLE 1. Minutes of arrival after 7 o'clock for the different routes and their delays

Routes	Minutes of arrival at different bus stops after 7 a.m. (min)	Average delay per route, (sec)	Standard road deviation of a route, (sec)
Route 2	4;20;34;49;69;89;109	48.497	139.187
Route 13	14;39;64;84;104	50.469	148.425
Route 21	26;56;76;96;116	-65.902	128.504

In Fig. 6 a model script file is given, which conducts simulations for determining the mean downtime, where „L1,L13,L21" are vectors from the time of arrival after 07:00 for the respective travel routes, while „N" marks the number of simulations.

Abosolutly the same way they were obtained T_mean2 and an error Error2 for holidays:

T_mean2=267.84s;

Error2=0.209s.

The wait time on a weekday is 270.97s and is comparable to that on a holiday day of 267.84s.

```
clear
L1=[4 20 34 49 69 89 109]*60;m1=48.497;s1=139.187;
L13=[14 39 64 84 104]*60;m13=50.469;s13=148.425;
L21=[26 56 76 96 116]*60;m21=-65.902;s21=128.504;
N=1000000;
n1=length(L1);
n13=length(L13);
n21=length(L21);
x=zeros(1,N);
for i=1:N
L1n=L1+randn(1,n1)*s1+m1;
L13n=L13+randn(1,n13)*s13+m13;
L21n=L21+randn(1,n21)*s21+m21;
L=sort([L1n L13n L21n]);
W=rand(1,1)*120*60;
z=L(find(L>W,1,'first'))-W;
if isempty(z)
   z=124*60-W;
end
x(i)=z;
end
Mt=mean(x)
Err=1.96*max([s1,s13,s21])/sqrt(N)
```

FIGURE 6. *Results from simulation with software product Mathlab R2017b*

3. CONCLUSIONS

Maintaining and improving the quality of the transport service for mass urban passenger transport is essential to ensure its attractiveness and use as an alternative to the use of personal transport. The regularity of the movement of vehicles by mass urban transport is an important indicator for assessing the quality of the transport service, so a study of the waiting time of the passengers at a selected bus stop during a work day and a holiday day is made.

The total travel time for passengers is a random quantity, which we determine as the sum of the mathematical expectations of walking time, waiting time and travel time of the vehicle. Therefore, statistical and stochastic approaches need to be used to quantify the quality of the transport service.

In order to determine the average waiting times for passengers, it is appropriate to use the Monte Carlo simulation method. In the Monte-Carlo survey and simulation, the average wait time for passengers at the bus stop is 270.97s on one work day and 267.84s on a holiday. This shows that the two times are practically comparable, due to the difference of about 3s. From the results obtained, adjustments may be made to the urban passenger transport timetables as necessary.

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REFERENCES

1. Q. I. Ahmed, H. Lu and S. Ye. *Urban transportation and equity: a case study of Beijing and Karachi*. (Transportation Research Part A: Policy and Practice, Elsevier, vol. 42(1), 2008), pp. 125-13

2. J.B. Ingvardson, J.K. Jensen, and O.A. Nielsen. *Analysing improvements to on-street public transport systems: a mesoscopic model approach*. (Public Transport, Vol. 9, 2017), pp. 385-409.
3. P. Miller, AG. de Barros, L. Kattan and SC. Wirasinghe. *Analyzing the sustainability performance of public transit*. (Transp Res Part D Transp Environ 44, 2016), pp. 177–198
4. P. Miller, AG. de Barros, L. Kattan and SC. Wirasinghe. *Public transportation and sustainability: a review.* KSCE J Civ Eng **20**(3):1076 (2016).
5. J. B. Ingvardson, O. A. Nielsen, S. Raveau and B. F. Nielsen *Passenger arrival and waiting time distributions dependent on train service frequency and station characteristics: a smart card data analysis*. (Transp Res Part C Emerg Technol 90, 2018), pp.292–306, ISSN 0968-090X
6. X. J. Eberlein, N.Wilson and D. Bernstein. *The Holding Problem with Real-time Information Available*. (Transportation Science, Vol. 35, No. 1, February 2001), pp. 1–18.
7. W. Fan and R. B. Machemehl. *Characterizing Bus Transit Passenger Waiting Times.* 2nd Material Specialty Conference of the Canadian Society for Civil Engineering. Monreal, Quebec, Canada, (2002) pp.1-10.
8. Charles River Associates. *Strategies for Improved Traveler Information.* Transit Cooperative Research Program (TCRP), Revised Draft Interim Report for Project A-20A, National Academy Press, Washington, DC,(2000).
9. A. Tavassoli, M. Mesbah and A. Shobeirinejad. *Modelling passenger waiting time using large-scale automatic fare collection data: an Australian case study*. (Transp Res Part F Traffic Psychol Behav 58, 2018), pp. 500–510.
10. I. Psarros, K. Kepaptsoglou and M. Karlaftis. *An empirical investigation of passenger waiting time perceptions using hazard-based duration models*. (J Public Transp 14(3), 2011), pp. 109–122.
11. M. D. Salek and R. B. Machemehl. *Characterizing Bus Transit Passenger Wait Times*. Southern Region University Transportation Center, SWUTC/99/167211-1 (1999) P. 218.
12. M. Frumin and J. Zhao. *Analyzing passenger incidence behavior in heterogeneous transit services using smartcard data and schedule-based assignment*. (Transp Res Rec J Transp Res Board 2274, 2012), pp. 52–60.
13. K. Lu, B. Han and X. Zhou. *Smart Urban Transit Systems: From Integrated Framework to Interdisciplinary Perspective*. Urban Rail Transit Vol.4, Issue 2, 2018), pp 49-67, ISSN: 2199-6679
14. M. Nygaard and T.Tørset. *Waiting Time Strategy for Public Transport Passengers*. Selected Proceedings from the Annual Transport Conference at Aalborg University Vol. 11 No. 1, 2016), pp.1-16, ISSN 1603-9696. https://journals.aau.dk/index.php/utd/article/view/3840/3309
15. V. Todorov, I. Dimov, R. Georgieva, et al. *Adaptive Monte Carlo algorithm for Wigner kernel evaluation*. Neural Comput & Applic 32, 9953–9964 (2020). https://doi.org/10.1007/s00521-019-04519-9.
16. I. Dimov, R. Georgieva and V.Todorov. *Balancing of Systematic and Stochastic Errors in Monte Carlo Algorithms for Integral Equations*. In: Dimov I., Fidanova S., Lirkov I. (eds) Numerical Methods and Applications. NMA 2014. (Lecture Notes in Computer Science, vol. 8962. Springer, Cham, 2015), pp.44-51.
17. Dimov I., Todorov V. *Error Analysis of Biased Stochastic Algorithms for the Second Kind Fredholm Integral Equation.* In: Margenov S., Angelova G., Agre G. (eds) Innovative Approaches and Solutions in Advanced Intelligent Systems. (Studies in Computational Intelligence, vol 648. Springer, Cham, 2016), pp.3-16.