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Price Forecasting and Risk Portfolio Optimization

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Abstract. Nowadays, the stocks trading is very popular. That is why the problem of forecasting assets' prices is of a special scientific interest. ARIMA (Autoregressive Integrated Moving Average) models for forecasting the stock prices are presented in this paper. For every model the expected return of the shares is calculated and the variance of the rate of returns is analyzed based on a given historical data. Quarterly data on stock prices of the four biggest banks in the United States, that are classified by total assets,areexamined for the period 01.01.2014 – 01.04.2019. An optimization problem is formulated, that is based on Harry Markowitz's model. The solution of this problem leads to finding an optimal risk portfolio for one period ahead and gives an estimate value of the expected rate of return. Depending on the coefficient of risk aversion, a comparative analysis of the structure of a complete portfolio of a risky and a risk-free asset is made. A Matlab programming code is developed, giving the results for an optimal risk portfolio with *n* assets.

INTRODUCTION

Stocks are part of every company's capital. Trading with them could be accomplished through stock markets. There, if a company wants to increase its capital, it can offer newly published shares to the investors, so they can trade with them. Nowadays, there is a growing interest in the stock markets. Investors must have very strong knowledge of how equities are traded on the stockexchange in order to be successful. By developing more and more online trading platforms, it becomes more easily to get into this and to start.

Equities represent a portion of the ownership of a public company and make up its worth or market cap. Stocks trading is the purchase and the sale of company'sequityin the hope of making a profit. The trading of shares is one of the most popular and best-known markets in investing, alongside forex and commodities.

Investors can use different strategies but there are four, say, winning strategies for equities trading.

- **Active trading during the day.**That is short-term strategy that aims to extract all kind of advantages of equities' prices movement on the Exchange's trading day. It brings high level of risk and often the strategy is highly volatile because of the prices.
- **Trading through usage of a technical analysis.**This strategy is used both in the long and short term. Historical data on assets' prices are used in order certain relations to be tracked. Based on this data, predictions about the future moving of the prices can be made.
- **Dollar-Cost Average.**Long-term strategy in which the invested amount in anequityis fixed.
- **Buy and Hold.**One of the oldest strategies where investors buy so called 'blue-chip equities'.

The problem of forecasting asset prices is of a special scientific interest. Some widely used theories in asset pricing could be found in [1].In contrast to government bonds, the future payoffs of equities are not known today. This presents researchers with an additional layer of uncertainty that has to be addressed in order to understand the pricing of equities [2]. This aspect becomes particularly relevant when trying to ascertain whether or not equity markets are fairly valued. One way of getting at the valuation question is through the estimation of equity risk premium and comparison of these premiums with historical averages.

Portfolio management is the art and science of making decisions about investment mix and policy, matching

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investments to objectives, asset allocation for individuals and institutions, and balancing risk against performance. The current price ofa given risk is obviously of a major concern to financial managers. Risk management can quickly become a gamble if models are not understood and the complexity of dependencies and their impact on risk is underestimated [3, 4, 5, 6].

The portfolio of each investor is composed of different types of assets. In addition to their direct investment in financial markets, investors rely on pension funds, life insurance policies with savings components, housing, human capital [7]. In practice, different types of insurance contracts are often used as hedging instruments as they offset exposure of the investor to a particular source of risk [8,9,10].

The portfolio optimization problem [7] with *n* risky securities and one risk-free asset can be summed up briefly as follows. The available risk-return combinations are determined from the set of the risk assets. Then an optimal portfolio of risk assets is determined, including the portfolio weights that yield the steepest capital allocation line (CAL). Finally, an appropriate complete portfolio is selected by mixing the risk-free asset with the optimal risk portfolio.

The choice of a certain complete portfolio [11] depends on the level of risk the investor is able to endure. Several qualitative psychological methods suggest that one should determine what is the most appropriate risk for them [12]. Some tests (the PASS test by W.G. Droms, the Baillard, Biehl& Kaiser test, the test of Barnewal, the Bonpian test, *etc.*) help to determine the psychological profile of an investor. The oldest rule is based on the following equation: 100 – age = the percentage to invest in risky assets. A quantitative and practical method is to use the risk aversion coefficient, *i.e.*, to attribute a number from 1 (lowest risk aversion) to 5 (highest risk aversion) to an investor, which is made in the current work.

In this paper, technical analysis is used for creating an investment portfolio. ARIMA (Autoregressive Integrated Moving Averages) models are presented to forecast the prices of the Big Four American Banks' shares. A Matlab programming code is developed for optimizing a risk portfolio with *n* risk assets. This code is applied to the data for the considered American banks. Various options for choosing a complete portfolio are considered depending on investors' degree of risk aversion.

The data is analyzed using the software products SPSS [13, 14] andMatlab [15, 16].

PROBLEM FORMULATION

For analytical purposes, quarterly data on equityprices of the four biggest banks in the United Statesis examined for the period 01.01.2014 – 01.04.2019.The banks are classified by the financial results fortheir total assets by the end of 2018 shown in Table 1.

J. P. Morgan Chase & Co. is the largest multinational investment bank and financial services company in the United States. The giant's total revenue worth \$109.029 billion with an annual net income of \$32.474 billion. Morgan Chase's total assets worth \$2.623 trillion and total equity worth \$256.52 billion.

Bank of America is the second largest bank in the United States. Its total revenue is \$91.94 billion and net income of \$28.14 billion from which total assets are \$2.325 trillion and total equity \$264.74 billion.

The third largest bank in the United States, Citigroup Inc. has total revenue of \$72.854 billion and net income of \$18.080 billion. The total assets of Citigroup Inc. are \$1.917 trillion, and its total equity is only \$197 billion.

Wells Fargo is an American multinational financial services company and is the fourth largest bank by total assets in United States. Its total revenue is \$86.40 billion and the net income is \$22.39 billion from which the total assets are worth \$1.895 trillion, and the total equity is \$197.06 billion.

TABLE 1. The Big Four – financial results at the end of 2018

Bank	Total Revenue (billion \$)	Annual Net Revenue (billion \$)	Total Equity (billion \$)	Total Assets (trillion \$)
JP Morgan	109.029	32.474	256.52	2.623
Bank of America	91.94	28.14	264.74	2.325
Citigroup	72.854	18.080	197	1.917
Wells Fargo	86.40	22.39	197.06	1.895

The prices for the equities of each bank are taken from the financial website Yahoo Finance [17].

ARIMA MODELS

Due to the nature of the data, the various methods for modeling time series are suitable for the study and the forecasting of equityprices. These methods include classical time series models,autoregressive patterns (AP), moving average (MA), autoregressive-moving average (ARMA), autoregressive integrated moving averages(ARIMA) [18]. ARIMA and Brown models are used for the purposes of the study. Similar models are used in various areas, including finance, economics, energy industry, transport [19], *etc.*

Three parameters are used to build the ARIMA models – *p, d*and*q* [18]. The autoregressive element *p* is the impact of the data from *p* previous moments in the model. The integrated element *d* is the trend in the data while the element *q* shows how many members are used to smooth small fluctuations with the help of a moving average.

In the general case, one ARIMA model with parameters *p, d* and *q*, can be shown by [18, 20]

$$Y_t = C + \varphi_1 \Delta^d Y_{t-1} + \cdots + \varphi_p \Delta^d Y_{t-p} - \theta_1 \varepsilon_{t-1} - \cdots \theta_q \varepsilon_{t-q} + \varepsilon_t,$$

where $C,\ \varphi_i\ , i = \overline{1,p},$ and $\theta_j\ , j = \overline{1,q}$,are the parameters sought, ε_j e is a randomly distributed value with a zero mathematical expectation and dispersion σ^2. If there is no information about the distribution, then it is assumed by default to be normal [20].Δ is the difference operator, which is defined as

$$\Delta^0 Y_t = Y_t, \Delta^1 Y_t = Y_t - Y_{t-1}, \ldots, \Delta^k Y_t = \Delta^{k-1} Y_t - \Delta^{k-1} Y_{t-1}.$$

The following procedure is used in the current work in order to identify the time series. The smallest parameter values are sought, starting from ARIMA (0, 0, 0). When the value for a parameter is 0, the element is not needed in this model. The element *d* (trend) is analyzed before *p* and *q*. The purpose is to determine whether the process is stationary, and if not – to turn it into stationary by removing the trend before determining the values of *p* and *q*.

The graphs of auto-correlation functions (ACFs) and partially autocorrelation functions (PACFs) are examined for each combination of parameters (p, d, q). These functions depend on a fixed number of lags and are calculated for each moment *t*, exception some end ones, where they cannot be calculated. If for Y_tACF and PACF have jumps outside the confidential intervals, then a new choice of parameters is made. Large jumps and repeating models in autocorrelation and partial autocorrelation functions show the approximate values of *p* and *q* in the ARIMA models, which is used for the identification of these parameters in the current work.

The formula for the autocorrelation function ACF in a current moment *t* for *k*-lag has the following type

$$r_k = \frac{\frac{1}{n-k}\sum_{t=1}^{n-k}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\frac{1}{n-1}\sum_{t=1}^{n}(Y_t - \bar{Y})^2},$$

where *n* is the number of observations in the whole series, *k* is the delay (number of lags), $\bar{Y}$ is the average value of the whole time series, and the denominator is the dispersion of the whole time series. The standard autocorrelation error is based on the square of the autocorrelation of all previous autocorrelations.

The formulas for calculating partial correlations are much more complex and include recursive technique [21].

Apart from ARIMA models, Brown's model [22, 23] is also considered as an option for modeling equities' prices. This model is a special case of Holt's model. It is appropriate for those series in which there is a linear trend and no seasonality. Its smoothing parameters are level and trend, which are assumed to be equal. Brown's exponential smoothing is the most similar to an ARIMA model with zero orders of autoregression, two orders of differencing, and two orders of moving average, with the coefficient for the second order of moving average equal to the square of one-half of the coefficient for the first order.

The models are developed on 86.4% of the data (19 randomly selected cases), the rest 13.6%are used for validation (3 cases). The development samples for each bank are different. When a model works well on the development sample, it is tested on the validation sample - a check is made whether the predictions of the model match the actual observations from the validation sample. If the deviations are not large, the model is applied to the entire data set for the corresponding bank, the ACF and PACF graphs are re-viewed, and forecasts are made for the next two periods. From a practical point of view, it can be considered that a maximum relative error on the validation sample to about 15% is acceptable. Moreover, when the model is applied to the entire dataset, the error actually decreases.The normality of the models' residuals is tested for each of the four banks.

Equities Pricing Model for Bank of America

An ARIMA (1,0,3) model is built on the development sample of Bank of America. It could be seen from Figure 1 that there is a small jump beyond the 95% confidence intervals in the ACF residual graph. There are no jumps outside the confidence intervals in the PACF residual graph.

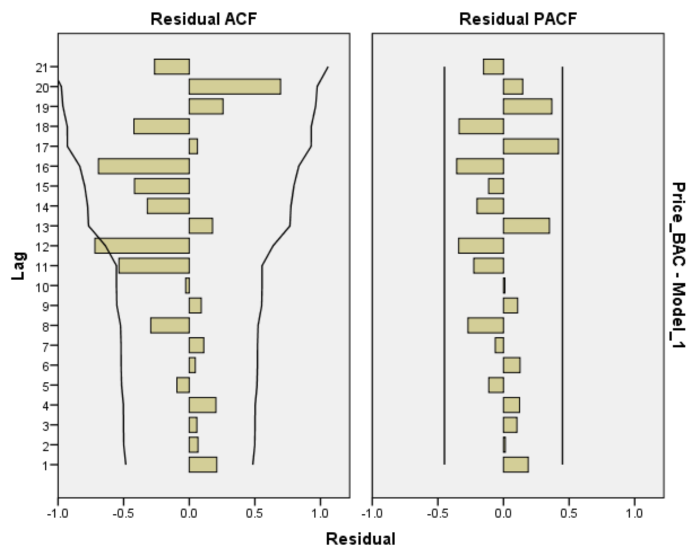

FIGURE 1. Errors and their confidence intervals of the ACF and PACF for ARIMA (1,0,3) on the development sample for Bank of America

Figure 2 represents the real (red) and the approximated (light blue) values for ARIMA(1,0,3) model on the development sample.

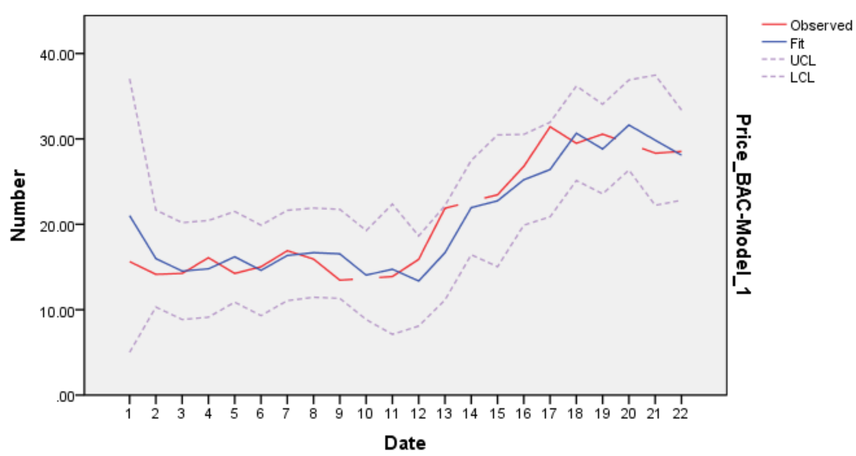

FIGURE 2. Real and approximated with ARIMA(1,0,3) values on the development sample for Bank of America.

Verification of the model on the validation sample shows that the maximum relative error is 15.67% (Table 2).

TABLE 2. Summary statistics for the relative error (%) on the validation sample for Bank of America.

Measure	Value
Mean	6.5514
Std. Deviation	7.9534
Minimum	1.03
Maximum	15.67

The obtained results show that it is appropriate to apply the ARIMA (1,0,3) model to all available Bank of America data and to make a forecast about the future periods. Figure 3 shows that there is a lack of any jumps outside the 95% confidence intervals in the model on all available data in both ACF and PACF residuals.

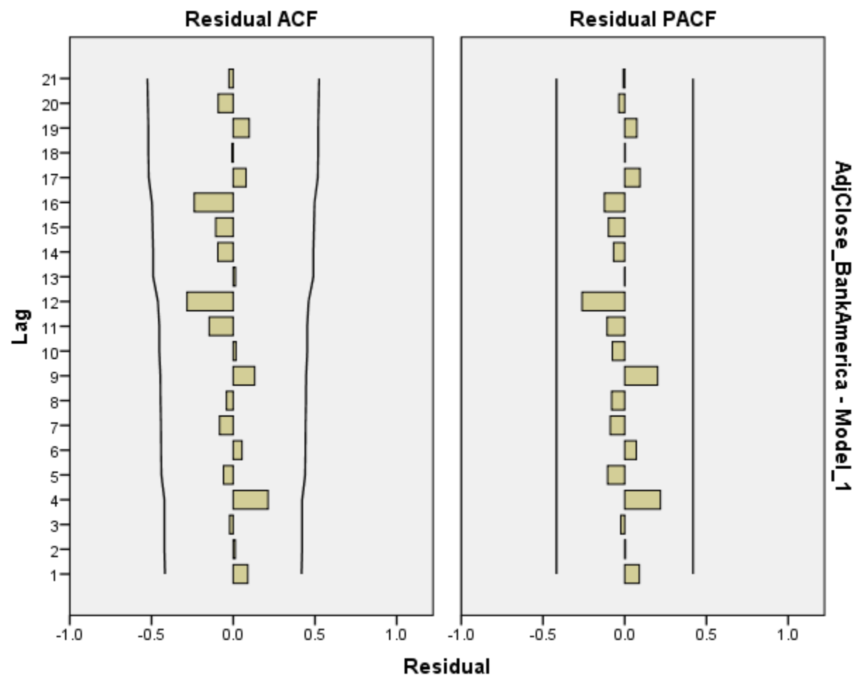

FIGURE 3. Errors and their confidence intervals of the ACF and PACF for ARIMA (1,0,3) on the totalsample forBank of America

Figure 4 shows the real (red), the approximated (light blue) and the forecasted (dark blue) values from the ARIMA (1,0,3) model for Bank of America equities' prices. The graph shows that the model predicts a slight decline in the bank's equity prices in the second and the third quarters of 2019. The normality test on model residuals shows that they are normally distributed.

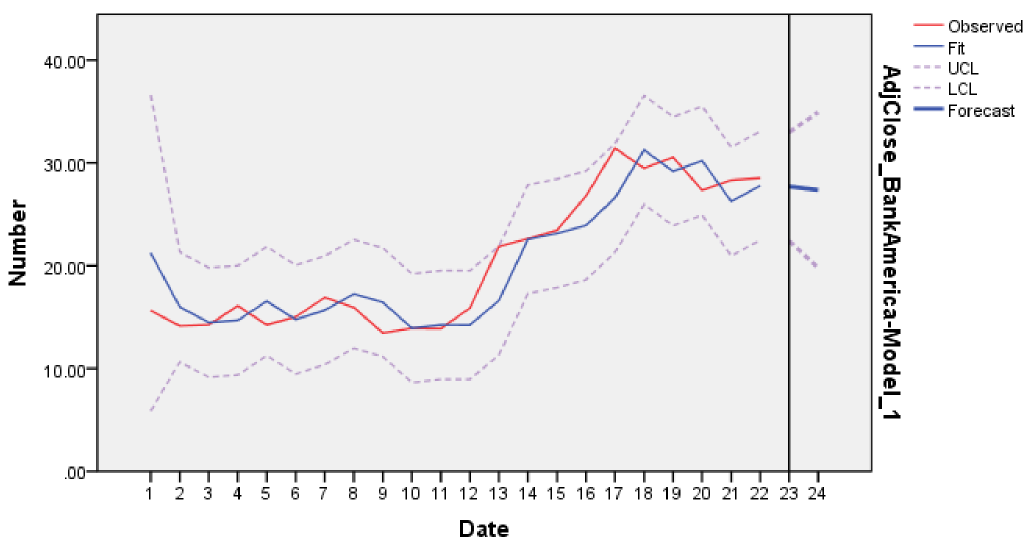

FIGURE 4. The real, approximated and forecasted values from ARIMA (1,0,3) on the total sample for Bank of America

EquitiesPricing Model for Citigroup

Similar to the Bank of America procedure, an ARIMA (1,1,0) model is built on the relevant development sample for the Citigroupequities' prices. It is verified that there is a small jump in the ACF and there are no jumps outside PACF's confidence intervals. The maximum relative error in applying the model to the validation sample is 30%.An error with such measure is not negligible and the results of this model should be considered with attention.

When applying the ARIMA (1,1,0) model to the entire Citigroup data, the ACF and PACF residuals fall within the 95% confidence intervals (see Figure 5).

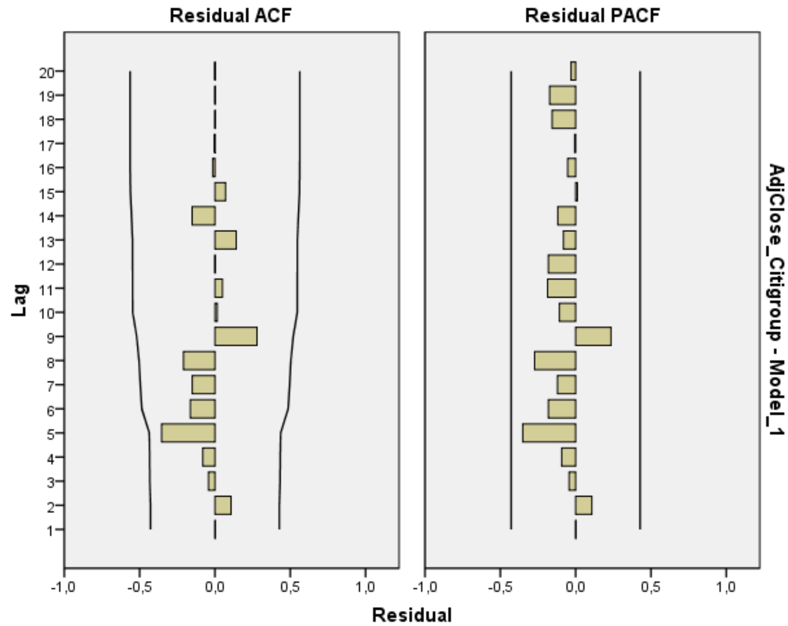

FIGURE 5. Errors and their confidence intervals of the ACF and PACF for ARIMA (1,1,0) on the totalsample for Citigroup

Figure 6 shows that the ARIMA (1,1,0)model predicts a slight increase in Citigroup equities' prices for the second and the third quarters of 2019. It has been verified that the model residuals are normally distributed.

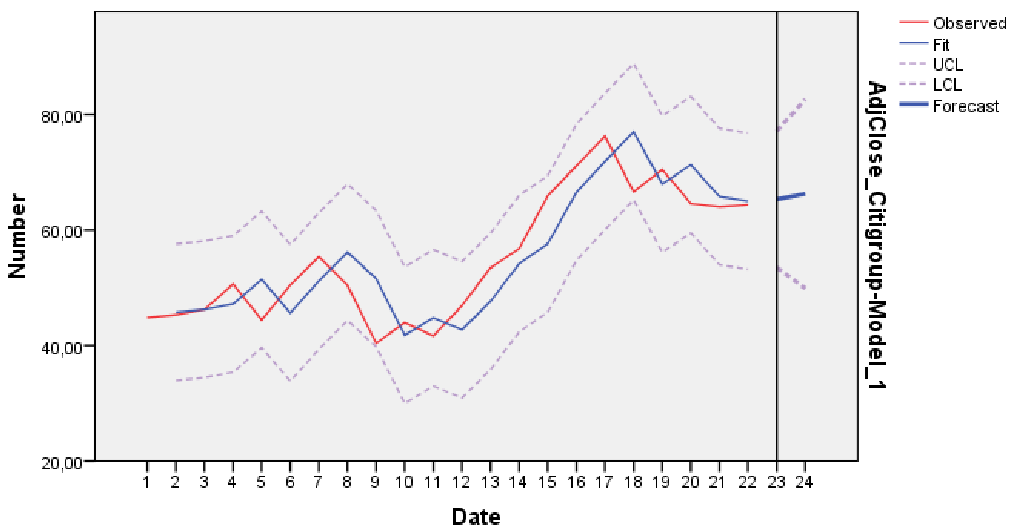

FIGURE 6. Real, approximated and forecasted values from ARIMA (1,1,0) on the total sample for Citigroup

Equities Pricing Model forWells Fargo

An ARIMA (1,1,0) model for the Wells Fargo equities' prices is built on the relevant development sample, following the same procedure as for the other banks. For the development sample, there are no jumps beyond the 95% confidence intervals for the ACF and PACF residuals. The maximum relative error of applying the model on the validation sample is 5.95%.

When applying the ARIMA (1,1,0) model to the entire data, there is a small jump beyond the 95% confidence intervals in both ACF and PACF residuals (see Figure 7).

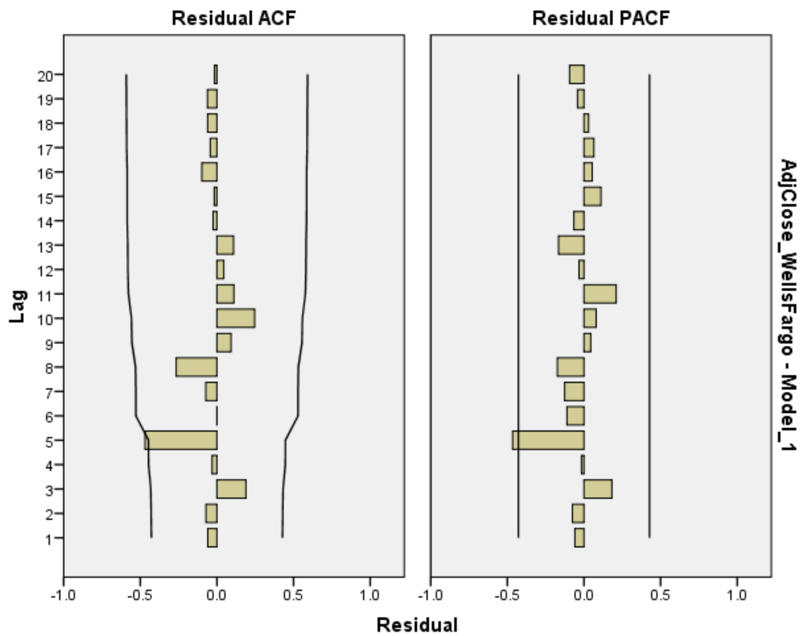

FIGURE 7. Errors and their confidence intervals of the ACF and PACF for ARIMA (1,1,0) on the totalsample for Wells Fargo

Figure 8 shows that the ARIMA (1,1,0)model predicts a slight increase in Wells Fargo equities' prices for the second and the third quarters of 2019. It has been checked that the model residuals are normally distributed.

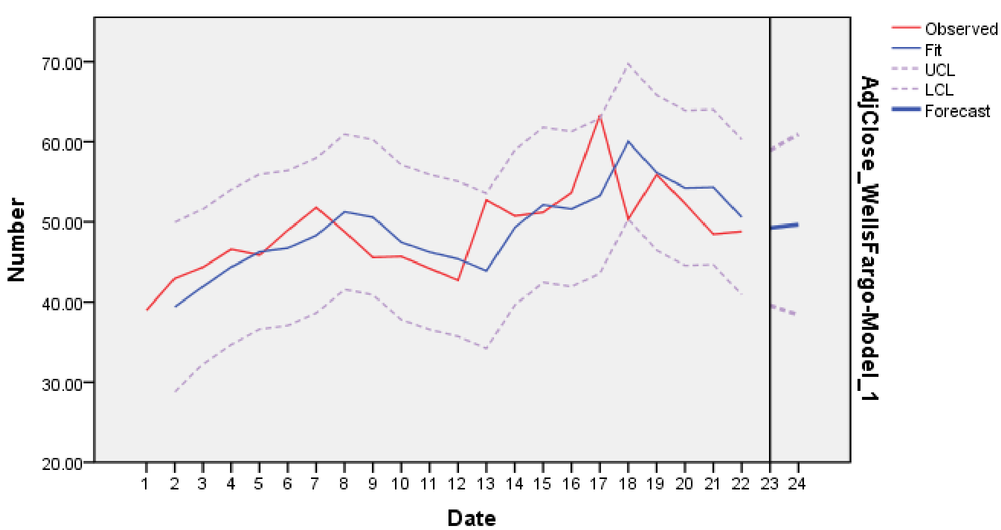

FIGURE 8. Real, approximated and forecasted values from ARIMA (1,1,0) on the total sample for Wells Fargo

Equities Pricing Model forJPMorgan

Like the procedure with the other banks, ARIMA (0,1,0) model is built on JP Morganequities' prices on the relevant development sample. On that sample for the ACF residual there are three jumps, and for PACF residual - two jumpsoutside the 95% confidence intervals. The maximum relative error of applying the model to the validation sample is 12.84%.

Figure 9 shows that when applying the ARIMA (0,1,0) model to all JP Morgan data, there is one jump outside PACF confidence intervals, and for ACF residual there are no jumps outside the confidence intervals.

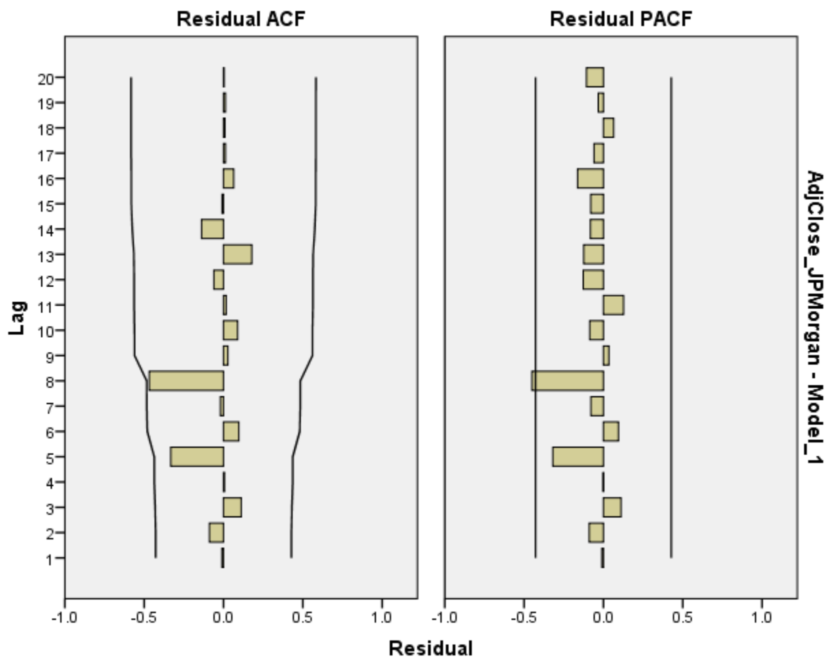

FIGURE 9. Errors and their confidence intervals of the ACF and PACF for ARIMA (0,1,0) on the totalsample for JP Morgan

Figure 10 shows that the ARIMA (0,1,0) model predicts a slight increase in JP Morgan equities' prices for the second and the third quarters of 2019. It has been verified that the model residuals are normally distributed.

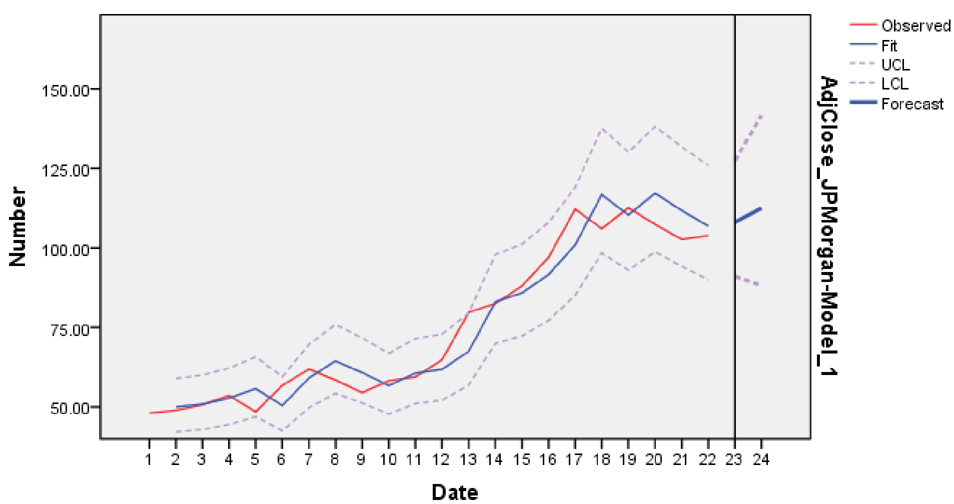

FIGURE 10. Real, approximated and forecasted values from ARIMA (0,1,0) on the total sample for JPMorgan

In JP Morgan equities' price modeling, besides ARIMA models, Brown's model is tested, and it is performing better: the maximum relative error on the validation sample is smaller, and on the total sample, the PACF has no jumps outside the confidence intervals and the ACF decreases to zero as long as the lagsincrease.

For Brown's model, there are no jumps in the ACF residual outside the 95% confidence intervals, and there is a jump in the PACF residual. The verification on the validation sample shows that the maximum relative error is 8.30%. Figure 11 shows that when applying the Brown's model to all JP Morgan data, both in ACF and PACF residuals, the model lacks any jumps outside the confidence intervals.

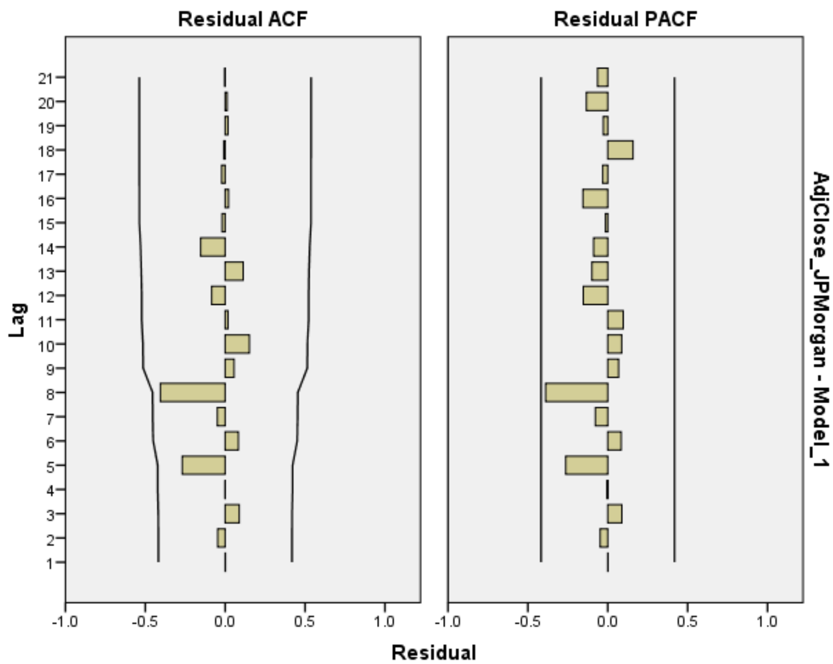

FIGURE 11. Errors and their confidence intervals of the ACF and PACF for Brown's model on the totalsample for JP Morgan

Figure 12 shows that the Brown's model predicts a slight decline in the equities'prices in the second and the third quarters of 2019. A check has been made to show that the model residuals are normally distributed.

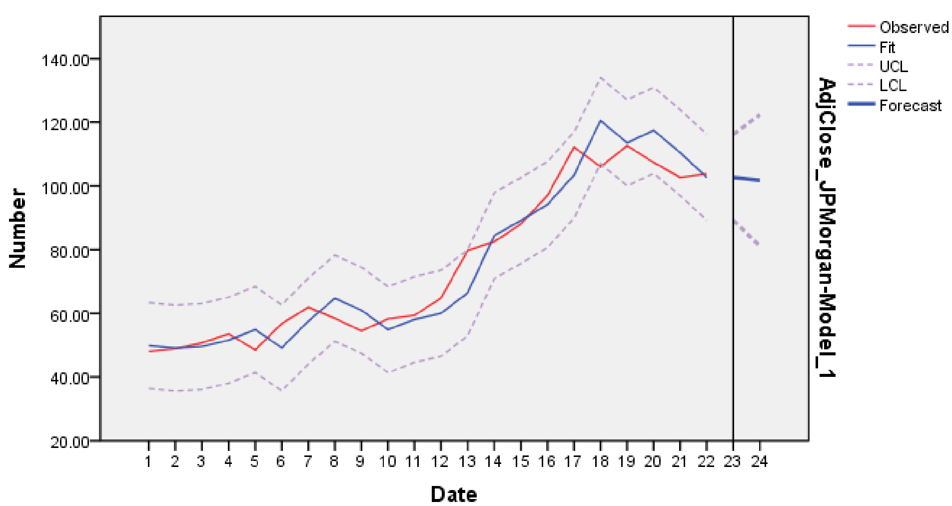

FIGURE 12. Real, approximated and forecasted values from Brown's model on the total sample for JPMorgan

Table 3 summarizes the forecasted prices of the four banks' equities for the second and third quarters of 2019, as well as the real equityprices for the first quarter of the year.

TABLE 3. Models' summary (in dollars).

Bank	**Model**	**Real Price 01.04.19**	**Estimated Price 01.07.19**	**Estimated Price 01.10.19**
JP Morgan	Brown's Model	103.85	102.69	101.74
Bank of America	ARIMA(1,0,3)	28.54	27.71	27.37
Citigroup	ARIMA(1,1,0)	64.36	65.32	66.25
Wells Fargo	ARIMA(1,1,0)	48.81	49.26	49.67

OPTIMIZING THE RISK PORTFOLIO

A Matlab programming code is developed in the present work to create an optimal risk portfolio of *n* risky securities. The goal is to maximize the Capital Allocation Line (CAL) for each eligible risk portfolio p. The code solves the following optimization task:

$$\min F = -\max S_P = -\frac{E(r_p) - r_f}{\sigma_P}$$

$$\sum_{i=1}^{n} w_i = 1,$$

where:

-w_i -weight of the *i*-th asset,

- $E(r_p)$ - the expected rate of return of the risk portfolio; this is the mean value of the expected rates of return of the risk assets, weighted with the corresponding proportion each of them takes in the risk portfolio:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i);$$

- σ_P–the standard deviation of the risk portfolio:

$$\sigma_P = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1, i\neq j}^{n} \sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho(r_i, r_j)};$$

- $\rho(r_i, r_j)$– thecorrelation coefficient between the *i*-th and the *j*-th assetrates of return.

The optimization problem solution is based on a modification of the Markowitz model [7] under the following assumptions: an existence of risk-free assets; borrowing at a risk-free rate; opportunity for short sales of risky assets.

The input data for the developed programming code includes the estimates of expected returns, their standard deviations, itscorresponding correlation matrix, and the return on risk-free asset. The output from the program is an optimal risk portfolio for *n* assets for one period ahead and an estimate of the expected return on the given risk portfolio.

The programming code itself is listed below.

Programming Code in Matlab

```
function [S,W,E,sigp]=fopti_portm(Er,s,Rs,rf)
% %% INPUT DATA   %%%
% Er – Expected Rate of return – n-dimensional vector
% s - Standard Deviation of Rate of return – n-dimensional vector
% Rs - Correlation Matrix – nxn matrix
```

```
% rf – Rate of return on risk-free asset
n=length(Er);
Er=Er(:);
s=s(:);
% As - Covariance Matrix
As=(s*s').*Rs;
% F = -S – optimization function
F=@(X)-(X(:)'*Er-rf)./sqrt(X(:)'*As*X(:));
% X - vector of weights
% Search that X, where min(F)
[W,S]=fmincon(F,ones(n,1)/2,[],[],ones(1,n),1,zeros(n,1),ones(n,1));
%%% OUTPUT DATA %%%
E=W'*Er;S=-S;
% W - weights in the optimal risk portfolio
% Er - estimated rate of returns in the optimal risk portfolio
sigp=sqrt(W'*As*W);
% sigp - standard deviation in the optimal risk portfolio
```

In the present work, the problem of creating a portfolio of equities of the four banks and treasury bills is solved. The horizon of the portfolio plan is one quarter. All estimates refer to the return for a quarter period of ownership. Only estimates for the second quarter of 2019, but not for the third, are used in the optimization task. A more accurate forecast for the third quarter can be made when the real data for the second quarter is known, and then an optimal portfolio canbe created for that period.[1]

The prices of the equities from 01.04.2019 (data for the first quarter of 2019) are taken as the prices for a moment of time *0*. The set of expected rates of return $E(r_i)$ is calculated as follows (Table 4):

$$E(r_i) = \frac{E(P_i^1) - P_i^0}{P_i^0},$$

whereP_i^0 – the prices at time *0* (as of 01.04.2019), $\mathrm{E}(P_i^1)$ - the estimated prices as of 01.07.2019 from the models obtained (Table 3).

For the purposes of the developed program, the sign before therates of return is ignored in the calculations. When interpreting the results, that sign is used to determine whether to play a short (negative) or a long (positive) position in that asset.

TABLE 4. Estimates of the equities' expected rates of return (RoR).

Bank	Expected RoR
JP Morgan	-1.1155%
Bank of America	-2.9178%
Citigroup	1.4842%
Wells Fargo	0.9302%

The variance of the rates of returns, shortly RoR, that is based on the given historical period, is examined and gives information about the estimates of the standard deviations of RoR in Table 5.

TABLE 5. Estimates of standard deviations of the rate of return of equities.

Bank	Std. Deviation
JP Morgan	8.4444%
Bank of America	12.1010%
Citigroup	10.0766%
Wells Fargo	9.5568%

[1]Following the release of data for the second quarter of 2019, the obtained ARIMA models should be validated on the new information, so that new estimates can be obtained. Hence, the new estimates should be entered as an input data in the risk portfolio optimization program for the third quarter of 2019 year.

The correlations between the rates of returns of the analyzed securities are presented in the form of a 4x4 matrix in Table 6.

TABLE 6. Correlation Matrix.

	JP Morgan	**Bank of America**	**Citigroup**	**Wells Fargo**
JP Morgan	1.0000	0.8266	0.3988	0.6317
Bank of America	0.8266	1.0000	0.3226	0.6348
Citigroup	0.3988	0.3226	1.0000	0.1939
Wells Fargo	0.6317	0.6348	0.1939	1.0000

Treasury bills with annual yield of 2.74% are used as a risk-free asset in the current work. Furthermore, the relative interest rate is taken, when assessing the yield on annual treasury bills on a quarterly basis, which is equivalent to 0.685% quarterly yield

$$r_{f_k} = \frac{r_f}{k} = \frac{2.74}{4}\% = 0.685\%.$$

Another approach is to use the conform interest rate

$$r_{f_k} = \sqrt[k]{100 + r_f} - 100 = \sqrt[\frac{1}{4}]{100 + 2.74} - 100 = 0.68\%,$$

which in our case corresponds to approximately 0.68% quarterly yield, and gives similar results to those obtained with the relative interest rate.

Using the Matlab programming code, developed on the input data, that is described above, the following results are obtained. The optimal risk portfolio for the second quarter of 2019 include 86.99% Bank of America equities in a short position and 13.01% Citigroup equities in a long position. In the portfolio for that quarter it is not profitable to hold equities of JP Morgan and Wells Fargo. The expected rate of return on the risk portfolio is 2.7313% and the standard deviation estimate is 11.0196%. The slope of CAL is $S = 0.1857$.

Due to the large relative error on the validation sample for the Citigroup model, a subject of future research by the authors could be to build a risk portfolio with a larger set of assets in order to obtain better risk diversification.

COMPARATIVE ANALYSIS

According to the separation principle, all investors with the same input lists (estimates of rates of return, standard deviations, correlations and returnon the risk-free asset) will hold the same risk portfolio, with the differences in their complete portfolio being only how much each of them allocates to the optimal risk portfolio and how much - to the risk-free asset [7].

Once the optimal risk portfolio is found, there are various options for compiling the complete portfolio of an investor - depending on his riskaversion. In this section a short comparative analysis is presented, based on the structure of a complete portfolio of risk-free and risky assets.

The optimal position in a risky asset is set by the formula

$$y^* = \frac{[\mathrm{E}(r_P) - r_f]}{0.01A{\sigma_P}^2},$$

where

$\mathrm{E}(r_P)$–expected rate of return for the optimal risk portfolio;

r_f – return on the risk-free asset;

σ_P – standard deviation of the optimal risk portfolio;

A – coefficient of risk aversion, $\mathrm{A} \in [1; 5]$. For investors prone to greater risk, A takes lower values, while non-risky individuals prefer higher A's.

Table 7 shows the different structure of the portfolio, depending on the coefficient of risk aversion. The expected return and the standard deviation of the complete portfolio are calculated as follows.

Expected rate of return for the complete portfolio:

$$\mathrm{E}(r_C) = y\mathrm{E}(r_P) + (1 - y)r_f = r_f + y[\mathrm{E}(r_P) - r_f].$$

Standard deviation for the complete portfolio:

$$\sigma_c = y\sigma_p,$$

where y is the weight of the risk portfolio.

TABLE 7. Structure of the complete portfolio depending on the coefficient of risk aversion.

A	Investments in the Risk Portfolio (%)			Investment in the Risk-Free Asset (%)	Expected Rate of Return (%)	Standard Deviation (%)
	Bank of America	Citigroup	Risk Portfolio (Total)			
1	146.5909	21.9238	168.5146	-68.5146	4.1333	18.5696
1.5	97.7273	14.6158	112.3431	-12.3431	2.9839	12.3798
2	73.2954	10.9619	84.2573	15.7427	2.4092	9.2848
2.5	58.6364	8.7695	67.4059	32.5941	2.0643	7.4279
3	48.8636	7.3079	56.1715	43.8285	1.8344	6.1899
3.5	41.8831	6.2639	48.1470	51.8530	1.6702	5.3056
4	36.6477	5.4809	42.1287	57.8713	1.5471	4.6424
4.5	32.5758	4.8719	37.4477	62.5523	1.4513	4.1266
5	29.3182	4.3848	33.7029	66.2971	1.3747	3.7139

It can be noticed that cautious investors ($A \in [4; 5]$) will hold portfolios with less standard deviation (between 3.71% and 4.64%), but with a lower expected rate of return (between 1.37% and 1.55 %). In contrast, risky investors' portfolio ($A \in [1; 2]$) have an expected rate of return between 2.41% and 4.13%, and a standard deviation between 9.28% and 18.57%. Investors with A=1 and A=1.5 should borrow at a risk-free rate to finance a leverage position in the risky asset. For A=1 the leverage position in the risky asset is 168.52%, and the short position in the risk-free asset is 68.52%.

In practice, borrowing at a risk-free rate is possible for government investors. Non-government investors are subject to higher interest rates when borrowing, the cost of the loan to the non-government investor will exceed the interest-free rate. This price will vary depending on the creditor and investor profile, and the calculations with the new borrowing rate will be similar to those in the paper.

CONCLUSIONS

The present work examines quarterly data on equity prices of the Big Four American Banks (classified by total assets) for the period 01.01.2014 - 01.04. 2019. Based on the following data:

1. ARIMA models for evaluating and predicting the prices of the four banks' equities are developed. The models are used to predict stock prices for two quarters ahead (2nd and 3rd quarter of 2019).
2. A Brown's model is also considered as an option for modeling equities prices.
3. Expected rates of return on the equities of each of the four banks are estimated.
4. The standard deviations of the rates of return and the correlation matrix between the returns on the equities of the Big Fourare estimated, using the data of the period under review.
5. A programming code in Matlab is developed that returns an optimal risk portfolio with n assets, where the input data is: expected rates of return, their standard deviations, the corresponding to them correlation matrix, and the return on the risk-free asset.
6. The structure of the optimal risk portfolio (the percentage share of equities of each bank), for one quarter ahead is obtained. The expected rate of return and standard deviation of the portfolio are calculated.
7. Depending on the coefficient of risk aversion, a comparative analysis of the structure of the complete portfolio (consisting of one risky and one risk-free asset) is made.

Such an approach could be used in future studies on the subject as well as in the practice of investors and funds'financial managers.

A subject of future research by the authors could be to build a risk portfolio with a larger set of assets in order to obtain better risk diversification.Furthermore, neural networks could be trained to predict the prices of these assets, and the results could be compared to those from the ARIMA models.

Note: By the time of publishing of this paper, the real prices for the second quarter of 2019 became available. They show that ARIMA(1,1,0) is the most precise model. There are generally slight differences between the estimated and the real prices, which can be explained with the turbulent and complex political situation in the USA.

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