

R E P O R T

on a Thesis for awarding the degree “Doctor of Sciences”

Title: Some Classes of Non-commutative Rings and Abelian Groups

Author: Dr. Peter Vassilev Danchev

Scientific field: 4. Natural sciences, mathematics and informatics

Professional field: 4.5. Mathematics

Overview

This thesis is devoted to problems from ring theory and the theory of abelian groups. The goal is to present new characterization and structure results for various classes of rings and abelian groups. It summarizes the research by the author from the last decade. My general impression is that the author is well acquainted with the state of the art and the most recent results in ring theory and group theory. A big part of the treated problems are considered in the field as important theoretically. The author demonstrates deep knowledge of his field of research and capacity to apply his knowledge to the solution of important problems.

Description of the results

This thesis amounts 225 pages of text and consists of an introduction, four chapters, conclusion and a list of references including 110 items. The first two chapters are introductory. The first of them describes the main goals of this thesis, as well as the main algebraic objects under investigation. The author presents some publications and results that serve as a motivation for his research. Chapter 2, which is very short, contains a list of some abbreviations used further throughout the thesis.

The main results are contained in chapters 3 and 4. Each of these contains two sections. The first section of chapter 3 contains characterization results for various classes of rings: clean, weakly-clean, exchange, weakly exchange, UU-, JU-, invo-clean rings etc. A typical result is e.g. Theorem 1.49 saying that a ring is an invo-clean ring iff and only if it is the direct sum of two rings R_1 and R_2 . The first of them is an invo-clean ring of characteristic at most 8, which is nil-clean, and the second ring is an imbedding in a direct sum of several copies of \mathbb{F}_3 . The second section of chapter 3 contains as a main result a characterization of the group ring $R[G]$ as a UU-ring. It is proved that if the group ring $R[G]$ is a UU-ring then R is a group ring and G is a 2-group. This condition is sufficient if G is a locally finite ring.

Chapter 4 contains characterization results for abelian groups. It is again subdivided in two sections. In the first of them (section 3) the author investigates generalizations of different algebraic objects like transitive and fully transitive abelian groups. The second of them contains results on simply presented abelian p -groups. The thesis ends up with a list of open problems which are considered as significant.

Remarks and comments

I have the following remarks, questions and comments related to this thesis:

- (1) The structure of the main text is a bit confusing. I do not think it good to have a single chapter devoted to abbreviations (of just one page). A similar remark can be made about the open questions and the list of references.
- (2) In this connection the enumeration is not set properly. For instance the problems with numbers 4.85, 4.86 etc. are in section 5. A more natural numbering would assign them numbers 5.1, 5.2 etc.
- (3) Some of the results are called propositions and some – theorems. What is the difference between a proposition and a theorem? Both are used in the text.
- (4) The characterization results are presented as internal problems to abstract algebra. Are they connected in some way to other fields outside abstract algebra?
- (5) The numbering of the theorems and the propositions in the thesis is different from the numbering in the author's summary.
- (6) I noticed certain inconsistencies in the impact factors of the listed papers, but they still meet the minimal national requirements by quite a margin.

All the above remarks are insignificant mathematically and do not change my very good impression of the deep research carried out by the author.

Publications related to the thesis

The results of this thesis are published in 16 papers. Seven of the papers are in journals with an impact factor:

- Mediterranean J. Math. (Q1,2018)
- Results in Mathematics
- Archiv der Mathematik (Basel)
- Journal of Group Theory (2)
- Houston J. of Mathematics

- J. of Commutative Algebra

Two of the remaining papers are in journals with a SJR, and the remaining seven are in other international journals not listed in the most important databases. In six of the publications Peter Danchev is the only author. In the remaining eleven he has one coauthor. I accept that the contribution of Dr. Danchev is significant in all presented publications.

The candidate presents a list of 30 citations of his results in papers appearing in refereed journals. I accept that his results are well-known and highly valued in the professional community.

The presented publications meet the minimal national requirements as given in the corresponding documents.

Author's summary

The author's summary is made according to the regulations and reflects properly the main results and contributions of this thesis.

Conclusion

This thesis is focused on problems from ring theory and the theory of abelian groups that are important theoretically. The obtained results mark a considerable progress in these fields of abstract algebra.

I am deeply convinced that the presented thesis "Some Classes of Non-commutative Rings and Abelian Groups" by Peter Vassilev Danchev contains results that are an original contribution to the theory of rings and the theory of finite abelian groups. The candidate demonstrates deep knowledge of his field and the capacity to develop it in a new and important way. With this, he meets the national requirements prescribed by the law and the specific regulations of the BAS and the IMI-BAS for the professional field 4.5 Mathematics. I assess **positively** the presented Thesis and recommend that this panel awards **Peter Vassilev Danchev** the scientific degree "Doctor of Sciences" in the scientific field 4. Natural sciences, mathematics and informatics, professional field 4.5 "Mathematics".

Sofia, 10.06.2020

Member of the Scientific Panel:

(Prof. Ivan Landjev)