REPORT

by **Valentina Petkova** (Corresponding member of BAS,
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of the Dissertation thesis of

Prof. Ludmil Vasilev Katzarkov

entitled

"Symplectic Topology, Non-commutative Geometry and Mirror Symmetry"

submitted for acquiring the scientific degree "Doctor of Sciences"

Higher education area: 4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5 Mathematics,
Scientific specialty: "Geometry and Topology"

The thesis presented contains **343** pages and consists of two parts, with 3 and 2 chapters respectively. It concludes with a summary of the main contributions and a discussion of the possible applications and generalisations. The list of references contains **256** titles. The thesis Abstract provides a concise conceptual exposition of the motivation, the methods explored and the results presented in the dissertation as well as their potential applications.

The thesis is based on **10** works published in the period 2004 - 2022; **7** of the papers appear in **6** prestigious journals with quartile **Q1**: *Topology, Geometry & Topology, Inventiones Mathematicae* (**2** papers), *Annals of Mathematics, Journal of the American Mathematical Society, Publications Mathematiques de l'Institut des Hautes Etudes Scientifiques.* One paper is in a journal with **Q4**: *Central European Journal of Mathematics,* and two of the papers are published in proceedings of conferences. One of the latter two, paper **4.** in the list, is actually the mostly sited, although it receives the least number of points, according to the rules adopted in the Regulations for the application of the Act on development of the academic staff in the Republic of Bulgaria. Altogether the publications of the dissertation have more than **475** citations up to November 2023, according to data from Scopus and Web of Sciences. I find this really impressive for a dissertation in mathematics! The minimum national requirements for the scientific degree "Doctor of Sciences" are met by a huge margin.

The dissertation fully meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the .Bulgarian Academy of Sciences. The scientific papers submitted by the applicant do not repeat those of previous procedures for acquiring a scientific degree and holding an academic position. There is no evidence for plagiarism in the presented scientific papers.

The thesis solves classical problems in algebraic geometry, exploring ideas of theoretical physics. They concern basic structures explored in the study of String theory. This theory is expected to provide an unified theory of elementary particles and their interactions. Consistent supersymmetric string theories exist with 10 target dimensions, 6 of which are 'compactified' to reproduce the 4-dimensional space time. In particular, compactification on Calabi-Yau three-folds was shown to give rise to realistic models, so that certain topological invariants of these manifolds describe families of particles in the effective field theory, arising in the low energy limit of the closed string. Further developments involved D-branes - submanifolds describing boundary conditions for open strings.

Mirror symmetry is an example of the various dualities demonstrating the equivalence of two different theoretical description of physical systems. It was first noticed in the study of some 2-dimensional N=2 superconformal field theories, examples of rational conformal field theories (CFT) described by a finite set of primary fields. These theories and their massive deformations have been related via Landau- Ginzburg (LG) models description to singularity theory, in particular to Arnold's ADE classification of isolated simple singularities. This classification matches the ADE classification of the modular invariant partition functions encoding the fields spectrum of a minimal (super)conformal theory with a given central charge. Topologically twisted (2,2) supersymmetric theories were proposed to realise string theories with two types of targets (A and B models) given by two "mirror" Calabi-Yau manifolds with related Hodge numbers.

It should be noted that the phenomenological predictions of the present (super)string theory are far from being confirmed by experiments. At the same time this theory has motivated and led to very rich and deep developments in pure mathematics. They not only provide mathematical precision of the original physicists constructions, but also open new perspectives. The work of Ludmil Katzarov on problems in algebraic geometry contributes to these developments.

The aim of the thesis is further development of the approach of Homological Mirror Symmetry (HMS) proposed by Kontsevich for the purposes of the birational geometry. HMS reformulates the mirror symmetry as an equivalence between certain derived categories determined from each of the two manifolds. In the first sections of the thesis this conjecture of Kontsevich is verified for various examples and used to show nonrationality of the considered manifolds. Then a general birational transformation is considered. The main conclusion of the first part is that birational transformations correspond to creation of new singular fibers of the LG models. In other words birational geometry is transformed in singularity theory. In the second part new Hodge structure which suits best this setup is introduced and developed, namely the Noncommutative Hodge structure with applications to spectra of singularities.

This is an excelent thesis which brings a new cutting edge approach to Birational Geometry. Prof. Dr. Ludmil Katzarkov is an internationally recognized mathematician with impressive personal contributions to the topic of his dissertation. His work definitely contributes to the fruitful relationship between physics and mathematics. I strongly recommend to the Scientific Jury of the Institute of Mathematics and Informatics to award Ludmil Vasilev Katzarkov the scientific degree "Doctor of Sciences".

Valentina Petkova

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