STATEMENT REPORT
under the procedure for acquisition
of the scientific degree "Doctor of Science"
by candidate Prof. PhD Lyudmil Katzarkov,
of the Thesis entitled: "SYMPLECTIC TOPOLOGY,
NON-COMMUTATIVE GEOMETRY, AND MIRROR SYMMETRY",

Scientific Field: **4. Natural Sciences, Mathematics and Informatics**
Professional Field: **4.5. Mathematics**
Scientific Specialty: **"Geometry and Topology"**,
Section: **"Analysis, Geometry and Topology"**
Institute of Mathematics and Informatics of Bulgarian Academy of Sciences

The statement report has been prepared by **Prof. DSci. Vesselin Stoyanov Drensky, Full Member of the BAS**, retired professor at IMI – BAS, member of the Scientific Jury by Order No. 569 / 22.12.2023 of the Director of IMI – BAS.

1. General characteristics of the dissertation thesis and the presented materials

The presented D.Sci. Thesis is written in English. It contains 318 pages of main text, ending with a summary of the contributions, 2 pages of contents, 2 pages of introduction, 16 pages of bibliography, including 256 titles, indexes with names and notions which appear in the main text. The thesis is divided into two parts, which consist of 3 and 2 chapters, respectively.

2. Short CV and personal impressions of the candidate

Prof. PhD Lyudmil Katsarkov received a solid education in world-renowned universities under the scientific guidance of distinguished mathematicians - leaders in the field of algebraic geometry. He graduated from the Department of Higher Algebra at the Moscow State University under the supervision of Vasiliy Alekseevich Iskovskikh and received his PhD from the University of Pennsylvania under the supervision of Ron Donagi. After that, he was sequentially an assistant professor, associate professor and professor at the University of Miami, and then a professor at the University of California Irvine, the University of Vienna, the National Research University Higher School of Economics in Moscow and the Institute of Mathematics and Informatics at the BAS. In the CV given in the documentation, there is a detailed list of the awards and honors received, the lectures given, the supervised 10 doctoral students and at least 25 postdoctoral students, the seminars and conferences organized, participation in editorial boards, etc. A list of 80 papers indexed in the Scopus database is also presented. In the English version of the abstract, a long list of talks at representable seminars and conferences is given, where the results of the dissertation are presented.

I have known Prof. Katsarkov for many years and have a very high opinion of his scientific and organizational qualities. He has essential contributions for the organization and financing of the International Center for Mathematical Sciences at IMI – BAS. With his recent involvement as Editor-in-Chief of Serdica Mathematical Journal, he made several important steps for the increasing of the role of the journal. I have some critical remarks about the exposition of the CV.

- It would be helpful to write in more detail the names of the universities where the applicant worked. Most colleagues who do not know Prof. Katsarkov will have a hard

time deciphering U of M (University of Miami), UW (University of Vienna) and HSE (National Research University Higher School of Economics in Moscow).

- The abbreviations "2C, 2A, 3A, 6C" of the courses given by the applicant do not say anything for the Bulgarian colleagues. Some of the data is out of date. For example, three doctoral students who defended their degrees in 2004 are listed (Nirschl, Tsai, and Boutchackchiev, with outdated information about their employment), but three others who defended after 2007 (Clarke, Liu, and Haiden) are not listed.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented DSci thesis and the publications to it, included in the procedure

Topic of the dissertation. The research presented in the dissertation is at the meeting point of two areas of theoretical physics (the conformal field theory and the theory of mirror symmetry) and of algebraic geometry, differential geometry and algebraic topology. According to MSC 2020 the papers on which the dissertation is based are reviewed in several areas of algebraic geometry (14 Algebraic geometry), differential geometry (53 Differential geometry) and algebraic topology (57 Manifolds and cell complexes). As it is stated in the dissertation and in the abstract, the motivation of the research is to use ideas and methods from theoretical physics to solve important classical problems of algebraic geometry. The starting point of the research is that birational geometry is a "mirror" of singularity theory.

Actuality of the developed problems. I am sure that important and current problems are studied and solved in the dissertation. The applicant uses known and develops new techniques, overcoming serious principal and technical difficulties.

Scientific contributions. In the first part of the dissertation, the categorical foundations of homological mirror symmetry are developed. The effect that birational transformations have on the properties and singularities in the "mirror" side is investigated. Achieving this goal, the applicant starts from the rational surfaces and reaches the general case. One of the main results is the proof of the homological mirror symmetry conjecture for Del Pezzo surfaces. In addition, results related to the study of the SYZ-"mirror" of objects important for algebraic geometry (SYZ=Strominger-Yau-Zaslow) have been obtained.

The second part of the dissertation is devoted to non-commutative Hodge theory and the related mirror symmetry. Quantum and non-commutative spectra are introduced and studied. Irrationality is then proved for a number of important manifolds.

Application of the obtained results. The obtained results are already being used by other researchers. As evidence, I shall point to the large number of citations, including in papers where the results of the dissertation are essentially used. I am confident that the obtained results will continue to be used in future research by both mathematicians and theoretical physicists.

As an additional credit to the dissertation, I shall note that in many places in the text, as well as at the end of the dissertation, unsolved problems are formulated, some of which are a serious challenge to mathematicians and physicists working in the field.

It is obvious to me that the applicant knows very well the results in the field of his research. The list of references contains publications from 1970 to the present, including unpublished results, preprints, and accepted papers. I have some critical remarks about the bibliography.

- It seems that the applicant copied and pasted some of the referencese from his papers, but made no efforts to update the data. I noticed 28 papers given as in preparation, preprints, or accepted for publication which are already published (on of them - [127], already in 2001). These are papers with numbers 1 (2014), 2 (2021), 15 (2015), 42 (2008), 72 (2017), 127 (2001), 129 (2008), 130 (2009), 131 (2009), 150 (2019), 154

(2008), 159 (2009), 161 (2008), 162 (2007), 169 (2018), 176 (2018), 182 (2022), 187 (2009), 188 (2011), 19 (2009), 197 (2006), 206 (2008), 212 (2019), 231 (2008), 237 (2007), 241 (2014), 243 (20068) and 251 (2016) (parentheses indicate when they were published according to Zentralblatt für Mathematik).

- In two of the articles in French, the accents typical for the French language are omitted.

As a result of the "copy and paste", there are other places in the dissertation that sound strange. For example, the expression "the second author" occurs several times in a text where no specific paper is mentioned.

In conclusion, I shall state that, in my opinion, with minimal efforts by the applicant, the dissertation may serve as the basis of a monograph, which would undoubtedly be very useful both for specialists in the field and for popularizing the subject.

4. Approbation of the results

According to the information presented in the abstract, the dissertation is based on 10 papers published in 2004 - 2023, but only 5 of these papers are cited in the dissertation itself. The other 5 articles are also directly or indirectly related to the thesis. For example, the results of one of the uncited papers (No. 9 in the list in the abstract) form the basis of the third chapter of the first part of the dissertation. All papers are published in outstanding editions: two papers in Inventiones Mathematicae and one paper in Topology, Geometry & Topology, Proceedings of Symposia in Pure Mathematics of AMS, Annals of Mathematics, Central European Journal of Mathematics, Journal of the American Mathematical Society, Publications Mathématiques de l'Institut des Hautes Études Scientifiques and Springer Proceedings in Mathematics and Statistics. Of the publications, 7 are in quartile Q1, 1 is in quartile Q4, 1 is indexed in Web of Science and 1 is with SJR of Scopus. The volume of publications (a total of 578 pages) and the attached list of 475 citations to these 10 articles are impressive. All papers are joint - 1 with four co-authors of the applicant, 3 - with three co-authors and 6 - with two co-authors: 6 are with Auroux (Berkeley), 4 are with Orlov (The Steklov Institute), 2 with Donaldson (Imperial College London), Yotov (Florida International University) and Abouzaid (Columbia University), 1 with Kontsevich (Institut des Hautes Études Scientifiques in the Paris suburb of Bures-on-Yvette), Pantev (University of Pennsylvania), Kapustin (California Institute of Technology), Ballard (University of Madison-Wisconsin), Favero (University of Vienna), Efimov (The Steklov Institute), Lee and Svoboda (University of Miami) and Petkov (FMI - SU). (The universities and scientific institutes of the co-authors are at the time of publication of the papers.) The list of co-authors is another indirect evidence of the quality of the publications. Among the co-authors are two Fields Medalists (Donaldson and Kontsevich) and two invited speakers at the International Congresses of Mathematicians (Auroux and Orlov). Unfortunately, there are no co-author declarations in the documentation for the joint papers, so I assume that the co-author contribution is equal. The only information in this regard is a reply from the applicant to a question of mine during the pre-defense: "The idea of using homological mirror symmetry to solve the problem of irrationality of the general four-dimensional cubic is mine. Maksim Kontsevich contributed to the definition of the partitioning." Having joint papers speaks to the ability to work in a team, and this is a quality that I personally value highly. In addition, collaborative work increases the efficiency factor because research uses methods from different areas of mathematics. On the other hand, I would recommend the applicant to publish also papers without co-authors. (According to Zentralblatt für Mathematik, Prof. Katsarkov's last paper without co-authors was from 2010.)

My inspection shows that:

a) The scientific works meet the minimum national requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria and respectively the additional requirements of Institute of Mathematics and Informatics for obtaining the scientific degree "Doctor of Sciences" in the scientific field and professional direction of the procedure. Moreover, with minimum requirements of 100 points in two of the groups of indicators, the applicant has submitted evidence for 406 and 2850 points, respectively.

b) The results presented by the applicant in the dissertation and scientific works to it do not repeat such from previous procedures for acquiring a scientific title and academic position.

c) There is no plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure.

5. Qualities of the abstract

The abstract and the reference to the contributions are written in shorthand, very briefly, but nevertheless they give a clear and adequate idea of the content and main results of the dissertation. Unlike all other abstracts of theses I have seen, in the submitted abstract the only literature cited is in the dissertation's list of papers on which the dissertation is based. I have some grammatical remarks in the Bulgarian version of the abstract. However, I am sure that the abstract and the summary of contributions meet all the requirements.

6. Critical notes and recommendations

I do not have critical remarks other than those in the previous points of my statement report. I am sure that the things I discussed are easily corrected and do not spoil my overall excellent impression of the results and their presentation in the dissertation.

7. Conclusion

Having become acquainted with the DSci thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented DSci thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, are at a very high scientific level and meet the requirements of a thesis in the area of mathematics. I **strongly recommend** to the respectable Scientific Jury to award Professor Professor PhD Lyudmil Katsarkovv with the scientific degree "Doctor of Sciences" in the Scientific Ffield 4. Natural Sciences, Mathematics and Informatics, Professional Field 4.5. Mathematics, Scientific Specialty: "Geometry and Topology".

Date: January 26, 2024 Signature:

(Vesselin Drensky, Prof. DSci, Full Member of the BAS)