REER REVIEW

for the competition for the academic position "Professor"
for the needs of the Institute of Mathematics and Informatics
at the Bulgarian Academy of Sciences
announced in Newspaper of State, No. 84 of 21.10.2022
Area of Higher Education: **4. Natural Sciences, Mathematics and Informatics**
Professional Field: **4.5. Mathematics**
Scientific Speciality: **Algebra and Number Theory (Transformation Semigroups)**

The peer review is written by **Prof. D.Sci. Vesselin Drensky, Full Member of the BAS**, retired professor at IMI – BAS, member of the Scientific Jury for the competition by Order No. 536/20.12.2022 of the Director of IMI – BAS.
The only applicant who has applied for the position is **Assoc. Prof. Dr. Habil. Jörg Koppitz** at the Institute of Mathematics and Informatics – BAS.

1. Biographical data. Assoc. Prof. Dr. Jörg Koppitz was born in Halle (Saale), Germany, where he graduated as a teacher in mathematics and physics at the Pedagogical University (now part of the Martin Luther University Halle-Wittenberg). Under the supervision of Reinhard Thron he defended his Ph.D. thesis (doctor rerum naturalium) on the topic "Über Halbgruppen mit vereinigungshalbdistributivem Unterhalbgruppenverband" at the University of Potsdam. The diploma is legalized by the BAS. From then until 2017, the entire scientific career of Assoc. Prof. Dr. Koppitz was connected with this university, where he held various academic positions. In 2002 he completed his habilitation there, defending a thesis for habilitation (doctor rerum naturalium habilitatus) on "M-solide Varietäten von Halbgruppen". According to the requirements, this scientific degree is close to the Doctor of Science degree in Bulgaria. From October 2015 until he started working at IMI – BAN, Assoc. Prof. Dr. Koppitz was a private Associate Professor at the University of Potsdam. He has lectured and led exercises of problem solving in a number of basic and special courses at the University of Potsdam. He supervised 7 successfully defended doctoral students (one from Germany, Bulgaria and Indonesia and 4 from Thailand). The connection with the University of Potsdam continues also today. Currently, Assoc. Prof. Dr. Koppitz is the supervisor of a Ph.D. student there. He is also the cosupervisor of a Ph.D. student in Thailand. After winning a competition in 2017, Assoc. Prof. Dr. Koppitz started working as an Associate Professor at IMI – BAS. I would especially note that the relations of Assoc. Prof. Dr. Koppitz with Bulgaria date back to 1997, when he published a paper in Serdica Mathematical Journal and in 2001 he published a joint paper with Slavcho Shtrakov. In 2006, Ilinka Dimitrova defended her Ph.D. thesis in Potsdam under the supervision of Assoc. Prof. Dr. Koppitz. Three times, for totally 21 months, he had a scholarship in Bulgaria under the Feodor Lynen fellowship and the Humboldt Foundation. Assoc. Prof. Dr. Koppitz is the author of 83 papers in refereed journals, 13 papers in proceedings of scientific conferences and one monograph. Of the papers, 7 are without coauthors (the last one was published in 2015), and the rest are joint with his students or with other established experts in the field. Again, the cooperation with Bulgarian algebraists impresses: 17 papers are joint with Ilinka Dimitrova (including those with other coauthors, one of the papers is joint with Kalcho Todorov), 7 are with Slavcho Shtrakov, and in the list of publications presented in the documentation there is one omitted paper with Slavcho Shtrakov and Dimiter Kovachev (referenced in Zbl 1287.03114). The presence of many joint papers speaks of successful

teamwork. This is a quality which I personally value very much. Moreover, many of the papers are coauthored with students of Assoc. Prof. Dr. Koppitz, i.e. he is also a successful team leader. But I would recommend the applicant to publish more papers as the only author, which would only increase his authority both in the Bulgarian algebraic collegium and abroad. Assoc. Prof. Dr. Koppitz also presented a list of 49 conferences at which he participated with a talk (including 4 times as an invited speaker). In addition to the regular participation in seminars at the University of Potsdam, he has repeatedly delivered series of lectures in Brno, Szeged, Blagoevgrad, Lisbon and Lugansk. Since 2008 he is a member of the Editorial Boards of two journals: Asian-European Journal of Mathematics (published by World Scientific, Singapore) and the published in Poland Discussiones Mathematicae, General Algebra and Applications.

2. Description of the presented documentation. The documentation presented by the applicant Assoc. Prof. Dr. Koppitz is in accordance with the requirements of the law and the accompanying rules of the Institute of Mathematics and Informatics. It contains: the statement that he wants to apply for the position, CV, copies of the diplomas for M.Sci., Ph.D., habilitation and Associate Professor, lists of all publications and the publications presented for the competition, a document confirming the scientific position he has at IMI, the announcement for the competition in Newspaper of State, proofs that he covers the minimal scientific requirements of the law for the position, description of the scientific and pedagogical activity of the applicant. The scientific activity will be discussed below. The applicant has presented for the competition 17 scientific publications in Bulgarian and foreight journals. All papers are in journals with impact factor with total impact factor 9.755. The papers were published in the period 2016 – 2021, are in the field of the competition and I accept them for reviewing. All papers are joint, 10 are with one coauthor (3 with Ilinka Dimitrova and with Ananya Anantayasethi, 1 with Tiwadee Musunthia, Yurii Zhuchok, Anatolii Zhuchok and Slavcho Shtrakov), 4 are with two coauthors (Anatolii Zhuchok and Yuliia Zhuchok, Vitor Fernandes and Tiwadee Musunthia, Somnuek Worawiset and Somchit Chotchaisthit, Ilinka Dimitrova and Laddawan Lohapan) and 3 are coauthored by three (2 with Ilinka Dimitrova, Vitor Fernandez and Teresa Quinteiro, 1 with Dara Phusanga, Jintana Joomwong and Surapol Jino), in 9 of the articles as coauthors are students of Assoc. Prof. Dr. Koppitz (Ilinka Dimitrova, Tiwadee Musunthia, Dara Phusanga and Laddawan Lohapan). The applicant has declared that the joint scientific works submitted for participation in the competition were written with equal participation of the coauthors. The papers are published as follows: 2 in Semigroup Forum, Bulletin of the Malaysian Mathematical Sciences Society, Journal of Algebra and Its Applications, Ukrainian Mathematical Journal, Asian-European Journal of Mathematics, Thai Journal of Mathematics and 1 in Algebra Universalis, Mathematica Slovaca, International Journal of Pure and Applied Mathematics, C.R. Acad. Bulg. Sci., Algebra and Discrete Mathematics.

3. General characteristic of the scientific work and achievements of the applicant. The main scientific interests of Assoc. Prof. Dr. Koppitz are in the field of semigroup theory. In addition, he also has works devoted to the properties of other algebraic systems. Semigroups are algebraic systems with one binary operation which satisfies the associative law. The well-known Cayley theorem, which for finite groups is included in the standard undergraduate algebra course of most universities in Bulgaria and abroad, states that every group is isomorphic to a group of invertible transformations of a set. The semigroup analogue of this theorem states that every semigroup is embedded in the semigroup of the set of all transformations of a set. Semigroups of set transformations are a typical (and perhaps the most

important example) of semigroups, which are studied both from the point of view of algebra and combinatorics, and because of their numerous applications in other areas of mathematics, theoretical computer science, and other sciences. An essential part of the research of the applicant is devoted to semigroups of transformations of various finite and infinite sets. There are possibilities for an exchange of ideas and fruitful cooperation with the group of Bulgarian algebraists who study analogous problems for semirings. Another important direction in the research of the applicant is the study of varieties of semigroups. Varieties are classes of algebraic systems which are defined by the identital relations they satisfy. This is one of the traditional directions in Bulgarian algebra, and for many years the world has been talking about the Bulgarian school in the theory of varieties of algebras, founded in the 1960s by Mihail Gavrilov together with Georgi Genov who joined the group in the early 1970s. At the moment, a number of Bulgarian algebraists in the country and abroad are actively working on this topic. Here also there is an opportunity for exchange of ideas and cooperation between Assoc. Prof. Dr. Koppitz and colleagues from the group. It is well known that there are many problems for varieties of algebraic systems which sound similarly no matter what operations the algebraic systems are defined by. From this point of view, it is natural that the applicant studies not only semigroups, but also other algebraic systems – doppelsemigroups, clones, terms, etc., as well as algebraic hypersystems, when the result of the operations is not an element but a subset of the algebraic system.

4. Main scientific and scientific-applied contributions. I shall briefly discuss the main results contained in the submitted publications of the applicant, as well as my evaluation of them. In the summary of his scientific contributions, the applicant has divided his publications into four groups: I. Transformation semigroups; II. Doppelsemigroups; III. Semigroups under point of view of universal algebra; IV. Semihypergroups.

I. Transformation semigroups (papers Nos 1 – 4, 7, 9, 14, 15). One of the popular topics in semigroup theory is the study of semigroups of fully or partially defined transformations of finite or infinite sets. One reason for this popularity is the versions of the theorem of Cayley for classes of semigroups which can be embedded in such transformation semigroups. Particularly interesting are the cases when the transformations preserve the order of linearly or partially ordered sets. Papers Nos 1, 9, 14 and 15 study transformation semigroups of partially ordered sets which are “fences”, i.e. with the so-called zag-zag order defined by the inequalities

$$a_1 > a_2 < a_3 > a_4 < \dots \text{ (or } a_1 < a_2 > a_3 < a_4 > \dots).$$

Papers Nos 1 and 15 study the inverse semigroup of partial automorphisms of a finite fence with n elements. Paper No. 15 describes the Green relations of this semigroup. In semigroup theory, Green relations are equivalence relations which characterize the elements of a semigroup in terms of the principal ideals they generate. The famous sentence of John Mackintosh Howie, a prominent semigroup theorist, speaks of the importance of these relations: “On encountering a new semigroup, almost the first question one asks is 'What are the Green relations like?'”. The Green relations are especially useful for understanding the nature of divisibility in a semigroup and give the important information how far the semigroup is from beeing a group. It is established that the considered semigroup is generated by transformations of rank (= the number of elements in the image) $> n - 3$, and using the GAP system for computer calculations it is shown that for n odd the semigroup cannot be generated by transformations of rank $\geq n - 1$. For n even it is given a minimal generating set which consists of $n + 1$ explicitly given transformations of rank $\geq n - 1$. Paper No. 1 develops further the methods of paper No. 15 and closes the case of n odd. Paper No. 14 is devoted to

analogous questions, but for the monoid of all zig-zag order-preserving partial transformations of the fence $\mathbb{N}$ of the set of natural numbers. One considers the relative rank modulo the set Y of all idempotents and all surjective transformations, i.e. how many transformations do we need to add to the set Y to obtain a generating set of the entire monoid. It has turned out that it suffices to add only the transformation which sends every n to $n + 2$. The same transformation together with the idempotents of finite rank is sufficient to generate all transformations of finite rank. Paper No. 9, which is the last of the cycle of papers devoted to the zig-zag order, studies the complete transformations of the fence which preserve this order. A new description of the elements of this semigroup is given, which has turned out to be very useful for determining the rank of the semigroup, finding a minimal generating set and a formula for the number of idempotents. As in the case of papers No. 1 and No. 15, the answers depend on the parity of the number of elements in the fence.

Papers Nos 2 and 3 study the semigroup $T(X,Y)$ of transformations of a set X with images in a fixed subset Y of X. The special cases when the set X is finite chain (in paper No. 2) and an infinite chain (in paper No. 3) are considered. Paper No. 2 finds the relative rank of the semigroup modulo the subsemigroup $OP(X,Y)$ of all orientation-preserving transformations and characterizes the minimal relatively generating sets. Analogous problems are also solved for the relative rank and relative generators of the semigroup $OP(X,Y)$ with respect to the subgroup $O(X,Y)$ of order-preserving transformations, generalizing results of Catarino and Higgins from 1999. Paper No. 3 calculates the relative rank of $OP(X,Y)$ with respect to $O(X,Y)$ for a class of infinite chains which contains also the classical infinite chains.

In the last two papers No. 4 and No. 7 of the first part of publications submitted for participation in the competition, methods from semigroup theory are applied to solving algebraic graph theory problems. The aim is to study the structure of monoids of endomorphisms of an unoriented finite path. In paper No. 4, an algebraic description of the monoids of all injective partial endomorphisms and of all partial automorphisms is given. Rank and power formulas are given and the Green relations are described. In paper No. 7 one considers the monoids of all endomorphisms and of all weak endomorphisms. Again the rank and the cardinality are calculated, the regular elements are characterized, and it is established when these two monoids are regular.

II. Doppelsemigroups (papers Nos 8, 10, 11). Paper No.11 deals with n-tuple semigroups. These are sets with n semigroup operations related by the associative law

$$(x*y)\circ z = x*(y\circ z) \text{ for any two operations } * \text{ and } \circ.$$

Questions that are natural for all classes of algebraic structures are considered and solved – the description of the free product of two objects in the general case, and in the commutative case the free objects, their subobjects and groups of automorphisms, as well as their minimal commutative congruences. I fully agree with the conclusion formulated in the summary by the applicant, that the paper gives a fundamental description of the variety of all commutative n-tuple semigroups.

For $n = 2$ the n-tuple semigroups are called doppelsemigroups. As in the case of arbitrary n, analogs of theorems from semigroup theory are sought for doppelsemigroups. A theorem of Zaretskiy from 1959 gives that any ordered semigroup can be embedded in the ordered semigroup of all binary relations on a suitable set. This is an analogue of the theorem of Cayley for ordinary semigroups. In paper No. 10, a doppelsemigroup variant of the theorem of Zaretskiy is proved. It has turned out that the theorem of Zaretskiy is a consequence of the main result of the paper. The theorem of Cayley for arbitrary semigroups also follows from there

(but it does not follow from the theorem of Zaretskiy).

Free objects are one of the main objects for any variety of algebraic systems. They are characterized by the property that any mapping of the free generators in an algebraic system in the variety extends uniquely to a homomorphism. In paper No. 8, the variety of rectangular doppelsemigroups is studied. This variety is defined by the identities $x*y*z=x*z$ for each of the two semigroup operations. The paper gives a description of the free objects in this variety and the minimal congruence of the free doppelsemigroup, the factorization of which yields a rectangular doppelsemigroup. Among other results, we shall note the description of maximal subobjects, idempotents and endomorphisms of the free objects. As a consequence, the description of free rectangular semigroups is obtained.

III. Semigroups under point of view of universal algebra (papers Nos 5, 6, 13, 16, 17). In universal algebra, there is a general combinatorial method of presenting the expressions in an algebraic system by rooted trees, whose vertices (or nodes) are denoted by operations in the system, and whose leaves are elements of the system or nullary operations. In the case of semigroups and groupoids, the trees are binary and there is only one operation. Paper No. 17 is devoted to the study of stable and s-stable varieties of semigroups, commutative and idempotent groupoids. It has turned out that there are exactly 10 stable varieties of semigroups and they are defined in terms of their identital relations. This is analogous to the description of solid varieties of semigroups given by Polak in 1999. Varieties of commutative and idempotent groupoids are also stable. The paper uses methods typical for both combinatorics and mathematical logic and theory of algorithms. The results are also related to theoretical informatics.

The same holds also for papers Nos 6, 13 and 16. Starting from the semigroup $T(X,Y)$ of transformations of a set X with images in a fixed subset Y of X in a natural way one introduces the semigroup $T_P(X,Y)$ of all nonempty subsets of $T(X,Y)$ with operation $AB=\{ab \mid a\in A,b\in B\}$. The important case $|Y|=2$ is considered. It is remarkable with the property that the semigroup of nondeterministic Boolean operations can be embedded in $T_P(X,Y)$. Paper No. 16 gives the description of the idempotent and the regular elements in $T_P(X,Y)$ and of the maximal subsemigroups with important properties. Paper No. 13 studies the Green relations of the semigroup $T_P(X,Y)$. The obtained results give important information for the congruence structure of the semigroup $T_P(X,Y)$. Paper No. 6 is the last paper in the cycle of three papers devoted to the properties of the semigroup $T_P(X,Y)$. It contains the characterization of the ideals and the principal ideals of $T_P(X,Y)$.

Paper No. 5 is also related to theoretical informatics. It studies the monoid of generalized hypersubstitutions for algebraic systems. A paper of the applicant with Dana Phusanga from 2018 gives a natural and useful approach to hypersubstitutions of an algebraic system. The paper submitted for participation in the competition continues these studies and is an indirect proof of the usefulness of this approach. The main result describes the idempotents and regular elements of the monoid of generalized hypersubstitutions for algebraic systems.

IV. Semihypergroups (paper No. 12). As the applicant comments in the summary, the opinion on the study of hyperstructures is controversial in the algebraic community. On one hand, it is an area that is actively worked on, and on the other hand, this area is considered to be in the periphery of modern algebra. The presented paper deals with semihypergroups. These are sets with a single binary operation which satisfies an analogue of the associative law and the operation maps every two elements not to an element, but to a subset of the considered set. The main idea of the paper is that the study of semihypergroups can be replaced by the study of

ordinary semigroups. This is illustrated for the two-element semihypergroups. It is proved that their classification reduces to a classification of three-element semigroups. It has turned out that there are exactly 17 nonisomorphic two-element semihypergroups. The proofs use different techniques. The main role is played by the so-called alternative identities and alternative varieties (introduced by Lyapin) instead of the ordinary notions of identital relations and varieties of algebraic systems.

In conclusion of my comments on the scientific contributions of the applicant, I want to note that he knows very well the main problems in the field and the literature on the objects under consideration and uses a rich arsenal of methods. I am convinced in the correctness of the arguments in the proofs. I have not noticed any significant inaccuracies.

The summary of the results correctly reflects the main contributions in the publications submitted for participation in the competition.

The applicant has presented a list of 103 citations of 34 of his publications; 20 of these citations are of 11 papers submitted fot the participation in the competition. In my opinion, this is a good achievement considering that the papers were published in the last 7 years. The list of citations can be extended. For example, it gives 8 citations to the monograph with Klaus Dieter Denecke and according to Zentralblatt für Mathematik the monograph is cited 31 times (7 of these citations are authocitations).

It can be concluded from the applied documentation that:

a) The scientific publications satisfy the minimal requirements of the law and the accompanying rules of IMI – BAS for the academic position "Professor" in the scientific field of the competition. With minimal requirements for the groups of indicators C, D, E and F respectively of 100, 220, 140 and 150 points, the applicant has submitted evidence of 150, 308, 156 and 175 points;

b) The scientific publications submitted for the completion have not been used in previous applications;

c) No plagiarism has been established in the presented for the competition works.

5. Significance of contributions to science and practice. The results obtained in the research papers of the applicant are interesting and meaningful. They contain new facts about objects which appear naturally in a number of areas of mathematics and its applications, and many of which have been previously studied by other authors. The results and the methods for obtaining them have been used and can be successfully used in other studies of this kind in the future.

6. Critical remarks and recommendations. I do not have essential critical remarks to the publications of the applicant. I want especially to mention that the documentation for the competition is prepared extremely accurately.

7. Personal impressions for the applicant. I personally know Assoc. Prof. Dr. Koppitz from the time when he applied for a position at IMI – BAS (I was a member of the Scientific Jury for the competition), as a member of the team of the Department of Algebra and Logic, and from his talks at the Algebra and Logic Seminar. I have presented some of his papers in C.R. Acad. Bulg. Sci. after positive reports of experts in the field. I have excellent impressions of him as a colleague and scientist.

CONCLUSION

In the presented scientific publications Assoc. Prof. Dr. Jörg Koppitz has obtained interesting results in current areas of algebra. Most of the results have already been used or can be used in a similar kind of research by other authors. A substantial part of the results have been published in respectable journals and reported at high level scientific forums. I have every reason to

confidently propose Associate Professor Dr. Jörg Koppitz for the academic position of "Professor" in the Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics, Professional Field: 4.5. Mathematics, Scientific Speciality: Algebra and Number Theory (Transformation Semigroups).

Sofia, March 20, 2023 Referee:

(Prof. D.Sci. Vesselin Drensky, Full member of the BAS)