Modeling the effect of measures to limit the spread of infectious diseases

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Modeling the Effect of Measures to Limit the Spread of Infectious Diseases

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Abstract. This article aims to model and study the effect of the strength, time and duration of the restrictive measures for the spread of an infectious disease. The inconveniences, economic losses and gaps in education are the price that society pays to prevent the spread of the virus. It is important that restrictive measures cover the shortest possible time interval, at the most appropriate time, in order to have minimal negative consequences for society, and at the same time to be effective against the spread of the virus. We consider as a basis the SIS compartmental model for the spread of a virus and apply numerical experiments assuming that, unlike the classic model, the transmission rate α is a monotonically decreasing function of time. Numerical experiments show that earlier introduction, greater stringency and a shorter period of adaptation to restrictive measures until they enter into force would lead to a smaller proportion of infected people, a shorter period of implementation of measures and small economic losses.

INTRODUCTION

Infectious diseases have posed a threat throughout human history. Despite the rapid development of medicine and technology, they continue to be a serious problem today. The civilized way of life weakens the immune system of the human body, and frequent trips outside the local communities create a precondition for the spread of viruses. After the first outbreak of a new infection, it was soon spread to all parts of the world. Medicine does not always manage to find a solution to the problem immediately, without side effects on health. Often, to deal with another virus, it is enough to introduce measures to limit its spread, plus means to increase the body's natural resistance. Such is, for example, the strategy used during each winter period for another influenza virus strain. Enhanced hygiene measures through the use of various disinfectants in public places, limiting close contact with infected people are logical and time-proven techniques. Extraordinary flu holidays for students, absence from work for 3-5 days for people who have shown symptoms are approved and accepted by society. They bring inconvenience, but do not drastically change everyday life for a long period of time, do not lead to large losses. Naturally, hypothetically, if every member of our society isolates itself, viruses will not be able to spread so easily. But it will also no longer be a society. The solution is probably to maintain a reasonable balance in restrictive measures for the spread of a virus, which also depends on the likelihood of cure with natural or medicinal remedies. A 14-day quarantine period for transcontinental travelers could also be established as a practice.

This article aims to model and study the effect of the strength, time and duration of the restrictive measures for the spread of an infectious disease. The inconveniences, economic losses and gaps in education are the price that society pays to prevent the spread of the virus. It is important that restrictive measures cover the shortest possible time interval, at the most appropriate time, in order to have minimal negative consequences for society, and at the same time to be effective against the spread of the virus.

1. METHODS AND MODELS

Compartmental models are widely used among epidemiologists to simulate disease dynamics. The origin of such models is the early 20th century ([1]). Depending on the disease, the compartments can be susceptible (S), exposed (E), infectious (I), or recovered (R). Some infections, for example, those from the common cold, influenza and the recent one COVID-19, do not confer any long-lasting immunity. Such infections do not give immunity upon recovery from infection, and surviving individuals become susceptible to the disease again. The notation SIS is used to describe a disease with no immunity against reinfection, to indicate that the passage of individuals is from the susceptible class to the infective class and then back to the susceptible class. Another commonly used model is SIR (Susceptible – Infectious - Recovered), which suggests that re-infection with the disease is not possible, i.e. the patient dies or recovers, developing immunity against future infection.

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Although the new world-famous virus COVID-19 appeared only at the end of 2019, there are already publications using the SIS and SIR models to simulate its spread ([2] – [8]). Recently, cases of people with established re-infection with COVID-19 have been reported in the media for only a few months after successfully recovering from the initial infection. Therefore, in this article we will use the SIS distribution model as a basis.

Let $I(t)$ denote the proportion of infected persons at the time t. Then $I(t)$ satisfies the differential equation (see [6])

$$\frac{dI(t)}{dt} = \alpha I(1-I) - \gamma I . \tag{1}$$

Here α and γ are the disease transmission rate and the recovery rate.

In the classical SIS model, α and γ are assumed to be positive constants. However, after the introduction of *measures* to *limit* the *spread* of the *virus* (MLSV), the value of α should decrease, not suddenly, but gradually until the MLSV are observed by all members of the population. In the next Section, we apply numerical experiments assuming that the transmission rate α is a monotonically decreasing function of time.

Assume that, during an initial period $[0, t_0)$, no MLSV have been introduced and the value of the transmission rate is α_0. Then at the time t_0 the official implementation of the measures begins and from time t_1 the measures are applied by all, which reduces the value of the transmission rate to α_1 (Fig. 1).

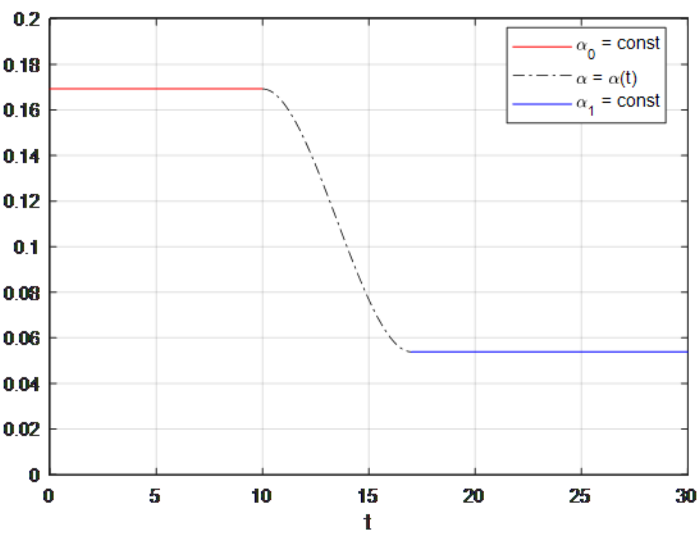

FIGURE 1. The transmission rate $\alpha = \alpha(t)$ before and after the introduction of the MLSV measures

Polynomials are a flexible tool for interpolating functions ([9]). In the interval $[t_0, t_1)$ the transmission rate α can be modelled as a function of time $\alpha = \alpha(t)$ with a polynomial $P_n(t)$, for which the following conditions are met

$$\left| \begin{array}{l} P_n(t_0) = \alpha_0 \\ P_n(t_1) = \alpha_1 \\ P_n'(t_0) = 0 \\ P_n'(t_1) = 0 \end{array} \right. . \tag{2}$$

They would guarantee a continuous first derivative for $\alpha(t)$ in the interval $[0, t^*]$, where t^* is the moment in which the measures are stopped. We will assume that t^* is determined by the condition "reaching a given proportion I^* of infected", I^* is a constant, $I^* \in (0, I(t_0))$.

The simplest polynomial that would satisfy (2) is of degree three, $P_3(t) = b_3t^3 + b_2t^2 + b_1t + b_0$, with coefficients

$$\begin{array}{c} b_3 = \dfrac{2(\alpha_0 - \alpha_1)}{(t_1 - t_0)^3}, \quad b_2 = -\dfrac{3(t_1 + t_0)(\alpha_0 - \alpha_1)}{(t_1 - t_0)^3}, \\ b_1 = \dfrac{6t_1t_0(\alpha_0 - \alpha_1)}{(t_1 - t_0)^3}, \quad b_0 = \dfrac{-\alpha_1 t_0^3 + 3\alpha_1 t_0^2 t_1 - 3\alpha_0 t_0 t_1^2 + \alpha_0 t_1^3}{(t_1 - t_0)^3}. \end{array} \tag{3}$$

The economic losses suffered from the imposed restrictions in the interval [t_0, t*] depend on the severity of the measures, i.e. on the difference $\alpha_0 - \alpha(t)$ and on the duration of their application - $t^* - t_0$.

In his article ([6]), Kahale' assumes that $t_0 = t_1$ and uses the model

$$L = c(\alpha_0 - \alpha_1)(t^* - t_0)$$

for the total economic losses, giving a maximally simple, linear and directly proportional dependence. In this case L can also be written as

$$L = c\int_{t_0}^{t^*}(\alpha_0 - \alpha_1)dt\ .$$

At a time-dependent transmission rate α then we will have

$$L = c\int_{t_0}^{t^*}\left(\alpha_0 - \alpha(t)\right)dt = c\left[\int_{t_0}^{t^*}\alpha_0 dt - \int_{t_0}^{t_1}P_3(t)dt - \int_{t_1}^{t^*}\alpha_1 dt\right]. \tag{4}$$

After performing transformations in (4), using equations (3) for the coefficients of the polynomial $P_3(t)$, the following representation is reached:

$$L = c(\alpha_0 - \alpha_1)\left(t^* - \frac{t_0 + t_1}{2}\right). \tag{5}$$

Alternatively, we can choose the function

$$\alpha(t) = \frac{(\alpha_0 - \alpha_1)}{2}\cos\left(\pi\frac{(t - t_0)}{(t_1 - t_0)}\right) + \frac{(\alpha_0 + \alpha_1)}{2}, \tag{6}$$

to model the values of the transmission rate α in the interval [t_0, t_1). It also satisfies the conditions (2). The values of the polynomial $P_3(t)$ and the function given by (6) are close to each other, because the polynomials of the third degree give a good approximation of the trigonometric functions. For values of the parameters $\alpha_0 = 2.2/13 =$ 0.1692, $\alpha_1 = 0.7/13 = 0.0538$, $t_0 = 10$, $t_1 = 17$ the largest difference in absolute value between the two functions is 0.00115.

For the economic losses at α given by (6) the representation (5) would also be obtained. In general, such a value for L would be obtained whenever the following condition is met

$$\int_{t_0}^{t_1}\alpha(t)dt = (t_1 - t_0)\frac{(\alpha_0 + \alpha_1)}{2}\ .$$

The same economic losses (5) would occur if for instance in the period [t_0, t_1) the transmission rate α is equal to the average $\frac{\alpha_0 + \alpha_1}{2}$ or is modelled by the straight line connected the points (t_0, α_0) and (t_1, α_1).

2. NUMERICAL RESULTS

In the general case, when α and γ are not constants, the differential equation (1) is unsolvable in quadratures. The solution must be found numerically. Matlab's ode113 solver was used, implementing a multi-step Adams-Bashforth-Moulton PECE method with a variable order of accuracy. All numerical experiments were performed at an initial proportion of infected $I(0) = 0.001$, a recovery rate $\gamma = 1/13$ and reaching a proportion of $I^* = 0.0001$ to drop the restrictive measures. For the transmission rate at the beginning α_0 we assume value $\alpha_0 = 2.2/13 =$ 0.1692, and for the constant c – $c = 1$. Similar parameters are considered in [6].

All results regarding the economic losses L and the moment t^* of termination of the constraints depend on the changes of the moment t_0 at which the constraints are introduced, the adaptation time $t_1 - t_0$, as well as on the value of the transmission rate α_1 after the moment t_1 in which the time for adaptation with the introduced MLSV measures ends.

Fig. 2 shows the proportion $I(t)$ of infected persons over time, modelled with the differential equation (1). After reaching the proportion I^* the measures are dropped. This happens at different times t^*, assuming $t_0 = 10$, $t_1 - t_0 = 7$ (i.e. $t_1 = 17$) and different severity of the adopted MLSV measures, determined by three values for α_1, respectively $\alpha_1 = 0.2/13 \approx 0.0154$, $\alpha_1 = 0.5/13 \approx 0.0385$ and $\alpha_1 = 0.7/13 \approx 0.0538$. It can be seen from the graphs that the weaker measures (larger values for α_1) lead to a later moment of time t^* reaching the proportion I^* and dropping the restrictive measures.

Delaying the time of implementation of the measures by 10 days in Fig. 3, assuming $t_0 = 20$, shifts the time of completion of the measures t^* by more than 10 days at all three selected values for α_1. Therefore, the earlier the measures are introduced, the shorter the interval $t^* - t_0$ in which they are applied and the less inconveniences and economic losses for society. It is clear from both figures that stricter measures introduced at an earlier stage can drastically reduce the duration of their implementation.

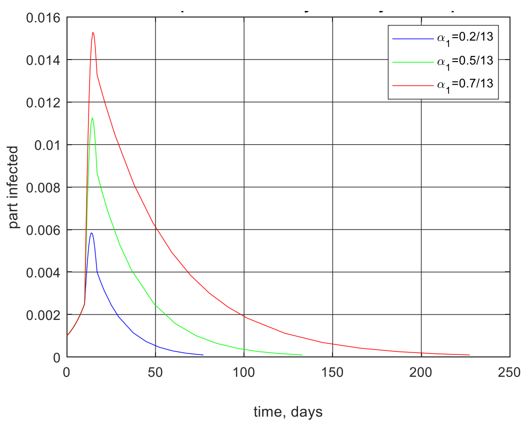

FIGURE 2. The proportion $I(t)$ of infected persons, assuming $t_0 = 10$ and 7 days for adaptation

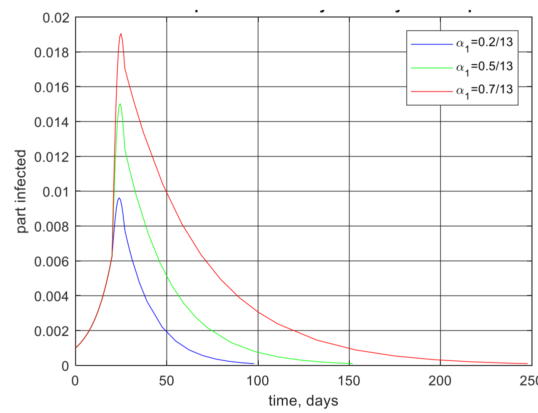

FIGURE 3. The proportion $I(t)$ of infected persons, assuming $t_0 = 20$ and 7 days for adaptation

Increasing the time to adapt to the measures by 7 days in Figures 4 and 5 leads to higher values of the proportion of infected $I(t)$ and delayed (by more than 7 days) end t^* of the MLSV measures.

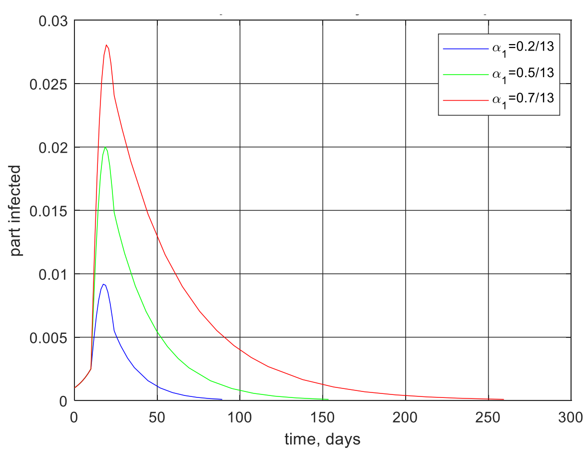

FIGURE 4. The proportion $I(t)$ of infected persons, assuming $t_0 = 10$ and 14 days for adaptation

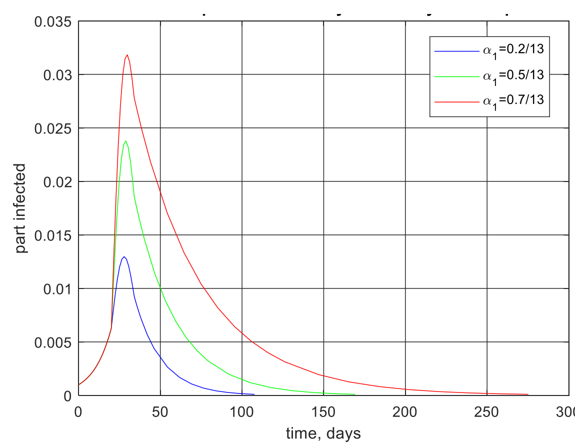

FIGURE 5. The proportion $I(t)$ of infected persons, assuming $t_0 = 20$ and 14 days for adaptation

Graphs of economic losses are presented in Fig. 6 and 7. Figure 6 assumes that $t_0 = 15$ and $t_1 - t_0 = 7$. The economic losses at three different values for α_1 are shown. Here again it is seen that with stricter measures (smaller values for α_1) the interval $[t_0, t^*]$ of application of the measures is shorter and the economic losses are smaller.

The graphs in Fig. 7 are at a fixed value for α_1, $\alpha_1 = 0.5/13 \approx 0.0385$ and $t_1 - t_0 = 7$. They show a linear dependence of economic losses L from the moment t_0 the measures are introduced. Three different values for t_0 are considered: 10th, 15th and 20th day. The shortest interval $[t_0, t^*]$ and the least economic losses are observed at the value $t_0 = 10$, representing the earliest introduction of the MLSV measures.

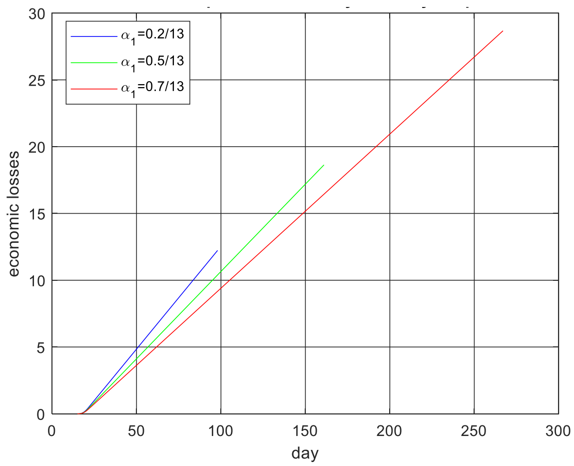

FIGURE 6. The total economic losses L, assuming $t_0 = 15$ and 7 days for adaptation

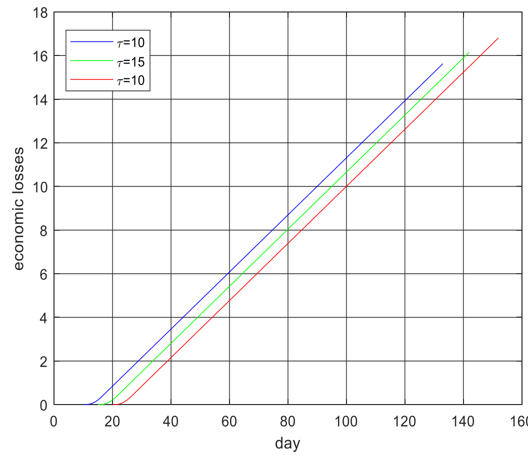

FIGURE 7. The total economic losses L, assuming $\alpha_1 = 0.5/13$ and 7 days for adaptation

3. CONCLUSIONS

We consider as a basis the SIS compartmental model for the spread of a virus and apply numerical experiments assuming that, unlike the classic model, the transmission rate α is a monotonically decreasing function of time. Several possible representations of α as a function of time are discussed. A model of the total economic losses in production due to the restrictive measures taken for the spread of the virus is derived.

Numerical experiments show that earlier introduction, greater stringency and a shorter period of adaptation to restrictive measures until they enter into force would lead to a smaller proportion of infected people, a shorter period of implementation of measures and small economic losses.

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