

Peer Review on the thesis: SYMPLECTIC TOPOLOGY, NON - COMMUATTIVE GEOMETRY, AND MIRROR SYMMETRY by L. Katzarkov

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January 12, 2024

1 Introduction

This is a peer review for the Thesis SYMPLECTIC TOPOLOGY, NON - COMMUATTIVE GEOMETRY, AND MIRROR SYMMETRY, submitted for awarding of the Scientific Degree "Doctor of Science" in Professional Field 4.5 Mathematics, Scientific Specialty "Geometry and Topology".

This peer review is written by Prof. F. Bogomolov. I am an algebraic geometer – professor in New York University and a member of the National Academy of Sciences (NAS) of USA and of Academia Europaea.

I have made serious contributions in the areas:

- Complex algebraic geometry;
- Arithmetics;
- Birational Geometry;
- Symplectic Geometry.

2 Evaluation of Ludmil Katzarkov

Ludmil Katzarkov is a professor in the University of Miami (UM) and Director of the Institute of the Mathematical Sciences of the Americas (IMSA), Miami. He is also a Scientific Director of the International Center for Mathematical Sciences – Sofia (ICMS-Sofia). Katzarkov has published more than 100 papers, in the highest level journals possible. The papers on which the thesis is based are published in the highest level journals:

- Annals of Mathematics;
- Inventiones Mathematicae;
- Journal of AMS.

Katzarkov has given more than 120 talks. He has received several prestigious awards and grants:

- Simons Investigators Award;
- ERC Advanced Grant;
- Russian Megagrant;
- NSF Career Award;
- Sloan Research Fellowship;
- Clay research Fellowship.

He is also a member of Academia Europaea.

3 Timeliness of the topic

Birational geometry is one of major directions of modern mathematics. Three Fields medals were awarded in this direction.

There have been many new approaches to Birational Geometry. But no one has explored an approach to non-rationality based on Homological Mirror Symmetry. This thesis paves the road for such an approach.

New applications based on non-commutative and quantum spectra are considered – most notable non-rationality of generic four-dimensional cubic.

4 Scientific achievements

The main idea in this dissertation is to use ideas of theoretical physics in order to solve classical problems in Algebraic geometry – problems of non-rationality.

There are two major parts of the thesis:

1. **Mirror Symmetry** – in the Hori Vafa interpretation and its categorical upgrade made by Kontsevich – Homological Mirror Symmetry.
2. **Conformal Field Theory.**

The Homological Mirror Symmetry in its geometric interpretation is the basis of the thesis.

The main conclusion of the first part is that birational transformation corresponds to creation of new singular fibers of the LG models.

In other words, birational geometry is transformed in singularity theory.

The thesis starts with detailed construction of the Homological Mirror Symmetry for two-dimensional Fanos - Sections 2 and 3.

Then general birational transformation is considered in Sections 3 and 4.

The new Hodge structure which suits best this set up is introduced – Non-commutative Hodge structure.

In this set up the author brings Conformal Field Theory – a quantum field theory that is invariant under conformal transformations.

The development of conformal field theory begins with the 1983 article by Belavin, Polyakov and Zamolodchikov.

In the two-dimensional quantum theory, we have the Witt algebra of infinitesimal conformal transformations which is centrally extended, with a central charge and other renormalization charges – spectra of dimensions.

Alexander Zamolodchikov has proven the Zamolodchikov C-theorem, and tells us that renormalization group flow in two dimensions is irreversible.

Computing the charges of conformal field theories is a challenging exercise in general. In the case of massive theories, one can use geometry in order to compute them.

The theory of spectra of singularities was developed in a parallel way to the theory of central charges. In fact, it was developed in the same city - in Moscow by Arnold and Varchenko. The spectra of singularity corresponds to the charges of conformal field theories and the Zamolodchikov C-theorem is the semi-continuity theorem in the theory of spectra of singularities.

The full correspondence between charges of conformal field theories spectra of singularity and asymptotics of solutions of ODE was indicated by Vafa and Cecoti in the nineties.

The dissertation combines parallel of Conformal Field theory with singularity theory and the singularity of LG models.

This leads to new birational geometry invariants:

- Non-commutative Hodge theory and splitting on Atoms - see Section 3.
- Spectra of Singularity - see Section 4.

5 Recommendations and critical remarks

I do not have essential critical remarks on the exposition. The text is written in sufficiently detailed and understandable manner and may be used as an introduction to the subject of Homological Mirror Symmetry. The abstract and the abstract of the contributions are written sufficiently detailed and give clear and adequate information.

6 General Conclusion

This is an excellence thesis which brings a new cutting edge approach to Birational Geometry. The applications will be immense:

- over the field of complex numbers;
- over algebraically non-closed fields.

In particular, the thesis leads to the solution of the long standing problem in algebraic geometry – non-rationality of generic 4-dimensional cubic. Many algebraic geometers have tried to do it, for instance Beauville, Voisin, Kollar, Kuznetsov, Thomas.

We expect that this method will lead to non-rationality of many other high dimensional Fano manifolds.

I confirm that the scientific contributions comply with the Law on the Development of Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at the Bulgarian Academy of Sciences for awarding of the scientific degree "Doctor of Science" in Professional Field 4.5 Mathematics, Scientific Specialty "Geometry and Topology".

In particular, the applicant satisfies the minimal national requirements in the professional field and no plagiarism was found in the presented scientific works.

That is why, **I evaluate most positively** the applicant. I support his thesis in the highest terms possible.

7 Final Recommendation

Based on the above discussions, **I strongly recommend** the scientific Jury to propose the appropriate authority of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences to award Prof. Ludmil Vassilev Katzarkov the scientific degree "Doctor of Science" in Professional Field 4.5 Mathematics, Scientific Specialty "Geometry and Topology".

Signature:

F. Bogomolov