REVIEW

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on

dissertation **„Application of cellular neural networks to the study of partial differential equations arising in financial mathematics"**

of **Pavel Todorov Stoinov**,

for awarding the educational and scientific degree "Doctor"

Neural networks are a modern and very popular tool for modeling many processes in biology, engineering, economics, informatics, sociology, psychology and even politics. They are a powerful tool for the approximate solution of complex computational problems, including differential equations and systems of equations used to model financial instruments in financial markets. The subject of the presented research are stochastic models in finance, leading to partial integral and integro-differential equations, and cellular neural networks and related techniques for solving these equations. The main tasks to be solved are:

- Presentation of modern stochastic models in finance and the partial differential and integro-differential equations arising from these models, as well as offering some new stochastic models and techniques;
- Introducing cellular neural networks and related techniques as computational models based on artificial intelligence, as well as proposing some new methods and techniques related to the use of cellular neural networks;
- Application of cellular neural networks and related techniques to the numerical solution of differential and integro-differential equations related to the valuation of various types of derivative financial instruments.

One chapter of the thesis statement corresponds to each of these three tasks set in the dissertation work, a total of 3 chapters in 128 pages. 55 titles and 8 independent publications of the author are cited.

The topic of the dissertation implies the presentation of many basic concepts in the field of mathematics, stochastic analysis and neural networks. The first two set tasks suppose a systematized approach in the description of existing models and methods, as well as the presentation of new ones.

In **Chapter 1**, consisting of 69 pages, there are only two paragraphs, the first of which begins immediately after its title. It contains in detail the description of various types of stochastic processes and their properties, definition of stochastic differential equations, partial differential and integro-differential equations, derivation of the Black- Scholes partial differential equation for share price and its analytical solution, its existing modifications, definition and properties of the Fourier transform and inverse Fourier transform in the one-dimensional and multi-dimensional case, the Laplace and Mellin transforms, examples of their use, the boundary value problem reached when evaluating barrier options. The author's proposals in Chapter 1 are related to the consideration of a distribution called the ST distribution. It is a special case of a generalization of the Gamma distribution, with 4 included parameters (whereas in the classical Gamma distribution they are only 2), with values of the first and third of the parameters equal to 1. When, in addition, the fourth parameter of the generalized Gamma distribution assumes a fixed value of -1, the so-called Lindley distribution is obtained. In this sense, the ST distribution is a generalization of the Lindley distribution. When the last, fourth parameter, in the generalized Gamma distribution assumes the value 0, with arbitrary values of the remaining three parameters, the classical Gamma distribution is obtained. This statement is formulated and proved in Lemma 1.4, but contains errors in its formulation. It can be easily verified that the following characterization of this ST distribution also holds: if the random variable X has the classical Gamma distribution, the truncated at zero distribution of X-1 is precisely the ST distribution. In Chapters 1 and 2 there are a total of 14 graphs of the density and distribution function of the ST distribution at different values of its parameters. Equations (1.111) and (1.116) give the same representation of the ST(2,β) density, the second of which is, in my opinion, redundant. Pages 26, 28, and 29 use conflicting notations for the parameters in the classical Gamma distribution and the Erlang distribution (which is the Gamma distribution with an integer value for its first parameter). Indeed, two types of notations are found in the literature: $\Gamma(\alpha,\beta)$ or $\Gamma\left(\alpha,\frac{1}{\beta}\right)$, but mixing them in the same text leads to confusion and incorrect formulas, such as formula (1.101). In two successive theorems, the moment generating function of the distribution ST(2,β) and the characteristic function of the general distribution ST(n,β) are derived. A mixture of the negative binomial distribution with an ST distribution is considered, which is then generalized to a mixture of the negative binomial with a generalized Gamma distribution. A process (ST-process) is defined, which is analogous to the Poisson process, but unlike it, the intervals between two jumps are not exponentially distributed, but ST(n,β) distributed. It is shown that the ST-process can be considered as a Poisson process, in which all jumps are omitted, except for jumps at certain times. Another contribution stated in the thesis is a proposal for tempering using the ST distribution. In this regard, on page 40 I noticed several inaccuracies: in formula (1.155) in the notation of the second indicator; in formula (1.159) from the proof of Theorem 1.10 I noticed two errors (respectively in its second and fourth lines), the first of which will lead to a different result in the formulation of the theorem as well (the binomial coefficients in the expansion of $(1 + x)^n$ by powers of x are omitted), and the second is no longer present in the next fifth line of the equation. The end-of-proof mark is not placed in the correct place. The author's proposals in Chapter 1 (occupying about 15 pages with graphs) are in the area of random processes and function transformations, they are not further developed into concrete examples and applications, as is done with some of the existing methods and approaches, and thus remain somewhat off-topic of the dissertation. About two-thirds of the remaining material in Chapter 1 (approximately 35.5 pages) turned out to be a translation from three sources not listed in the literature cited. These are the textbooks given at the end of the review [1], [2] and [3].

Paragraph 1 of **Chapter 2** discusses basic concepts and types of cellular neural networks. In paragraph 2 (about 11 pages) the author's proposals in relation to the theory of neural networks are discussed. One of them is to use an activation function based on the density of the ST distribution. An attempt is made to explain the motivation behind this proposal. The author states that he "can show" that formula (2.100) is true, but it is not clear how, since nothing is said about the values of $P(C_1)$ and $P(C_2)$ involved in the formula, and furthermore, the left and the right sides of formulas (2.100) and (2.101) involve functions of different arguments - of *x* and *u*, respectively. Following are a total of 11 plots of densities and distribution functions of the ST distribution for different values of its parameters. Another suggestion of the author is to use the ST distribution to calculate the probability of obtaining a value of 1 from the dependent variable in logistic regression. Here again we come across a wrong formula – (2.115), part of the proposition itself. Its right-hand side is probably copied from formula (2.107), referring to the previous proposal to use the ST distribution as an activation function. The author's third proposal is to use the density of the ST distribution in Kohonen cellular neural networks to model the variable neighborhood radius and the variable learning rate. However, these proposals are not illustrated with examples of their application and effectiveness, as the topic of the dissertation would suggest.

The last **Chapter 3** is dedicated to the use of cellular neural networks for the approximate solution of integro-differential equations from financial models. Unfortunately, it turns out that 75% of the material presented here, including the MATLAB codes used, is again a direct translation from uncited sources [1] – [3]. These are 21 of the 28 total pages that make up Chapter 3. In its remainder, Chapter 3 refers to two sources that are properly cited. Chapter 3 also cites a publication by the author, which, however, after checking, found that for the most part (two-thirds of it) it uses sources [1] and [3], without citing them in the literature of the article.

In conclusion, I will point out that PhD student Pavel Todorov Stoynov has undoubtedly invested significant time and effort in mastering extensive and complex parts of mathematics such as the theory of stochastic processes, stochastic analysis, mathematical analysis, probabilities, finance, neural networks, numerical methods. The author's proposals in Chapters 1 and 2 are relevant and also testify to a high degree of knowledge in the various fields. 6 publications are presented, 5 of which are in journals with SJR and 1 is refereed in Zentralblat fur Mathematik. According to the attached list, there are a total of 3 citations to the two 2019 publications. However, a large part of the dissertation uses uncited sources in direct translation - it is a total of 56.5 pages out of all 128 pages contained in Chapters 1, 2 and 3, i.e. 44% of the material presented. The author's proposals from Chapters 1 and 2 occupy, together with the total of 15 figures included in them, only 26 pages or 20%. Besides being inaccuracies, they only hint at various possible applications without actually considering them. Thus, they remain somewhat outside the subject of the dissertation.

Due to the stated considerations, I give a **negative evaluation** to the dissertation work "Application of cellular-neural networks for the study of partial differential equations arising in financial mathematics" by Pavel Todorov Stoinov.

References:

I. Boyarchenko and S.Z. Levendorskii Non-Gaussian Merton-Black-Scholes Theory. World Scientic, River Edge, NJ, 2002.

2. Cont and P. Tankov. Financial Modeling with Jump Processes. Chapman & Hall/CRC Press, New York, 2004.

3. Omur Ugur. An Introduction to Computational Finance. Imperial College Press, 2009.

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