S T A N D P O I N T

on the acquisition of the academic degree "Doctor of Sciences"

in Professional Direction 4.5 Mathematics (Geometry and Topology)

by Professor Ludmil Vassilev Katzarkov

The standpoint is written by Prof. Azniv Kirkor Kasparian, Section of Algebra, Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", Professional direction 4.5 Mathematics, as a member of the scientific juri for the acquisition of the academic degree "Doctor of Sciences" by Prof. Ludmil Vassilev Katzarkov, according to the Order N 569/15.10.2023 of the Director of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences.

1 General description of the presented materials

1.1 Data of the application

The presented documents comply with the requirements of the Law on the Development of Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Bulgarian Academy of Sciences.

Prof. Ludmil Katzarkov has presented 10 articles for the acquisition of the scientific degree "Doctor of Sciences". These are cited 475 times and reported 69 times at conferences, workshops and seminars. All the articles are joint works. To the best of my knowledge, the contributions of the co-authors in the presented articles are equipollent. There is no doubt of plagiarism. The scientific contributions of Prof. Ludmil Katzarkov comply with and exceed considerably the minimal national requirements of Decree 26/13.02.2019 on the Amendments of the Rules of Implementation of the Law on Development of Academic Staff of Republic Bulgaria, as well as the specific requirements of the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Bulgarian Academy of Sciences. Five of the presented 10 articles are reflected by the dissertation.

1.2 Brief biography of the applicant

Prof. Ludmil Katzarkov has obtained his Ph.D. degree at the University of Pennsylvania. He has been a member of Simons Laufer Mathematical Sciences Institute, an Assistant Professor, an Associated Professor and a Professor at the University of California at Irvine, as well as a Professor ar the University of Miami, a Professor at the University of Washington, a Professor at the Highest School of Economics of Russia and a Professor at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. Prof. Ludmil Katzarkov has held visiting positions at Institute des Hautes Etudes Scientifiques, Max Planck Institute at Bonn, Imperial College of London, Universite de Nice and Munich University. He has received 29 prestigious awards. Prof. Ludmil Katzarkov has taught undergraduate courses in Calculus, Linear Algebra and Mathematical Biology, graduate courses in Algebra, Algebraic Geometry, Modern Homotopy Theory, Symplectic Lefschetz Theory, Homological Mirror Symmetry, Noncommutative Geometry and Mathematical Biology. He has advised 10 graduate students and 25 postdocs. Prof. Ludmil Katzarkov has organized 9 conferences an two seminars. He is an Associate Editor of two scientific journals and an Editor-in-Chief of one journal. Prof. Ludmil Katzarkov has published 80 article in highly prestigious scientific journals. Eight of them are standalone and 72 are joint works. The scientific contributions of Prof. Ludmil Katzarkov have 1090 noticed citations.

1.3 Content analysis of the scientific contributions of the presented materials

The articles of Prof. Ludmil Katzarkov for the procedure develop mirror symmetry, relating the algebraic geometry, mathematical physics, category theory, differential equations, arithmetic and etc. They contribute extraordinarily to the development of mathematics and shape the contemporary establishment of the subject. The works of Prof. Ludmil Katzarkov discuss almost all distinguished achievements of the contemporary algebraic geometry and reveal their common background. Motivated by the non-rationality of complex algebraic varieties, they develop an essentially novel Homological Mirror Symmetry on categorical level and create a noncommutative Hodge theory. The presence of 475 citations of the ten articles, presented for the procedure, and the 29 prestigious awards are convincing testimonies for the enormous contributions of the research of Prof. Ludmil Katzarkov to mathematics and mathematical physics. His works introduce a vast amount of new notions, raise challenging conjectures, and prove some of them by developing a lot of sophisticated new methods, combining techniques from algebraic geometry, algebraic topology, mathematical physics, differential equations and etc.

Five of the articles of Prof. Ludmil Katzarkov, presented for the procedure, are reflected by the dissertation. Since these are the ones with most citations and the dissertation relates them to 10 works in progress, let us start by analyzing them and then consider the other 5 articles. Three chapters of the dissertation develop mirror symmetry, while the other two ones discuss noncommutative Hodge theory and, respectively, various spectra from the viewpoint of mirror theory. At its dawn, Gromov-Witten's mirror theory has counted holomorphic curves on mirror pairs. Based on the quantum field theory and Hori-Vafa's interpretation, Kontsevich suggests a Homological Mirror Symmetry, expressed as an equivalence of categories. A series of works of Prof. Ludmil Katzarkov, presented by his dissertation, realize the Homological Mirror Symmetry as an equivalence of the bounded derived category of coherent sheaves on X with categories of vanishing cycles on the mirror Y of X. The discussion is concentrated on weighted projective planes X, Hirzebruch surfaces X and Del Pezzo surfaces X. The Homological Mirror Symmetry leads naturally to the notion of noncommutative deformations of Fano manifolds and generalizes such manifolds.

The first chapter of the dissertation introduces the notion of a noncommutative weighted projective space $\mathbb{P}_\theta(\overline{a})$. It constructs a full exceptional collection of objects of the bounded derived category $D^b(\mathrm{coh}(\mathbb{P}_\theta(\overline{a})))$ of coherent sheaves on $\mathbb{P}_\theta(\overline{a})$ and shows that $D^b(\mathrm{coh}(\mathbb{P}_\theta(\overline{a})))$ is equivalent to the category $D^b(\mathrm{mod}-B)$ of the right projective finitely generated modules over the endomorphism algebra B of that collection. Further, it provides a fully faithful functor of the bounded derived category $D^b(\mathrm{coh}(\mathbb{F}_n))$ of coherent sheaves on a Hirzebruch surface $\mathbb{F}_n$ in the bounded derived category $D^b(\mathrm{coh}(\mathbb{P}(1,1,n)))$ of coherent sheaves on the weighted projective plane $\mathbb{P}(1,1,n)$. The Homological Mirror Symmetry for a noncommutative weighted projective plane $\mathbb{P}_\theta(a,b,c)$ asserts the equivalence of the bounded derived category $D^b(\mathrm{coh}(\mathbb{P}_\theta(a,b,c)))$ of coherent sheaves on $\mathbb{P}_\theta(a,b,c)$ with the category $\mathrm{Lag}_{\mathrm{vc}}(f,\{\gamma_i\})$ of the Lagrangian vanishing cycles on the mirror (X,W) of $\mathbb{P}_\theta(a,b,c)$, where X is the hypersurface in $(\mathbb{C}^*)^3$ with equation $x^a y^b z^c = 1$ and $W = x + y + z : X \to \mathbb{C}$. The Hirzebruch surfaces $\mathbb{F}_n$, $n \geq 0$ have mirrors $X = \left((\mathbb{C}^*)^2, W_b = x + y + \frac{b}{x^n y}\right)$. For $n \geq 3$, sufficiently large $R >> 2$ and b close to 0, the category $D^b(\mathrm{coh}(\mathbb{F}_n))$ is proved to be equivalent to the full subcategory of $D(\mathrm{Lag}_{\mathrm{vc}}(W_b,\{\gamma_i\}))$, arising from restriction to the open domain $\{(x,y) \in (\mathbb{C}^*)^2 \mid |W_b(x,y)| < R\} \subset (\mathbb{C}^*)^2$.

The second chapter of the dissertation proves the Homological Mirror Symmetry for Del Pezzo surfaces X_k, $k \leq 8$, which are blow ups of $\mathbb{P}^2$ at k points. The elliptic fibration with three singular fibres, which is the mirror of $\mathbb{P}^2$ and corresponds to the superpotential $W_0 = x + y + \frac{1}{xy}$ admits a natural compactification to an elliptic fibration $\overline{W_0} : \overline{M} \to \mathbb{P}^1$, where $\overline{W_0}^{-1}(\infty)$ has 9 rational components. Let $\overline{W_k} : \overline{M} \to \mathbb{P}^1$ be the deformation of $\overline{W_0}$, which

displaces k of the 9 critical points of $\overline{W_0}^{-1}(\infty)$ towards finite values of the superpotential. Then $\left(M_k := \overline{M} \setminus \overline{W_k}^{-1}(\infty), W_k := \overline{W_k}|_{M_k}\right)$ is the Landau=Ginzburg mirror of X_k. The second chapter of the dissertation establishes that for any Del Pezzo surface X_k there is a complexified symplectic form $B + i\omega$ on M_k, with respect to which the bounded derived category of the Lagrangian vanishing cycles of $W_k : M_k \to \mathbb{C}$, denoted by $D^b(\mathrm{Lag}_{\mathrm{vc}}(W_k))$, is equivalent to the bounded derived category $D^b(\mathrm{coh}(X_k))$ of coherent sheaves on X_k. There arises an explicit mirror map, which associates to a cohomology class $[B + i\omega] \in H^2(M_k, \mathbb{C})$ the k points on $\mathbb{P}^2$, whose blow up results in X_k. The noncommutative deformations of X_k arise naturally from the deformations of the symplectic structure on M_k, which do not correspond to deformations of the complex structure on X_k. The parameters μ of the noncommutative deformations $X_{k,\mu}$ of X_k are determined completely by the cohomology class $[B + i\omega] \in H^2(M_k, \mathbb{C})$, which specifies the composition tensors in $D^b(\mathrm{Lag}_{\mathrm{vc}}(W_k))$. In such a way, the mirror map is expressed by theta functions.

The third chapter discusses the Lagrangian torus fibrations on the mirrors of hypersurfaces and complete intersections. It explains how birational transformations of a variety create new singular fibres of the Lagrangian fibration of its homological mirror. In terms of Kontsevich's Homological Mirror Symmetry, Strominger-Yau-Zaslow conjecture or briefly, SYZ-conjecture amounts to the realizability of the moduli space of Lagrangian torus fibrations on X by rank 1 local systems. Due to the lack of Lagrangian torus fibrations on a manifold X of general type, a pair (Y, W) is said to be a SYZ-mirror of a Kähler manifold X if there is an open dense subset $X^o \subset X$ with a Lagrangian torus fibration $\pi : X^o \to B$, such that Y is a completion of a moduli space of unobstructed torus-like objects of the Fukaya category $\mathcal{F}(X^o)$ and W restricts to the superpotential, induced by the deformation of $\mathcal{F}(X^o)$ to $\mathcal{F}(X)$. Let $H = \{v \in V \mid f(v) = 0\}$ be a smooth nearly tropical hypersurface in a toric variety V and X be the blow up of $V \times \mathbb{C}$ at $H \times 0$, equipped with an S^1-invariant Kähler form, with respect to which the exceptional divisor has sufficiently small area. When the first Chern class $c_1(V)$ of V in numerically effective, the dissertation constructs explicitly the SYZ-mirror (Y^o, W^o) of X, supported by an open subset Y^0 of a toric variety Y, whose polytope depends on the tropicalization of f. Moreover, the open Calabi-Yau manifold Y^o is SYZ-mirror to the conic bundle $X^o = \{(x, y, z) \in V^o \times \mathbb{C}^2 \mid yz = f(x)\}$ over the open subset $(\mathbb{C}^*)^n \simeq V^o \subset V$. Conversely, X^o is SYZ-mirror of Y^o and the replacement of Y^o by its toric compactification Y amounts to endowing X^o with a superpotential. If the toric variety V is replaced by an abelian variety V and X^o is the complement of the proper transform of V under the blow up of $V \times \mathbb{C}$ at $H \times 0$, resulting in X, then Y^o is again a SYZ-mirror of X^o. Endowing Y^o with an appropriate superpotential, one obtains SYZ-mirror of X, while Y with another superpotential is generalized SYZ-mirror of the hypersurface H, in the spirit of the Lefschetz Hyperplane Theorem. Let $H_i = \{v \in V \mid f_i(v) = 0\}$, $1 \leq i \leq d$ be smooth nearly tropical hypersurfaces in an n-dimensional toric variety V, whose tropicalizations intersect transversally and X be the blow up of $V \times \mathbb{C}^d$ along $H_i \times \{y \in \mathbb{C}^d \mid y_i = 0\}$, $1 \leq i \leq d$. The third chapter of the dissertation constructs explicitly SYZ-mirror Y^o of the affine part X^o of X. If the toric variety V is affine, then an appropriate superpotential turns Y^o into SYZ-mirror of X and another superpotential on the toric compactification Y of Y^o supplies SYZ-mirror to the complete intersection $H_1 \cap \ldots \cap H_d$.

The fourth chapter of the dissertation introduces the notion of a noncommutative Hodge structure or, briefly, a nc-Hodge structure, in order to discuss some Hodge theoretic aspects of mirror symmetry. While Simpson's non-abelian Hodge theory defines Hodge and weight filtrations on non-linear objects like the cohomologies with non-abelian coefficients or the homotopy type, the nc-Hodge structures comprise new types of filtration data on vector spaces like the periodic cyclic homology of an algebra. A natural source of nc-Hodge structures are the de Rham cohomologies of noncommutative spaces of categorical origin. Let $\mathcal{M}$ be a finite dimensional vector space over the field of the meromorphic Laurent series in u with pole at most at $u = 0$ and

∇ be a meromorphic connection on $\mathcal{M}$. The algebraization functor realizes the equivalence of the category of the pairs $(\mathcal{M}, \nabla)$ with the category of the finite rank algebraic vector bundles $M \to \mathbb{C}^*$ with connection ∇, which have a regular singularity at ∞. The locally constant sheaf of the locally ∇-horizontal sections of $M \to \mathbb{C}^*$ induces a locally constant sheaf $\mathbb{S} \to S^1$ of $\mathbb{C}$-vector spaces with a local filtration of subsheves $\{\mathbb{S}_{\leq\omega}\}_{\omega\in \mathrm{Del}}$, labeled by Deligne's local system Del on S^1. A rational pure nc-Hodge structure is a triple $(H, \mathcal{E}_B, \simeq)$ of a $\mathbb{Z}_2$-graded algebraic vector bundle $(H, \nabla) \to \mathbb{C}$, a local system $\mathcal{E}_B \to \mathbb{C}^*$ of finite dimensional $\mathbb{Z}_2$-graded $\mathbb{Q}$-vector spaces and an analytic isomorphism $\mathcal{E}_B \otimes \mathcal{O}_{\mathbb{C}^*} \simeq H|_{\mathbb{C}^*}$ of holomorphic vector bundles, which satisfies the nc-filtration axiom on the poles of ∇ at $0, \infty$, the $\mathbb{Q}$-structure axiom on the compatibility of $\mathcal{E}_B$ with $\{\mathbb{S}_{\leq\omega}\}_{\omega\in \mathrm{Del}}$ and the oppositeness axiom on the real structure of $\mathbb{S}$, induced by $\mathbb{S}\cap\mathcal{E}_B$. A variation of a pure nc-Hodge structure is a family of compatible rational pure nc-Hodge structures, which is parameterized by a complex manifold S and satisfies Griffiths transversality axiom. The fourth chapter of the dissertation explains how a nc-Hodge structure of exponential type can be glued out of nc-Hodge structures with regular singularities and additional gluing data. It establishes that the nc-Betti data of a nc-Hodge structure can be given in four equivalent ways : by an appropriate perverse sheaf of $\mathbb{Q}$-vector spaces, by an appropriate constructible sheaf of $\mathbb{Q}$-vector spaces, a finite set $U_1, \ldots, U_n$ of non-zero finite dimensional $\mathbb{Q}$-vector spaces with liner maps $T_{ij} : U_j \to U_i$, $T_{ii} = \mathrm{Id}_{U_i}$ or a local system $\mathbb{S} \to S^1$ with Deligne-Malgrange-Stokes filtration of exponential type. Let $(H, \mathcal{E}_B, \simeq)$ be a variation of nc-Hodge structure over a supermanifold S. Suppose that $x \in S$, $v \in T_xS$ and ξ_v is a local vector field around x, extending v. The holomorphic first order differential operator $\nabla_{u\xi_V} : H \to H$ restricts to an endomorphism $\nabla_{u\xi_v}|_{H_{(0,x)}}$ of the fibre $H_{(0,x)}$ of $H \to \mathbb{C} \times S$ over $(0, x) \in \mathbb{C} \times S$. A variation of nc-Hodge structure on S is of Calabi-Yau type at a point $x \in S$ if there exists an odd or an even generating vector $h \in H_{(0,x)}$, such that the linear map $T_xS \to H_{(0,x)}$, $v \mapsto \nabla_{u\xi_v}(h)$ is an isomorphism. A differential $\mathbb{Z}_2$-graded Lie algebra $\mathfrak{g}$ is homotopy abelian if the Lie algebra cohomology algebra $H^\bullet(\mathfrak{g}, \mathbb{C})$ is isomorphic to the algebra of formal power series on some supervariables. Any homotopy abelian differential $\mathbb{Z}_2$-graded Lie algebra gives rise to a moduli space $\oplus\mathbb{M}\mathrm{od}(\mathfrak{g}, d\mathfrak{g}) := \mathrm{Spf}(H^\bullet(\mathfrak{g}, \mathbb{C}))$ of formal supermanifolds. In the aforementioned terminology, the classical Tian-Todorov theorem asserts that if X is a compact Kähler manifold with vanishing first Chern class $c_1(X) = 0$, then the differential graded algebra $(\mathfrak{g}^{(1)}, d_{\mathfrak{g}^{(1)}}) := \left(\Gamma_{C^\infty}(X, T^{1,0}X \otimes_{C^\infty} A^{0,\cdot}_X), \overline{\partial}\right)$ of the deformations of X is homotopy abelian, so that the formal moduli space of X is smooth. In order to generalize this result, let us assume that A is a differential $\mathbb{Z}_2$-graded $\mathbb{C}$-algebra and $X = \oplus_{\mathrm{nc}}\mathrm{Spec}(A)$ is a smooth and compact affine Calabi-Yau nc-space, which satisfies the Degeneration Conjecture on the equality of the dimension of the periodic cyclic homology of A over the field of the formal power series of u, with the complex dimension of the Hochschield homology of A. Then the Hochschield cochain algebra of X is shown to be homotopy abelian, the formal moduli space $\oplus\mathbb{M}\mathrm{od}_X$ of X is a formal supermanifold and the negative cyclic homology of the universal family over $\oplus\mathbb{M}\mathrm{od}_X$ gives a vector bundle $H \to \oplus\mathbb{M}\mathrm{od}_X \times \mathbb{D}$ with a flat meromorphic connection ∇, providing the de Rham part (H, ∇) of a Calabi-Yau variation of nc-Hodge structure.

The fifth chapter of the dissertation introduces the noncommutative spectrum of a projective algebraic variety $X \subset \mathbb{P}^N$, describing the asymptotics of the limiting stability conditions on the localized Fukaya category, as well as the asymptotics of the quantum differential equation. It discusses the interrelations among various kind of spectra by the means of singularity theory, knot theory, Alexander polynomial and other topological, analytic and categorical tools. The results and the conjectures are illustrated by instructive examples. A work in progress of Katzarkov-Kontsevich-Pantev shows that the eigenspace decomposition of the quantum multiplication by the canonical class is a birational invariant. It provides sufficient conditions for non-rationality of a smooth projective Fano complete intersection. More general, the non-rationality of a projective variety X is related with the structures of the quantum and the cyclic cohomologies of X.

One of the articles of Prof. Ludmil Katzarkov, presented for the procedure but not discussed in

the dissertation, introduces and studies symplectic invariants of a compact symplectic 4-manifold (X, ω). A previous work of Auroux and Katzarkov has established that for any line bundle $L \to X$ with first Chern class $c_1(L) = \frac{1}{2\pi}[\omega]$ there exist branched coverings $f_k : X \to \mathbb{P}^2$ by approximately holomorphic sections of $L^{\otimes k}$, for sufficiently large $k \in \mathbb{N}$. The singularities of the branch curves $D_k \subset \mathbb{P}^2$ of f_k are nodes or cusps. The article, presented for the procedure, defines the affine stabilized fundamental group $G_k(X, \omega)$ and the projective stabilized group $\widetilde{G_k}(X, \omega)$ as quotients of $\pi_1(\mathbb{C}^2 \setminus D_k)$, respectively, of $\pi_1(\mathbb{P}^2 \setminus D_k)$, making use of a representation in the braid group B_k with k strings. It shows that $G_k(X, \omega)$, $\widetilde{G_k}(X, \omega)$ and their reduced subgroup $G_k^o(X, \omega)$ are symplectic invariants of (X, ω). In the case of a simply connected X is described the abelianization of $G_k^o(X, \omega)$. A special attention is paid to the case of a double cover $X_{a,b} \to \mathbb{P}^1 \times \mathbb{P}^1$, ramified over smooth algebraic curve of bi-degree $(2a, 2b)$, $a, b \in \mathbb{N}$, which is given by a section of the pull back of $\mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^1}(p, q)$ with $p, q \in \mathbb{N} \setminus \{1\}$.

Another article characterizes the near-symplectic oriented 4-manifolds (X, ω), making use of Donaldson's approximately holomorphic techniques. It proves that up to a blow up, any such (X, ω) decomposes into two symplectic Lefschetz fibrations over discs and a fibre bundle over S^1, relating the boundaries of these Lefschetz fibrations by fibrewise handle additions. Conversely, any such decomposition gives rise to a near-symplectic (X, ω). Any near-symplectic form ω on X is shown to be associated with a Riemannian metric on X, with respect to which ω is a self-dual harmonic form. Generic Riemannian metrics on certain compact 4-manifolds admit self-dual harmonic forms ω, defining near-symplectic structures on X.

One of the articles of Prof. Ludmil Katzarkov, presented for the procedure, discuses the Homological Mirror Symmetry for manifolds of general type. It conjectures that the presence of a fully faithful functor $\Phi : D^b(G) \to D^b(F)$ of the derived category $D^b(G)$ of a manifold G of general or of Calabi-Yau type in the derived category $D^b(F)$ of a projective Fano manifold F, implies the existence of a fully faithful functor $\Psi : \mathrm{DFuk}(G, \alpha_G) \to \mathrm{DFuk}(F, \alpha_F)$ between the corresponding Karoubi closures of the derived Fukaya categories with respect to a compatible pair of complexified Kähler classes α_G, α_F. This conjecture is proved for a curve G of genus 2 and a Fano threefold F of degree 4. The orthogonal complement of $\mathrm{DFuk}(G, \alpha_G)$ in $\mathrm{DFuk}(F, \alpha_F)$ is expected to be a direct sum of several copies of the category of graded modules over a Clifford algebra of a symmetric bilinear form on a complex vector space of dimension $\dim_{\mathbb{C}} F$.

One of the works of Prof. Ludmil Katzarkov computes the Orlov spectrum of the category of singularities of an isolated hypersurface singularity. It shows that the maximal gap of the Orlov spectrum of a triangulated category is bounded above by the maximal Rouquier dimension of its semi-orthogonal components. The article provides an explicit generator of the bounded derived category $D^b(\mathrm{coh}(X))$ of coherent sheaves on a hypersurface $X \subset \mathbb{P}^{n+1}$ of degree $n + 1$ and computes its generation time. It derives also an upper bound on the generation time of an exceptional collection.

The Landau-Ginzburg model of the Riemann sphere $C(n) := \mathbb{P}^1 \setminus \{p_1, \ldots, p_n\}$, punctured at $n \geq 3$ points is a noncompact toric threefold $X(n)$ with a superpotential $W : X(n) \to \mathbb{C}$. One of the articles of Prof. Ludmil Katzarkov shows that the derived wrapped Fukaya category of $C(n)$ is equivalent to the triangulated category of the singularities of the singular fibre $W^{-1}(0)$ Moreover, if D is a k-fold unramified cyclic cover of $C(3)$ then there is a $\mathbb{Z}_k$-action on $(X(3), W)$ and the derived wrapped Fukaya category of D is equivalent to the equivariant triangulated category of the singularities of $W^{-1}(0)$.

The scientific contributions of Prof. Ludmil Katzarkov comply with and exceed considerably the minimal national requirements of Decree 26/13.02.2019 on the Amendments of the Rules of Implementation of the Law on Development of Academic Staff of Republic Bulgaria, as well as the specific requirements of the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Bulgarian Academy of Sciences. More precisely, the five articles, reflected by the dissertation, rate the total of 186 points, versus the required 100 ones. The other five articles, presented for the procedure, earn 220 points, instead of the

required 100. The aforementioned ten articles have 475 citations, which amount to 2850 points, instead of the required 100. There is no doubt in plagiarism. To the best of my knowledge, the contributions of the co-authors in the presented joint articles are equipollent. The aforementioned circumstances have definitely convinced me that Professor Ludmil Vassilev Katzarkov complies and exceeds considerably the minimal national requirements for the acquisition of the scientific degree "Doctor of Sciences".

1.4 Conclusion on the application

After getting acquainted with the materials and the scientific works, presented for the procedure, and based upon the aforementioned analysis of their scientific significance and applicability, I confirm that the scientific contributions comply with the Law on the Development of Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at Bulgarian Academy of Sciences. for the acquisition of the scientific degree "Doctor of Sciences" in professional direction "4.5 Mathematics" (Geometry and Topology). In particular, the applicant satisfies the minimal national requirements in the professional direction and no plagiarism was found in the presented scientific works. That is why,

I evaluate positively the applicant.

2 General conclusion

Based upon the aforementioned, **I strongly recommend** the scientific Juri to propose the appropriate election authority of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences to award

Prof. Ludmil Vassilev Katzarkov
the scientific degree "Doctor of Sciences"

in Professional Direction 4.5 Mathematics (Geometry and Topology).

January 12, 2024

The standpoint is written by:

Prof. Azniv Kasparian