

REVIEW

by **prof. Dr. Sci. Stefan Petrov Ivanov** -
professor at Sofia University "St. Kliment Ohridski", FMI and IMI-BAS

of a dissertation for the award of a scientific degree "Doctor of Science"
in the field of higher education 4. *Natural sciences, mathematics and informatics*
professional direction 4.5. *Mathematics (Geometry and Topology)*

Author: Professor Ludmil Vasilev Katzarkov, PhD - IMI-BAN

Title: "Symplectic topology, non-commutative geometry and mirror symmetry"

By Order No. 569/22.12.2023 of the Director of IMI-BAS I have been appointed as a member of the jury for providing the procedure for the defense of a dissertation on "Symplectic Topology, Noncommutative Geometry and Mirror Symmetry" for the degree of Doctor of Science of IMI-BAS in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5. Mathematics (Geometry and Topology) .

The author of the dissertation is Professor Dr. Ludmil Vassilev Katzarkov from IMI-BAS.

1. General description of the submitted materials

The presented by prof. Katzarkov's set of materials is in accordance with the Regulations for the Development of the Academic Staff of IMI-BAS, includes the following documents:

- Request to the Director of IMI-BAS for disclosure of the procedure for the defense of a dissertation;
- CV in European format;
- a copy of the recognized diploma for the educational and scientific degree "Doctor", Certificate №000077/16.10.2019;
- Minutes of a unit meeting with preliminary discussion of the dissertation work;
- Abstract of the dissertation;
- list of scientific publications on the topic of the dissertation;

- copies of scientific publications on electronic form;
- reference for compliance with the specific requirements of IMI-BAS;
- Reference for participation in research and educational projects;
- Reference for participation in scientific forums;
- List of submitted scientific publications;
- List of citations of scientific publications on the dissertation;
- NACID Information Card.

Professor Katzarkov has attached 10 publications out of all his 80 publications on the topic of the dissertation, and these 10 articles have not been used in other scientific competitions. Five of these articles are co-authored with two co-authors and five are co-authored with three co-authors, with 3 of the presented articles being co-authored by a Fields medal winner, two of them with S. Donaldson and one with M. Kontsevich. A list of 475 citations of the articles on the topic of the dissertation is presented.

I have no notes or comments on the documents.

2. Brief biographical data

Prof. Dr. Ludmil Katzarkov was born in 1961 in Bulgaria. He received a Master's degree in Mathematics from Moscow State University under the guidance of the world famous mathematician V. Iskovskikh where he received the Moscow State University Best Student Paper Award for 1986 and 1987. In the period 1990-1995 he was a doctoral student at the University of Pennsylvania, USA. He defended his doctoral thesis on "Factorization Theorems" at the University of Pennsylvania, USA under the supervision of the world famous mathematician R. Donagi. He became an Assistant Professor at the University of California, Irvine in 1996, and in 1999 held the position of Associate Professor at the same university. In 2002 he became Professor of the University of California Irvine, 2007 - Professor of the University of Vienna, Austria, 2017 - Professor of the National Research University Higher School of Economics, Moscow, Russia.

In 2004, he became a professor at the University of Miami, USA, and in 2018 he took up the position of professor at IMI-BAN, where he has worked and held these two positions until now.

Professor Katzarkov was a visiting professor at some of the most prestigious centers in the world such as IHES in Paris, France, Max Planck Institute of

Mathematics in Bonn, Germany, King's College in London, England, etc. There is a list of awards and grants, 29 in number, and I will note the 3 currently active grants, the Russian Megagrant 2017-, Simons Investigators Award 2017-, the Contract “Vihren” with the Bulgarian NSF, 2019-.

Professor Katzarkov has a very rich pedagogical activity, having supervised over 10 doctoral students and over 25 post-doctoral students in various places around the world.

3. Actuality of the topic and appropriateness of the set goals and objectives

Professor Katsarkov works in the field of algebraic, differential and symplectic geometries and topologies and their connections with string theories.

The main objects of the dissertation is developing Homological Mirror Symmetry. Homological Mirror Symmetry is a new direction in Modern Mathematics. A major part of this dissertation is to give well defined mathematical theory of Homological Mirror Symmetry in the case of Fano manifolds. The main purpose of the dissertation is developing Homological Mirror Symmetry for the benefit of classical birational geometry to show that generic four-dimensional cubic and other Fano manifolds are not rational. The main idea is to use the mirror symmetry ideas of theoretical physics. It should be notated that many well known mathematicians and theoretical physicists are working in this directions including two Fields medal winners, E. Witten and M. Kontsevich.

With this, I believe that the topic of the dissertation work is sufficiently relevant on a global scale.

4. Characterization, evaluation and contributions of the dissertation work

The dissertation submitted for review contains an introduction, an exposition in two chapters of 3 and 2 paragraphs, respectively, and a list of cited sources. The total volume of the text is 343 pages in English, and the list of cited literature contains a total of 256 titles. The dissertation work is based on 10 articles published in the most prestigious international scientific journals in mathematics in the world, and I will mention **Annals of Mathematics, Inventiones, Publications Mathematiques de l'Institut des Hautes Etudes Scientifiques, Journal of the AMS**, etc. with a total

Impact Factor of 15.152. Moreover, 475 citations of the results of the dissertation are presented.

The introduction describes the problems and goals addressed in the dissertation. The main idea in the dissertation is to use ideas of theoretical physics in order to solve classical problems in Algebraic geometry - problems of nonrationality. The Homological Mirror Symmetry in its geometric interpretation is the bases of the considerations in the theses.

There are two major parts of the theses:

1) Mirror Symmetry - in the Hori Vafa interpretation and its categorical upgrade made by Kontsevich - Homological Mirror Symmetry.

2) Conformal Field Theory

The main conclusion of the first part is that birational transposition correspond to creation of new singular fibers of the LG models (the mirror of the Fano). In other words birational geometry is transformed in singularity theory.

The theses starts with detailed construction of the Homological Mirror Symmetry for two dimensional Fanos in Section 2 and Section 3.

Then general birational transformation is considered in Section 3 and Section 4.

One of the main achievement in the theses is the introduction of the new Hodge structure, Noncommutative Hodge structures, which suits best this set up. In this set up the autor brings Conformal Field Theory - a quantum field theory that is invariant under conformal transformations. The development of conformal field theory begins with the 1983 article by Belavin, Polyakov and Zamolodchikov.

In the two-dimensional quantum theory one has the Witt algebra of infinitesimal conformal transformations which is centrally extended, with a central charge and other renormalization charges - spectra of dimensions. Alexander Zamolodchikov has proven the Zamolodchikov C-theorem, and tells us that renormalization group flow in two dimensions is irreversible.

Computing the charges of conformal field theories is a challenging exercise in general. In the case of massive theories one can use geometry in order to compute them. The theory of spectra of singularities was develop in a parallel way to the theory of central charges and was done in the same city - in Moscow by Arnold and Varchenko. The spectra of singularity corresponds to the charges of conformal field theories and the Zamolodchikov C-theorem is the semicontinuity theorem in the theory of spectra of singularities. The full correspondance between charges of

conformal field theories spectra of singularity and asymptotics of solutions of ODE was indicated by Vafa and Cecoti in the nineties.

One has the correspondence R-charges in Conformal Field theory and Asymptotics of Quantum differential equation. As it follows from the blow up procedure Sections 2,3,4 these asymptotics become birational invariants which is established in the MAIN THEOREM in the theses.

As a result of the above discussions, the dissertation combines parallel of Conformal Field theory with singularity theory and the singularity of LG models which leads to new birational geometry invariants:

- Noncommutative Hodge theory;
- Spectra of Singularity.

These new birational geometry invariants obtained in the theses confirm the non-rationality of known examples and lead to the solution of the long standing problem in algebraic geometry - nonrationality of generic 4 dimensional cubic established in Theorem~2.14. It should be noted that many algebraic geometers have tried to do it, e.g. Beauville, Voisin, Kollar, Kuznetsov, Thomas. One could expect that the method developed in the theses could lead to nonrationality of many other high dimensional Fano manifolds.

In conclusion, this is an excellence theses which brings a new cutting edge approach Birational Geometry. The applications could be immense over the field of complex numbers over algebraically non closed fields.

5. Critical remarks and recommendations - I have none.

6. Abstract and author's note.

The abstract (in Bulgarian and English) in volume of 14 pages in Bulgarian and 13 pages in English was made according to the requirements and correctly reflects the main results and contributions of the dissertation.

7. Personal impressions.

I know very well the person and scientist Ludmil Katzarkov, I have excellent impressions of him as a person, teacher and scientist and I value very highly his moral and professional qualities.

CONCLUSION:

The submitted dissertation, the abstract, related scientific works and documents show that Professor Ludmil Katzarkov is an established specialist in the field of mathematics and mathematical physics, algebraic, differential and symplectic geometries and topologies and their connections with string theories with significant contributions in these fields.

The dissertation contains scientific, scientific-applied and applied results, which represent an original contribution to science and meet all the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Regulations for the Implementation of ZRASRB and the relevant Regulations of IMI-BAS. The presented materials and dissertation results fully correspond to the specific requirements of IMI-BAS, adopted in connection with the Regulations of IMI-BAS for the application of RASRB.

The materials presented by the candidate do not repeat those from previous procedures for acquiring a scientific title and occupying an academic position.

No plagiarism was found in the dissertation and related publications.

The dissertation work shows that professor Ludmil Vasilev Katzarkov has in-depth theoretical knowledge and professional skills in scientific specialty 4.5. Mathematics (Geometry and Topology) by demonstrating qualities and skills for conducting research with original and globally significant scientific contributions.

Due to the above, I confidently give my positive assessment of the conducted research, presented by the above-reviewed dissertation work, abstract, achieved results and contributions, and I propose to the honorable scientific jury to award the scientific degree "Doctor of Sciences" to Ludmil Vasilev Katzarkov in the field of higher education: 4. *Natural sciences, mathematics and informatics*, professional direction 4.5. *Mathematics (Geometry and Topology)* .

18.01.2024 г.

Reviewer:.....

Prof. Dr. Sci. Stefan Ivanov