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SYMPLECTIC TOPOLOGY, NON-COMMUTATIVE GEOMETRY, AND MIRROR SYMMETRY

ABSTRACT OF DISSERTATION

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Contents

1 Introduction

Homological Mirror Symmetry is a new direction in Modern Mathematics. The main purpose of this dissertation is developing Homological Mirror Symmetry for the benefit of classical birational geometry – showing that generic four-dimensional cubic and other Fano manifolds are not rational.

In his address to the International Congress of Mathematicians 1994, Kontsevich (the main collaborator of Katzarkov) speculated that mirror symmetry for a pair of Calabi-Yau manifolds X and Y could be explained as an equivalence of a triangulated category constructed from the algebraic geometry of X (the derived category of coherent sheaves on X) and another triangulated category constructed from the symplectic geometry of Y (the derived Fukaya category). Witten described the topological twisting of the N = (2, 2) super-symmetric field theory into what he called the A and B model topological string theories [citation needed]. These models concern maps from Riemann surfaces into a fixed target – a Calabi-Yau manifold or Fano manifold. Most of the mathematical predictions of mirror symmetry are embedded in the physical equivalence of the A-model on Y with the B-model on its mirror X. To cover the case of open strings, one must introduce boundary conditions to preserve the super-symmetry. In the A-model, these boundary conditions come in the form of Lagrangian submanifolds of Y with some additional structure. In the B-model, the boundary conditions come in the form of holomorphic submanifolds of X with holomorphic (or algebraic) vector bundles on them. These are the objects one uses to build the relevant categories. They are often called A and B branes, respectively. Morphisms in the categories are given by the spectrum of open strings stretching between two branes. The closed string A and B models only capture the topological sector – a small portion of the full string theory. Similarly, the branes in these models are only approximations to the full dynamical objects that are D-branes. The mathematics resulting from this small piece of string theory is the main topic of the theses.

A major part of this dissertation is to give well defined mathematical theory of Homological Mirror Symmetry in the case of Fano manifolds.

2 Main Ideas

In fact, the main idea in this dissertation is to use ideas of theoretical physics in order to solve classical problems in Algebraic geometry.

Our considerations are based on two major parts of theoretical physics:

1) Conformal Field Theory – a quantum field theory that is invariant under conformal transformations.

2) Mirror Symmetry – in the Hori-Vafa interpretation and its categorical upgrade made by Kontsevich.

The development of conformal field theory begins with the two-dimensional case. It is consolidated with the 1983 article by Belavin, Polyakov and Zamolodchikov.

In the two-dimensional quantum theory we have the Witt algebra of infinitesimal conformal transformations which is centrally extended, with a central charge and other renormalization charges – spectra of dimensions.

Alexander Zamolodchikov has proven the Zamolodchikov C-theorem, and tells us that renormalization group flow in two dimensions is irreversible.

Computing the charges of conformal field theories is a challenging exercise in general. In the case of massive theories one can use geometry in order to compute them.

The theory of spectra of singularities was developed in a parallel way to the theory of central charges. In fact, it was developed in the same city – in Moscow by Arnold and Varchenko. The spectra of singularity corresponds to the charges of conformal field theories and the Zamolodchikov C-theorem is the semi-continuity theorem in the theory of spectra of singularities.

The full correspondence between charges of conformal field theories spectra of singularity and asymptotics of solutions of ODE was indicated by Vafa and Cecoti in the nineties.

In this dissertation, we connect the above correspondence with the Homological Mirror Symmetry – the second building block of theoretical physics we use.

The main body of the dissertation is developing the Homological Mirror Symmetry. The mirror Symmetry started as a theory allowing counting curves - done by physicists. For us that counting of curves – Gromov Witten theory is a tool which leads to higher structures and as a result to higher order applications.

The point we take is that Birational geometry (derived categories) is mirror to theory of singularities (the category of vanishing cycles).

We start with the very simple case – rational surfaces, where birational geometry is rather easy. First we develop the categorical foundations of Homological Mirror Symmetry. We establish the fact that the birational transformations lead to creation of new singularities on the mirror site. We extend this correspondence in general – this is the main content of the first part of the dissertation.

In the second part, we develop the theory of Non-commutative Hodge structures. We also show that the quantum differential equation and its asymptotics corresponds to the spectrum of singularities of the Landau Ginzburg model. This is the second part of the dissertation.

3 Structure of the dissertation

The dissertation is organized as follows. We begin with establishing Homological Mirror Symmetry (HMS) with and easy from the point of Birational geometry case – Fano surfaces.

3.1 Fano Surfaces

In the case of Fano manifolds, the statement of the HMS conjecture is the following:

Conjecture: The category of A-branes $D(FS(W))$ is equivalent to the derived category of coherent sheaves (B-branes) on Y.

Here $(FS(W))$ is the Fukaya Seidel category of the potential W.

We prove this conjecture for various examples.

There is also a parallel statement of HMS relating the derived category of B-branes on $W : X \to \mathbb{C}$, whose definition was suggested by Kontsevich, and the derived Fukaya category of Y. Since very little is known about these Fukaya categories, we will not discuss the details of this statement in the present work. Our hope in this direction is that algebro-geometric methods will allow us to look at Fukaya categories from a different perspective.

The case we will be mainly concerned here is the case of the weighted projective plane $P^2(a, b, c)$ (where a, b, c are co-prime positive integers). Its mirror is the affine hypersurface $X = \{x^a y^b z^c = 1\} \subset (\mathbb{C}^*)^3$, equipped with an exact symplectic form ω and the super-potential $W = x + y + z$. Our main theorem is:

Theorem 1 *HMS holds for $P^2(a, b, c)$ and its non-commutative deformations.*

Namely, we show that the derived category of coherent sheaves (B-branes) on the weighted projective plane $P^2(a, b, c)$ is equivalent to the derived category of vanishing cycles (A-branes) on the affine hypersurface $X \subset (\mathbb{C}^*)^3$. Moreover, we also show that this mirror correspondence between derived categories can be extended to toric non-commutative deformations of $P^2(a, b, c)$, where B-branes are concerned, and their mirror counterparts, non-exact deformations of the symplectic structure of X, where A-branes are concerned.

Observe that weighted projective planes are rigid in terms of commutative deformations, but have a one-dimensional moduli space of toric non-commutative deformations (P^2 also has some other non-commutative deformations). We expect a similar phenomenon to hold in many cases where the toric mirror ansatz applies. An interesting question will be to extend this correspondence to the case of general non-commutative toric varieties. We return to this question in section 3 of the dissertation.

We will also consider some other examples besides weighted projective planes, in order to demonstrate the ubiquity of HMS:

- as a warm-up example, we give a proof of HMS for weighted projective lines.
- we also discuss HMS for Hirzebruch surfaces $\mathbb{F}_n$. For $n \geq 3$, the canonical class is no longer negative ($\mathbb{F}_n$ is not Fano), and HMS does not hold directly, because some modifications of the toric mirror ansatz are needed. The direct application of the ansatz produces a Landau-Ginzburg model whose derived category of vanishing cycles is identical to that on the mirror of the weighted projective plane $P^2(1, 1, n)$. In order to make the HMS conjecture work we need to restrict ourselves to an open subset in the target space X of this Landau-Ginzburg model.
- we will also outline an idea of the proof of HMS (missing only some Floer-theoretic arguments about certain moduli spaces of pseudo-holomorphic discs) for some higher-dimensional Fano manifolds, e.g. P^3.

3.2 Blow - up

The previous considerations suggest general understanding of the mirror birational transformations.

3.3 Statement of the results

Our main result in the third part of the dissertation can be formulated as follows.

Let $H = f^{-1}(0)$ be a smooth nearly tropical hypersurface in a (possibly noncompact) toric variety V of dimension n, and let X be the blow-up of $V \times \mathbb{C}$ along $H \times 0$, equipped with an S^1-invariant Kähler form ω_ϵ for which the fibers of the exceptional divisor have sufficiently small area $\epsilon > 0$.

Let Y be the toric variety defined by the polytope $\{(\xi, \eta) \in \mathbb{R}^n \times \mathbb{R} \,|\, \eta \geq \varphi(\xi)\}$, where φ is the tropicalization of f. Let $w_0 = -T^\epsilon + T^\epsilon v_0 \in \mathcal{O}(Y)$, where T is the Novikov parameter and v_0 is the toric monomial with weight $(0, \ldots, 0, 1)$, and set $Y^0 = Y \setminus w_0^{-1}(0)$. Finally, let $W_0 = w_0 + w_1 + \cdots + w_r \in \mathcal{O}(Y)$ be the leading term, namely the sum of w_0 and one toric monomial w_i ($1 \leq i \leq r$) for each irreducible toric divisor of V. We assume:

Assumption 1 *$c_1(V) \cdot C > \max(0, H \cdot C)$ for every rational curve $C \simeq \mathbb{P}^1$ in V.*

This includes the case where V is an affine toric variety as an important special case. Under this assumption, our main result is the following:

Theorem 2 *Under our assumptions, the B-side Landau-Ginzburg model (Y^0, W_0) is SYZ mirror to X.*

In the general case, the mirror of X differs from (Y^0, W_0) by a correction term which is of higher order with respect to the Novikov parameter.

Equipping X with an appropriate super-potential, given by the affine coordinate of the $\mathbb{C}$ factor, yields an A-side Landau-Ginzburg model whose singularities are of Morse-Bott type. Up to twisting by a class in $H^2(X, \mathbb{Z}/2)$, this A-side Landau-Ginzburg model can be viewed as a stabilization of the sigma model with target H.

Theorem 3 *Assume V is affine, and let $W_0^H = -v_0 + w_1 + \cdots + w_r \in \mathcal{O}(Y)$. Then the B-side Landau-Ginzburg model (Y, W_0^H) is a generalized SYZ mirror of H.*

Landau-Ginzburg model

Unlike the other results stated in this introduction, this theorem strictly speaking relies on the assumption that Fukaya categories of Landau-Ginzburg models satisfy certain properties for which we do not provide complete proofs. We give sketches of the proofs of these results, and indicate the steps which are missing from our argument.

A result similar to Theorem can also be obtained from the perspective of mirror duality between toric Landau-Ginzburg models. However, the toric approach is much less illuminating, because geometrically it works at the level of the open toric strata in the relevant toric varieties (the total space of $\mathcal{O}(-H) \to V$ on one hand, and Y on the other hand), whereas the interesting geometric features of these spaces lie entirely within the toric divisors.

The main theorem of this part of the dissertation relies on a mirror symmetry statement for open Calabi-Yau manifolds which is of independent interest. Consider the conic bundle

$$X^0 = \{(\mathbf{x}, y, z) \in V^0 \times \mathbb{C}^2 \,|\, yz = f(\mathbf{x})\}$$

over the open stratum $V^0 \simeq (\mathbb{C}^*)^n$ of V, where f is again the defining equation of the hypersurface H. The conic bundle X^0 sits as an open dense subset inside X. Then we have:

Theorem 4 *The open Calabi-Yau manifold Y^0 is SYZ mirror to X^0.*

In the above statements, and in most of this chapter, we view X or X^0 as a symplectic manifold, and construct the SYZ mirror Y^0 (with a super-potential) as an algebraic moduli space of objects in the Fukaya category of X or X^0. This is the same direction considered. However, one can also work in the opposite direction, starting from the symplectic geometry of Y^0 and showing that it admits X^0 (now viewed as a complex manifold) as an SYZ mirror. For completeness we describe this converse construction in the next section.

Fukaya category

The methods we use apply in more general settings as well. In particular, the assumption that V be a toric variety is not strictly necessary – it is enough that SYZ mirror symmetry for V be sufficiently well understood. As an illustration we consider more examples.

The rest of this chapter is organized as follows.

First we briefly review the SYZ approach to mirror symmetry. Then we introduce notation and describe the protagonists of our main results, namely the spaces X and Y and the super-potential W_0.

After that we construct a Lagrangian torus fibration on X^0, similar to those previously considered by Gross. Later we study the Lagrangian Floer theory of the torus fibers, which we use to prove the main theorem. Then we consider the partial compactification of X^0 to X, and prove the main theorem.

We briefly consider the converse construction, namely we start from a Lagrangian torus fibration on Y^0 and recover X^0 as its SYZ mirror.

Finally, some examples illustrating our main results are given that blow up in general in nothing else but creating a new singular fiber.

In what follows, we need to develop theory of singular fibers which do not come from Blow ups. In order to do that we need to develop:

1) Non-commutative Hodge theory and theory of quantum spectra – eigenvalues of the quantum multiplication by canonical class.

2) Theory of non-commutative spectra.

3.4 Non-commutative Hodge theory

Due to its foundational nature this chapter of the dissertation comes out long winded and technical. It is organized in three major parts:

The first part introduces and develops the abstract theory of non-commutative Hodge structures. This theory is a variant of the formalism of semi-infinite Hodge structures that was introduced by Barannikov. We discuss the general theory of non-commutative Hodge structures in the abstract and analyze the various ways in which the Betti, de Rham and Hodge filtration data can be specified. In particular, we compare non-commutative Hodge and the ordinary Hodge theory and explain how non-commutative Hodge theory fits within the setup of categorical non-commutative geometry.

We start with discussion of the notion of a pure non-commutative Hodge structure. The non-commutative Hodge structures are analogues of the classical notion of a pure Hodge structure on a complex vector space. Both the non-commutative structures discussed presently and Simpson's non-abelian Hodge structures generalize classical Hodge theory. In Simpson's theory, one allows for non-linearity in the substrate of the Hodge structure: the non-abelian Hodge structures are given by imposing Hodge and weight filtrations on non-linear topological invariants of a Kähler space, e.g. on cohomology with non-abelian coefficients, or on the homotopy type. In contrast, the non-commutative structures discussed in this chapter consist of a novel filtration-type data (the twistor structure of which are still specified on a vector space, e.g. on the periodic cyclic homology of an algebra).

Similarly to ordinary Hodge theory non-commutative Hodge structures arise naturally on the de Rham cohomology of non-commutative spaces of categorical origin.

We will give several different descriptions of a non-commutative structure in terms of local data. We begin with the notion of a rational and unpolarized non-commutative Hodge structures, ignoring for the time being the existence of polarizations and integral lattices.

The non-commutative structures will be described in terms of geometric data on the punctured complex line, so we fix once and for all a coordinate u on $\mathbb{C}$ and the compactification $\mathbb{C} \subset \mathbb{P}^1$. We will write $\mathbb{C}[[u]]$ for the algebra of formal power series in u, and $\mathbb{C}((u))$ for the field of formal Laurent series in u. Similarly, we will write $\mathbb{C}\{u\}$ for the algebra of power series in u having positive radius of convergence, and $\mathbb{C}\{u\}[u^{-1}]$ for the field of meromorphic Laurent series in u with a pole at most at $u = 0$.

This leads to Quantum D module and its asymptotics – the Non-commutative spectra.

We also pay special attention to the non-commutative aspects of Hodge theory and its interaction with the classification of irregular connections on the line via topological data. One of the most useful technical results in this part is the gluing Theorem which allows us to assemble non-commutative structures out of some simple geometric ingredients. This theorem is used later in the chapter for constructing non-commutative structures attached to geometries with a potential.

The second part explains how symplectic and complex geometry give rise to non-commutative Hodge structures and how these structures can be viewed as interesting invariants of Gromov-Witten theory, projective geometry and the theory of algebraic cycles. In particular, we analyze the Betti part of the non-commutative Hodge theory of a projective space (viewed as a symplectic manifold) and use this analysis to propose a general conjecture for the integral structure on the cohomology of the Fukaya category of a general compact symplectic manifold. The formula for the integral structure uses only genus zero Gromov-Witten invariants and characteristic classes of the tangent bundle. Our conjecture is in complete agreement with the recent work of Iritani who made a similar proposal based on mirror symmetry for toric Fano orbifolds. We also discuss in detail the origin of the Stokes data for holomorphic geometries with potentials and investigate the possible categorical incarnations of this data.

In the third part we study non-commutative-Hodge structures and their variations under the

Calabi-Yau condition. We extend and generalize the standard treatment of the deformation theory of Calabi-Yau spaces in order to get a theory which works equally well in the non-commutative context and to be able to properly define the canonical coordinates in Homological Mirror Symmetry. We approach the deformation-obstruction problem both algebraically and by Hodge theoretic means and we obtain unobstructedness results, generalized pre Frobenius structures and some interesting geometric properties of period domains for non-commutative Hodge structure. We also study global and infinitesimal deformations and describe different constructions of Betti and de Rham non-commutative Hodge data for ordinary geometry, relative geometry, geometry with potentials and abstract non-commutative geometry.

Finally, we indicate the notions of quantum and non-commutative spectrum.

3.5 Spectra

The main idea of the spectra is interpretation of the classical singularity spectrum of Landau-Ginzburg (LG) models. We use the theory of LG models as generalized theory of singularity.

The above considerations lead to birational invariants. We will base our birational considerations on the following major notions and ideas:

1. **Quantum spectrum**. The quantum spectrum is defined. Let $K\cdot$ be the quantum multiplication by canonical class. It defines the following splitting of cohomology:

$$\mathcal{H} = \oplus_{\lambda_i} H_{\lambda_i}.$$

   Here λ_i are the eigenvalues of $K\cdot$. We call these eigenvalues *quantum spectrum*. The main theorem is:

   MAIN THEOREM: The splitting $\mathcal{H} = \oplus_{\lambda_i} H_{\lambda_i}$ is a birational invariant.

2. **Non-commutative spectrum**. The non-commutative spectrum is defined.

We extend these ideas and give some examples.

A) We build analogues with low dimensional topology and give several new directions for research.

B) We extend the definition of a non-commutative spectrum to multispectra. Possible applications are discussed.

Our considerations are only the tip of the iceberg. We propose a correspondence between non-rationality over algebraically non-closed fields and complexity of the discriminant loci of the moduli space of LG models. We consider some arithmetics applications. In fact, one can define several different spectra.

In addition to the **quantum spectrum** mentioned above, one can define several other spectra:

- **Non-commutative spectrum;**

  defined by the asymptotics of the quantum equation.

- **Givental spectrum;**

  defined by the solutions of the Givental's equation.

- **Spectrum of LG model – multiplier ideal sheaf;**

  defined as the Steenbrink spectrum of a new singularity theory of the LG model.

- **Asymptotics of stability conditions – stability spectrum;**

  defined as asymptotics of limiting stability conditions.

- **Serre dimension of the Kuznetsov's component;**

  defined as a categorical dimension.

- **Arnold-Varchenko-Steenbrink spectrum of the affine cone.**

  defined as the classical spectrum of the affine cone singularity over X.

- **R-charges** – the asymptotics of RG flow – the same as asymptotics of Kähler-Ricci flow.

4 Conclusions and Future Directions

The above developments lead to combining both sides of Homological Mirror Symmetry and creates new birational invariants. Several spectacular applications are discussed at the end.

What have we achieved in this dissertation?

We have felicitated the use of Homological Mirror Symmetry to the solution of deep problems in Birational geometry – non-rationality questions. The only algebraic geometry applications of Homological Mirror Symmetry before that were counting curves.

Birational geometry is a central part of Algebraic geometry – we still need to admit we do not understand the non-rationality of cubics.

The first steps in that direction were done by Riemann - the theory of elliptic integral proves non-rationality of one-dimensional smooth cubic.

In dimension two rationality questions were done by Enriques, Castelnuovo, Zariski.

Some spectacular results were obtained in dimension 3 and higher by Clemens, Griffiths, Voisin, Kollar, Tschinkel.

This dissertation offers a completely different method based on Homological Mirror Symmetry.

The Homological Mirror Symmetry is a subject with many faces from different subjects from logic to arithmetics.

In this dissertation, we concentrate on the connection with birational geometry and Hodge theory.

The reason for that is the application to non-rationality questions we have in mind.

We start with simple example related to rational surfaces where it is easy to investigate mirror site of the birational geometry.

In first part of the thesis, we prove Homological Mirror Symmetry for projective plane and for Del Pezzo surfaces – see Theorem 1.2 and Theorem 1.4.

In these two examples it becomes clear that birational transformations lead to theory of singularities on the mirror side.

In the third part, we expand this observation in any dimension. It becomes clear that birational transformations are nothing but new singular fibers in the Landau Ginzburg models – see Theorem 1.7.

In order to go deeper in birational transformations we need to expand Hodge theoretic invariants – see section Non-commutative Hodge Structure. We do this in the second part of the thesis. We introduce non-commutative Hodge theory – theory of quantum D-modules.

This leads to two spectra – see the section Interpretation of spectra:

1. The eigenvalues of quantum multiplication by canonical class - quantum spectrum.

2. The asymptotics of the solutions of the quantum differential equation.

The last part of the thesis suggests how these two spectra lead to spectacular birational applications.

Indeed we can use these spectra to show non-rationality of generic four-dimensional cubic – more than sixty years old problems in birational geometry.

Many other Fano's are considered. Of course, this is only the tip of the iceberg. We expect further applications in the following directions:

- Application of the above method to non-rationality questions of varieties over algebraically non-closed fields.
- We can extend the method to the case of orbofolds.
- In particular, the discrete torsion from K theory becomes a birational invariant.
- The method can be extended to the case of non-rationality questions of singular varieties.
- In fact it goes much deeper on splitting of non-commutative motive as a direct sum of atoms – categories with their stability conditions.
- The dynamics of the other numbers in the specter will bring new obstruction to rationality.

5 Publications on which the dissertation is based

The results included in the dissertation are based on the following publications:

1. Auroux, D., Donaldson, S.K., **Katzarkov, L.**, Yotov, M., *Fundamental groups of complements of plane curves and symplectic invariants*, (2004) Topology, 43 (6), pp. 1285–1318, DOI:10.1016/j.top.2004.01.006, Web of Science IF (2004): 0.727, Q1

Cited 22 times.

2. Auroux, D., Donaldson S.K., **Katzarkov, L.**, *Singular Lefschetz pencils*, (2005) Geometry and Topology, 9, pp. 1043–1114, DOI:10.2140/gt.2005.9.1043, Web of Science IF (2005): 1.275, Q1

   Cited 49 times.

3. Auroux, D., **Katzarkov, L.**, Orlov, D., *Mirror symmetry for Del Pezzo surfaces: Vanishing cycles and coherent sheaves*, (2006) Inventiones Mathematicae, 166 (3), pp. 537–582. DOI: 10.1007/s00222-006-0003-4, Web of Science IF (2008): 1.659, Q1

   Cited 73 times.

4. **Katzarkov, L.**, Kontsevich, M., Pantev, T., *Hodge theoretic aspects of mirror symmetry*, Proceedings of Symposia in Pure Mathematics, 78, American Mathematical Society, Providence, RI, 2008, 87–174. DOI: 10.1090/pspum/078/2483750, Web of Science

   Cited 132 times.

5. Auroux, D., **Katzarkov, L.**, Orlov, D., *Mirror symmetry for weighted projective planes and their noncommutative deformations*, (2008) Annals of Mathematics, 167 (3), pp. 867–943. DOI: 10.4007/annals.2008.167.867, Web of Science IF (2008): 3.447, Q1

   Cited 81 times.

6. Kapustin, A., **Katzarkov, L.**, Orlov, D., Yotov, M., *Homological Mirror Symmetry for manifolds of general type*, (2009) Central European Journal of Mathematics, 7 (4), pp. 571–605. DOI: 10.2478/s11533-009-0056-x, Web of Science IF (2009): 0.361, Q4

   Cited 22 times.

7. Ballard, M., Favero, D., **Katzarkov, L.**, *Orlov spectra: Bounds and gaps*, (2012) Inventiones Mathematicae, 189 (2), pp. 359–430. DOI: 10.1007/s00222-011-0367-y, Web of Science IF (2012): 2.259, Q1

   Cited 30 times.

8. Abouzaid, M., Auroux, D., Efimov, A.I., **Katzarkov, L.**, Orlov, D., *Homological mirror symmetry for punctured spheres*, (2013) Journal of the American Mathematical Society, 26 (4), pp. 1051–1083. DOI: 10.1090/S0894-0347-2013-00770-5, Web of Science IF (2013): 3.061, Q1

   Cited 27 times.

9. Abouzaid, M., Auroux, D., **Katzarkov, L.**, *Lagrangian fibrations on blowups of toric varieties and mirror symmetry for hypersurfaces*, (2016) Publications Mathematiques de l'Institut des Hautes Etudes Scientifiques, 123 (1), pp. 199–282. DOI: 10.1007/s10240-016-0081-9, Web of Science IF (2016): 3.182, Q1

   Cited 39 times.

10. **Katzarkov, L.**, Lee, K.S., Svoboda, J., Petkov, A., *Interpretations of Spectra*, In: Birational Geometry, Kähler-Einstein Metrics and Degenerations, Springer Proceedings in Mathematics and Statistics, 2023, 409, pp. 371–407, DOI: 10.1007/978-3-031-17859-7-20, Scopus SJR (2022): 0.181

    Cited 0 times.

The publications listed above have a total of **475** citations according to the SCOPUS and Web of Science databases as of November 08, 2023.

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