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An Application of Markov Chains in Stock Price Prediction and Risk Portfolio Optimization

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Abstract. Trading with securities and stock indices gains a lot of popularity these days. Every investor's dream is to know the future prices of financial instruments. This paper attempts to apply a Markov chain model to forecast the trends in the stock prices. Markov chain models are obtained for the prices of 3 different stocks based on probability transition matrix and initial state vector. Quantitative data on the daily closing share prices of the stocks is obtained for the period 01.01.2019-31.12.2019. The analyzed stocks are further mixed up in an optimal risk portfolio. An analysis of risk aversion coefficient and how it influences the choice of complete portfolio is made. Such an approach could be applied in future studies on this matter as well as in the practice of portfolio managers and investors.

INTRODUCTION

Nowadays, trading with securities and stock indices gains a lot of popularity among investors. Stock exchange trading gives investors the chance to hedge their market risk and at the same time offers a good investment opportunity, and in some cases they can take advantage of the conditions for arbitration. Portfolio managers must have very strong knowledge of how equities are traded on the stock exchange and also strong statistical and analytical background in order to take advantage of the good opportunities of the stock exchange, in the wright moment, and to be successful.

Portfolio managers could use different strategies for securities trading, most popular of them are the following:

- **Active trading during the day.** Represents the purchase and retention of securities for a period of 24 hours. The strategy is short-term and seeks to reap all the benefits of price movements during a given business day of the financial stock exchange. It brings high level of risk and this strategy is usually highly volatile.
- **Trading through usage of a technical analysis (TA).** This method could be applied in both short and long term. A summary model of the technical analysis states that most of the information related to prices is contained in the market price itself. Therefore, at the time of market processing, the delivery of information plays a specific role, thus leading to constant interactions between traders. The technical analysis concerns the identification of trends, using more or less complex procedures for forecasting of the change in the future price, based on its historical data.
- **Dollar-Cost Average.** Represents long-term strategy for averaging of the purchased value. The invested amount in an equity is fixed. The strategy aims to avoid one-time investments that do not have a well-defined asset price, because that could lead to large losses.
- **Buy and Hold.** This is one of the oldest strategies, in which the investors buy the so called ‘blue-chip equities’. The approach is ideal for people looking for investments with good long-term return.

The problem of predicting stocks’ prices is of a great scientific interest. Furthermore, this problem is subject of debate between experts in two opposing positions. Some widely used theories in stock pricing are given in [1].

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Today we do not know the payoffs of the equities tomorrow. This brings an additional layer of uncertainty to the researchers, which they have to deal with in order to understand the equities' pricing [2].

Experts who believe in efficient-market hypothesis (EMH) are quite skeptical about adequate market predictability, but many skilled traders who actually practice still believe otherwise. There are papers that give a critical assessment of TA [3]. But there are some later studies that show that TA methods are able to overcome market uncertainty [4]. Therefore, forecasting a stock market trend becomes an important activity. Finally, thanks to the ever-increasing power of the computer and the rapid development of large financially oriented databases, the number of modern works and studies focused on TA is increasing. These various methods and studies have the main goal of making a more accurate forecast of the future state of the market and helping traders in making effective decisions for their portfolio [5].

The change in the prices of financial assets can be considered as time series, and in the general case they can be interconnected. An appropriate tool for analyzing these series are the Markov processes. The basic idea behind them is that the past and the future state of the process are independent, and the present state is known. This means that if the current state of the process is known, no additional information about its previous states is needed to make the best possible prediction for the future states of this process. This simplification allows a significant reduction in the number of parameters when studying such a process.

A Markov chain is called a sequence of random events with a finite or countable number of results, characterized by the property - in a fixed current state the next does not depend on the previous states.

The formal definition is:

A number of discrete random variables is called a Markov chain if:

$$P(X_{n+1}= i_{n+1}|X_n= i_n, X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, ..., X_0= i_0)=P(X_{n+1}=i_{n+1}|X_n= i_n) \tag{1}$$

One of the main concerns of financial managers is the current price of a given risk. No matter whether it comes to portfolio management [6, 7], forex trading [8, 9], banking [10, 11], or insurance [12, 13], risk management can quickly become a gamble if models are not set up correctly, and/or the complexity of dependencies and their impact on risk is underestimated.

The task of optimizing a portfolio with *n* risky stocks and a risk-free asset [6] is solved in the present work, using the following steps. First, we determine the available risk-return combinations from the set of the risk stocks. Second, we form an optimal risk portfolio, choosing such weights of the risky stocks in the portfolio, that give the steepest capital allocation line (CAL). Finally, we mix the optimal risk portfolio with the risk-free asset in order to form an appropriate complete portfolio, based on the level of risk the investor is able to endure. Each investor has different preferences on the complete portfolio structure, assuming what is the most appropriate risk for him [14]. There are different tests which help to determine the psychological profile of an investor, for example the Baillard, Biehl & Kaiser test, the test of Barnewal, the Bonpian test, the PASS test by W.G. Droms, etc. The oldest rule is based on the following equation: 100 – age = % to invest in risky stocks. A quantitative and practical method is to use the risk aversion coefficient, i.e. to attribute a number from 1 (lowest risk aversion) to 5 (highest risk aversion) to an investor, which is made in the current work.

The present paper aims to try to predict the trend in the change of the stock market prices of a given financial instrument with the help of models based on Markov chains. A Matlab programming code previously developed by the authors [15] is used for optimizing a risk portfolio with *n* risky stocks. Depending on the risk aversion coefficient, different scenarios for choosing a complete portfolio are considered.

The calculations are made using the software product Matlab [16, 17].

PROBLEM FORMULATION

For the purposes of this analysis, daily data on stock's close prices of 3 companies are collected from [18] for the working days over the period 01.01.2019 – 31.12.2019. This forms a 252 days trading data panel for each stock price.

Apple Inc. (AAPL), formerly Apple Computer, Inc. is an American manufacturer of personal computers, computer peripherals, and computer software. Its total assets worth \$338.5 billion and total equity equal to \$90.49 billion. Apple reported net income of \$55.26 billion in its 2019 fiscal year, the second highest net income to date. It's total revenue worth \$260.174 billion.

The Home Depot Inc.(HD) is the largest home improvement retailer in the United States, supplying tools, construction products, and services. Its total revenue is $27.2 billion and net income of $11.121 billion from which total assets are $44.003 billion and total equity $3.116 billion.

Intel, in full Intel Corporation (INTC)., is an American manufacturer of semiconductor computer circuits. The corporation has total revenue of $71.9 billion and net income of 21.048 billion. The total assets of Intel. are $ 136.524 billion, and its total equity is in the amount of $77.504 billion.

In this paper, the changes in stocks' prices are categorized, Markov Chain Models are obtained to forecast the stocks' prices, and the predicted return rates are calculated for each of the 3 stocks. Furthermore, the variances of the rates of return are analyzed based on the historic data. This information is used as an input data to optimize a risk portfolio, formed from the 3 stocks.

MARKOV CHAINS

The process of transitioning from one state of a system to another with the associated probabilities of each transition is known as a chain [19]. Markov's property says that each step of the chain depends only on the previous state. It is easy to see how the calculation of probabilities from a series (chain) of events in a system is significantly simplified because of this Markov property [20]. Instead of looking at the entire path that an arbitrary variable has traveled to arrive at its current state, we only need to look at its state directly before a point of interest. The probabilities of transition from one state to another form a *mxm* transition probability matrix *T*, where:

$$T=[p_{ij}]=\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix} \tag{2}$$

Every row of *T* is the corresponding probability distribution associated with the transition from state *i* to state *j*. We assume that states *i* and *j* are interconnected if there is a path between them [19]. It must be true that state *i* is accessible from state *j* in a finite number of transitions and also that *j* is reachable by *i* in a finite number of transitions for any two states *i* and *j* in order for them to be interconnected. We will say that state *i* is periodic if all paths starting from state *i* leading back to *i* have a length of aliquot of some positive number *k*, which is possibly the lowest [19]. If all the states of the chain are interconnected and not periodic, then the chain is called ergodic (long-term). The chain has a stable distribution if vector π exists such that for a given transition matrix *T*, the following equation is true:

$$\pi\, T=\pi \tag{3}$$

From Markov's basic theorem, which states that if all elements of the stochastic matrix are strictly positive, then there exist numbers p_j, $0 \leq p_j \leq 1$, $\sum p_j = 1$, such that $\lim_{n\to\infty} p_{i,j}{}^n = p_j$, regardless of the index *i*, thus the limit state of such a chain is unique and is reached at exponential speed [21].

From that theorem and from the fact that in most cases all transition probabilities are positive $p_{ij} > 0$, therefore the existence of this stable vector is guaranteed [19, 22]. The stationary vector π can be considered as a distribution of an arbitrary variable in the long run. The vector π can be obtained as an arbitrary row from the matrix of the following boundary transition [22]:

$$\lim_{n\to\infty} T^n=\begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_m \\ \pi_1 & \pi_2 & \cdots & \pi_m \\ \vdots & \vdots & \vdots & \vdots \\ \pi_1 & \pi_2 & \cdots & \pi_m \end{bmatrix} \tag{4}$$

Models for forecasting the change in the trend of prices

Given the above wording for the transition matrix and its stable state, a system for classification of the change in the prices of financial assets can be created. The idea of using Markov chains to predict stock price behavior is popular, because potential investors are interested in market trends that could lead to an optimal investment strategy.

In this paper, the following three models, based on Markov analysis, are considered, depending on the number of states:

a) The value of the asset decreases or increases;

b) The value of the asset decreases, remains "unchanged" or increases;

c) The value of the asset falls within a given range.

After a specific choice of model, the probabilities for a given state of the model are calculated.

For model a) each day is classified according to whether the closing prices are lower or higher (or equal) than those of the previous day, and thus a distinction is made between the following two conditions (see Figure 1), namely:

- State 1: The closing value is lower than the closing value of the previous day;
- State 2: The closing value is greater than or equal to the closing value of the previous day.

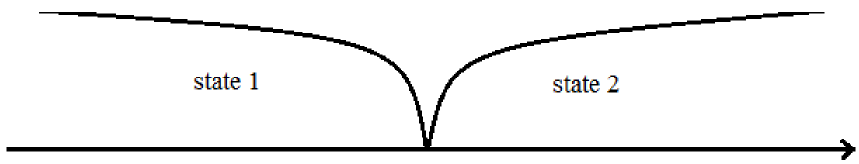

FIGURE 1. *Possible states in model a).*

For model b) each day is classified according to whether the closing prices are lower, higher or remain relatively unchanged compared to those of the previous day, and thus the following three conditions (see Figure 2) are distinguished:

- State 1: The closing value is lower than the closing value of the previous day. (This difference must exceed a certain value, for example, under -2.34 in the case with Apple Inc.);
- State 2: The closing value is "equal" to the closing value of the previous day. (The price difference must not exceed a certain interval, in this case between [-2.34,2.34] for Apple Inc.);
- State 3: The closing value is greater than the closing value of the previous day (This difference must exceed a certain value, for example, up than 2.34 in the case with Apple Inc.).

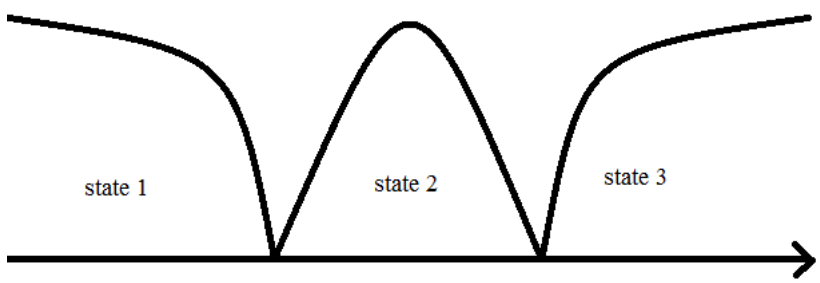

FIGURE 2. *Possible states in model b).*

In model c) 6 states are considered. Profits and losses are divided into three subcategories each: small, moderate and large (see Figure 3). The transitions for this model consist of moving from one category of profit or loss on one day to another category of profit or loss in the next day, namely:

- Status 1: Big jump down (loss greater than 4.68 for Apple Inc.)
- Status 2: Moderate jump down (loss between 2.34 and 4.68 for Apple Inc.)
- Status 3: Small jump down (loss below 2.34 for Apple Inc.)
- Status 4: Small jump up (profit below 2.34 for Apple Inc.)
- Status 5: Moderate jump up (profit between 2.34 and 4.68 for Apple Inc.)
- Status 6: Big jump up (profit greater than 4.68 for Apple Inc.)

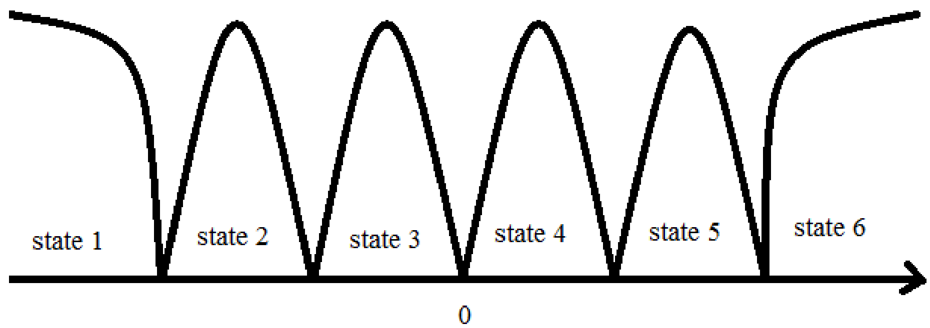

FIGURE 3. *Possible states in model c).*

The intervals in the brackets are exemplary and the choice of specific values depends on the preferences of the investor. It is recommended that the intervals be adjusted to the average daily rises or falls of shares. With Apple Inc. (AAPL) the average daily increases are 2.5142 and the average daily decreases are 2.1588. For this reason, the length of the intervals in Model a) and Model b) are 2.34 (approximate the average value). This model is designed to be more precise than the previous two, because the concept that the market is moving "up" or "down" is now more detailed and better defined.

After calculating the probabilities of being into a given interval, a random variable ξ is constructed, taking values of the means of these intervals. The value of the random variable in the semi-open intervals (the ones in the both ends) is calculated, and these intervals are assumed to be with equal length to the nearest (adjacent) closed interval. In this way, the mathematical expectation of the random variable describing the trend can be calculated. It has the meaning of average daily increases / decreases in assets.

Equities Pricing Model for Apple Inc.

Using the Apple Inc. data, the transition matrix T1 for model a), where the values are rounded to the fourth decimal, looks like this:

$$T_1 = \begin{pmatrix} 0.4340 & 0.5660 \\ 0.4097 & 0.5903 \end{pmatrix}$$

After that:

$$\lim_{n\to\infty} T_1^n = \begin{pmatrix} 0.4199 & 0.5801 \\ 0.4199 & 0.5801 \end{pmatrix},$$

where π_1= (0.4199, 0.5801).

The probability of a average decrease in the price of the equities is higher than that of an increase. It can be concluded that in the near future the trend is to decrease the price of the equities. Based on the given model a) the random variable ξ can be constructed as follows (Table 1):

TABLE 1. *The random variable ξ, describing the average rises / falls of the Apple Inc. equities' prices for model a).*

ξ	P
-2.1588	0.4199
2.5142	0.5801

The mathematical expectation is E [ξ] = 0.5520 average jump.
The transition matrix T2 for model b) looks like this:

$$T_2 = \begin{pmatrix} 0.1875 & 0.5000 & 0.3125 \\ 0.1090 & 0.6538 & 0.2372 \\ 0.1290 & 0.6290 & 0.2419 \end{pmatrix}$$

After the transition:

$$\lim_{n\to\infty} T_2^n = \begin{pmatrix} 0.1237 & 0.6287 & 0.2477 \\ 0.1237 & 0.6287 & 0.2477 \\ 0.1237 & 0.62878 & 0.2477 \end{pmatrix},$$

where π_2 = (0.1237, 0.6287, 0.2477). For the random variable ξ the following distribution is obtained (see Table 2).

TABLE 2. *The random variable ξ, describing the average rises / falls of the Apple Inc. equities for model b).*

ξ	P
-2.34	0.1237
0	0.6287
2.34	0.2477

The mathematical expectation is E[ξ] =0.5804.

The mathematical expectation is positive and is equal to 0.5804, close to that of model a). It can be concluded that the trend in the near future is for the average price of the equities to follow an increase.

For model c) the transition matrix has the form:

$$T_3 = \begin{pmatrix} 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0.1818 & 0.0909 & 0.2727 & 0.2273 & 0.1818 & 0.0455 \\ 0.0270 & 0.1081 & 0.2973 & 0.3108 & 0.1622 & 0.0946 \\ 0.0122 & 0.0732 & 0.3171 & 0.3780 & 0.1585 & 0.0610 \\ 0.0238 & 0.0952 & 0.2857 & 0.3571 & 0.1190 & 0.1190 \\ 0.05 & 0.1 & 0.3 & 0.3 & 0.2 & 0.5 \end{pmatrix}$$

After the transition:

$$\lim_{n\to\infty} T_3^n = \begin{pmatrix} 0.0361 & 0.0883 & 0.2965 & 0.3323 & 0.1671 & 0.0798 \\ 0.0361 & 0.0883 & 0.2965 & 0.3323 & 0.1671 & 0.0798 \\ 0.0361 & 0.0883 & 0.2965 & 0.3323 & 0.1671 & 0.0798 \\ 0.0361 & 0.0883 & 0.2965 & 0.3323 & 0.1671 & 0.0798 \\ 0.0361 & 0.0883 & 0.2965 & 0.3323 & 0.1671 & 0.0798 \\ 0.0361 & 0.0883 & 0.2965 & 0.3323 & 0.1671 & 0.0798 \end{pmatrix},$$

where π_3 = (0.0361, 0.0883, 0.2965, 0.3323, 0.1671, 0.0798). For the random variable ξ the following distribution is obtained (see Table 3).

TABLE 3. *The random variable ξ, describing the average rises / falls of the Apple Inc. equities for model c).*

ξ	P
-5.85	0.0361
-3.51	0.0883
-1.17	0.2965
1.17	0.3323
3.51	0.1671
5.85	0.0798

The mathematical expectation is E[ξ] =0.5743.

The greatest probabilities are observed with a relatively small change in the equities prices, both in the positive and in the negative direction. The mathematical expectation is positive and is approximately the same as that obtained in the previous two models. A similar conclusion can be made that the trend in the near future is for the price of the equities to follow a slight increase of 0.57 units on average.

By the time this article is completed, the data for January 2020 is already known. It shows that the real average rise in share prices of Apple Inc. is positive. Moreover, it is 0.458, which gives a relative error of the forecast compared to the real increase of about 20-25%.

Equities Pricing Model for The Home Depot Inc.

Similarly, the trend of average ups / downs in the near future was assessed for the equities of The Home Depot Inc. (HD). Because model c) is more accurate, a random variable ξ describing the average rises / falls of these prices (and its mathematical expectation) is calculated from this model. The results are shown in Table 4.

TABLE 4. *Random variable ξ, describing the average rises / falls of The Home Depot, Inc. equities' prices for model c), with an interval length of 1.74.*

ξ	P
-4.35	0.0590
-2.61	0.0923
-0.87	0.2768
0.87	0.3395
2.61	0.1734
4.35	0.0590

The mathematical expectation is E[ξ] =0.2596.

This data is validated with the one for January 2020. The average real increase is 0.422, and the positive trend has been successfully predicted.

Equities Pricing Model for Intel Corporation

A random variable ξ describing the average rises / falls of the prices and its mathematical expectation, calculated from model c) for the Intel Corporation equities, are presented in Table 5.

TABLE 5. *Random variable ξ, describing the average rises / falls of the Intel Corporation equities' prices in model c), with an interval length of 0.6.*

ξ	P
-1.5	0.0473
-0.9	0.1004
-0.3	0.3231
0.3	0.3015
0.9	0.1678
1.5	0.0598

The mathematical expectation is E[ξ] =0.0729.

This data is validated with the one for January 2020. The average real increase is 0.1545, and the positive trend has been successfully predicted.

RISK PORTFOLIO OPTIMIZATION

The task of compiling an optimal risk portfolio of the securities of the three companies, which is to be mixed with treasury bills, is solved in the present paper. Ready-made Matlab program code [15] is used for the purpose. The code solves the following optimization task in order to maximize the slope of Capital Allocation Line (CAL) for each eligible risk portfolio *p*:

$$\min F = -\,max\, S_P = -\frac{E(r_p) - r_f}{\sigma_P}$$

$$\sum_{i=1}^{n} w_i = 1,$$

where:

- w_i - weight of the i-th stock,

- $E(r_p)$ - the expected rate of return of the risk portfolio; this is the mean value of the expected rates of return of the risk stocks, weighted with the corresponding proportion each of them takes in the risk portfolio:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$

- σ_P – the standard deviation of the risk portfolio:

$$\sigma_P = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1, i \neq j}^{n} \sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho(r_i, r_j)}$$

- $\rho(r_i, r_j)$ – the correlation coefficient between the i-th and the j-th stock rates of return.

The solution of the given optimization problem is based on a modification of the Markowitz model [6], given the following constraints: risk-free stocks exist; borrowing is possible at a risk-free rate; short sales of risky stocks are allowed.

The input data for the programming code includes the estimates of expected returns based on Markov model c) (Table 6), and their standard deviation (Table 7), based on historical data, as well as the corresponding correlation matrix (Table 8), and the return on risk-free stock.

TABLE 6. *Estimates of the equities' expected rates of return (RoR) from Markov model c).*

Bank	Expected RoR
Apple Inc.	0.5743
The Home Depot Inc.	0.2596
Intel Corporation	0.729

TABLE 7. *Estimates of standard deviations of the rate of return of equities.*

Bank	Std. Deviation
Apple Inc.	0.0165
The Home Depot Inc.	0.0115
Intel Corporation	0.1707

TABLE 8. *Correlation Matrix.*

	Apple Inc.	The Home Depot Inc.	Intel Corporation
Apple Inc.	1.0000	0.2337	0.3029
The Home Depot Inc.	0.2337	1.0000	0.1179
Intel Corporation	0.3029	0.1179	1.0000

Treasury bills with an annual yield of 0.0278, which is equivalent to 0.000077 daily yield (0.0278 / 360), are used as a risk-free stock.

With the help of the program in Matlab [15] the following results were obtained from the input data described so far. The optimal risk portfolio consists of 58.60% shares in Apple Inc. in a long position and 41.40% shares in The Home Depot Inc. again in a long position without investing any shares in Intel Corporation. The expected rate of return on the risk portfolio is 0.4440, and the standard deviation estimate is 0.0117. The slope of the capital alocation line is S = 37.86.

RISK AVERSION

There are three main principles in financial portfolio management, with risk taking central role in all of them:

- Investors avoid risk in general. If there is a risk, they require a remuneration (risk premium) higher than that of alternative risk-free investments;

- Through the function of welfare or utility, the personal compromise choice of each investor between portfolio risk and expected return can be quantified. Investors can assign an utility value to each investment portfolio, depending on its risk and return:

$$U = E(r_c) – 0.005A\sigma_c^2,$$

where
$\mathrm{E}(r_c)$ – expected rate of return for the complete risk portfolio;
σ_c – standard deviation of the complete portfolio;
A – coefficient of risk aversion, A $\in [1; 5]$. For investors prone to greater risk, A takes lower values, while non-risky individuals prefer higher A's.
The multiplier 0.005 is a scale convention that allows the expression of the expected return and the standard deviation from the equation in percentages.

- The risk of an asset cannot be assessed separately from the portfolio in which it is included, i.e. the correct way to assess the risk associated with an individual asset is to assess its impact on the volatility of the entire investment portfolio.

The task of allocating the investor's capital between the risk-free assets and the risk part of the portfolio can be solved using the following steps:

- First, a compromise risk-return choice is determined, which the investor faces when choosing between the risky and the risk-free asset;
- Then the optimal combination of the two assets is determined based on the risk phobia or the degree of risk avoidance of the respective investor.

According to the separation principle, all investors with the same input lists (estimates of rates of return, standard deviations, correlations and return on the risk-free stock) will hold the same risk portfolio. The difference in their complete portfolio is only how much each of them is ready to spend on the optimal risk portfolio and how much - on the risk-free stock [6]. Once the optimal risk portfolio is found, there are various options for compiling the complete portfolio of an investor - depending on his risk aversion.

The optimal position in a risky asset is set by the formula

$$y^* = \frac{[\mathrm{E}(r_P) - r_f]}{0.01A\sigma_P{}^2},$$

where
$\mathrm{E}(r_P)$ – expected rate of return for the optimal risk portfolio;
r_f – return on the risk-free asset;
σ_P – standard deviation of the optimal risk portfolio;
A – coefficient of risk aversion.

Expected rate of return for the complete portfolio:

$$\mathrm{E}(r_C) = y\mathrm{E}(r_P) + (1 - y)r_f = r_f + y[\mathrm{E}(r_P) - r_f].$$

Standard deviation for the complete portfolio:

$$\sigma_c = y\sigma_p,$$

where y is the weight of the risk portfolio.

It is observed that cautious investors (A$\in$ [4; 5]) hold portfolios with less standard deviation, but with a lower expected rate of return. In contrast, risky investors' portfolios (A$\in$ [2; 3]) have higher expected rate of return, and higher standard deviation. Investors with A$\in$ [1; 2] should borrow at a risk-free rate to finance a leverage position in the risky stock, and they will have the highest expected rate of return, but also the highest standard deviation.

In practice, borrowing at a risk-free rate is possible for government investors [6]. Non-government investors are subject to higher interest rates when borrowing, the cost of the loan to the non-government investor will exceed the interest-free rate. This price will vary depending on the creditor and investor profile, and the calculations with the new borrowing rate will be similar to those in the paper.

CONCLUSIONS

Based on the examined data:

1. Three types of Markov Chain Models for forecasting the change in the trend of equities' prices are developed. Based on model c), which is the most accurate one, the examined data is validated, using the one for January 2020. The results show that the upward trend in closing prices is correctly predicted by the models for the three stocks. There are discrepancies in the value of the increase due to factors not reported by the model, including those of force majeure.
2. As a result of the obtained models, the expected rates of return on the stocks of the observed companies are estimated. Furthermore, based on historic data, the standard deviations of the rates of return and the correlation matrix between the RoRs on the stocks are estimated.
3. The percentage shares of equities of each company in the optimal risk portfolio are obtained, and the expected rate of return and standard deviation of this portfolio are calculated.
4. An analysis of risk aversion coefficient and how it influences the choice of complete portfolio is made.

Similar approach could be applied in future studies on this matter, as well as in the practice of portfolio managers and investors.

All predictions should be approached with some degree of reticence, due to the presence of many factors not reported in the model, as well as factors of force majeure [23]. An example of the last are recent events such as:

- The US air strike in early 2020, which led to the death of Iranian major general Kasem Soleimani;
- The Covid-19 pandemic, which brought great uncertainty not only in the financial markets, but also in the political, economic, social and other spheres.

A subject of future research by the authors could be to build a risk portfolio with a larger set of assets in order to obtain better risk diversification. Furthermore, different approaches (ARIMA, neural networks, approximation etc., see [24, 25]) could be tested and compared against Markov Chain Models or used as additional tools in risk portfolio analysis.

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