Opinion

On DrSci dissertation

Some classes of noncommutative rings and abelian groups
For the scientific degree "Doctor of Sciences" in
Area of higher education: 4. „Natural Sciences, mathematics and informatics"
Professional area: 4.5 „Mathematics"
Scientific specialty: „Algebra and number theory"
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This opinion is prepared and presented on the basis of order № 75 from 28.02.2020 of the Director of IMI-BAS for appointment of Scientific Jury and the minutes of the first meeting of the Jury conducted on 17.03.2020.

1. **Presentation of the candidate.** Peter Danchev has completed secondary education in 19898 in OMG Plovdiv and higher education (MS) in 1995 in FMI of Plovdiv University. He has defended PhD dissertation in professional area 4.5 (Mathematics) in scientific specialty Algebra and number theory in IMI-BAS in 2018 entitled "Associative rings with unity and weakly unipotent multiplicative groups" in 2018 after a PhD pursued in IMI. Since 2018 he works in IMI-BAS, before he has worked in TU Sofia, NUTI, 120 SOU, and others).

2. **Materials presented and preliminary defense.** All required for preliminary defense admission and defense admission materials are presented in IMI in time and correspondingly to the requirements of the Regulations of IMI. The preliminary defense is conducted on 14.02.2020, followed by a positive evaluation for the defense preparadness.

3. **Theme and actuality of the dissertation.** The dissertation is devoted to important questions from the theory of certain classes of clean rings and abelian groups. The problems under consideration are in two major directions – investigations of generalizations of clean rings and corresponding group rings and investigation of certain classes of abelian groups as in both directions significant structural and classifiacation results are obtained. These themes are actual and this is proved by the works from other authors in the same area and from the citations. The list of references includes 110 titles of other authors, classical and recent, which means that the candidate is well educated in the area.

4. **Methodology of the investigation.** The author uses both classical and ad hoc developed algebraic approaches. The proofs show the difficulties surmounted as some of them underline possible further generalizations (for example, by weakened conditions) and corresponding obstacles. Open problems are formulated which marks up to certain extent the level of the investigations in the corresponding area; similarly do the presented examples.

5. **Content and results on the dissertation.** The dissertation is written in English, has 230 standard pages and consists of two introductory parts, two main parts, each of them devoted to one of the two directions mentioned above (noncommutative rings and abelian groups), a section with open problems, reference section, list of authors' publications related to the dissertation with 16 items and list of 31 citations of these publications. Authors' reference for scientific contributions accordingly to the national requirements is presented.

   The first two parts contain a survey of the literature and some of the main definitions and notations need for the further exposition, respectively.

   In the first main part, entitled „Noncommutative rings" and divided into two parts, „Weakly exchange rings" and „Application to group rings", several main results are proved. Different classes of rings are considered, some of them introduced by the author, and classification results

are proved, for example (Theorems 1.6 and 1.8), that a ring R is weakly exchange (clean) if and only if R/J(R) is so and 2 belongs to J(R), (part 1.2) JU-rings (Jacobson units) contain some classes of clean rings, a structural description of the so-called (strongly) n-periodic clean rings is obtained. A criterion when the group ring R[G] is a UU ring is proved.

The second main part, „Abelian groups", is divided into several parts. First (strongly) socle-regular abelian p-groups are considered as a generalization of socle-regular p-groups as many structural results are proved (most of them for direct sums) and interesting examples are given. The relations to different kinds of transitivity are described in detail. Projectively (strongly) fully transitive and commutator fully transitive abelian groups are considered and important examples are given. In the last part an application of the set theory is considered as first total Σ-groups are introduced and equivalent characterizations are proved and then further generalizations are considered. Some variations of the idea for n-simply presented groups are investigated and this class is generalized as the generalizations are considered in detail in order to have description of the relations between them. This sets extension and generalization of the main homology theorem of Nunke from 1967. The behavior of specific classes with respect to taking subgroups and quotients is analyzed.

6. **Contributions of the dissertation.** The "autoreferat" presents a reference to the scientific contributions in the dissertation. I accept this reference and completely agree with its claims. The results obtained and their number meet, in my opinion, the high standards of IMI for a DrSci dissertation.

7. **Critical remarks and recommendations.** Writing the dissertation on the basis of papers leads to some repetition – for example the definition of clean ring appears in the beginning (of course!) but also in Definition 1.33, similarly for nil-clean ring. The great productivity of Danchev is remarkable but I think that he has met the moment when he has to focus on smaller number of prestigious publications – the dissertation shows that he has ability to do this. I would also recommend more formal style of his oral presentations in front of professionals. These remarks do not diminish the dissertation's values anyway and do not change my positive evaluation for the work of Danchev.

8. **Publications and citations related to the dissertation.** The dissertation is based on 16 papers published as follows:
   - 10 papers in journals with impact factor and/or impact rang;
   - 6 paper in other (international specialized) journals.
   A list of 31 citations of the papers of the dissertation is presented.

9. **Authorship of the results.** The authorship of the publications is as follows: 5 are single-authored and in 11 Danchev has one co-author. I think that the contribution of Danchev in the joint publications is equal to the contribution of the corresponding co-authors.

10. **Conclusion.** The dissertation presented complies with the requirements of the Law for development of academic personnel Republic of Bulgaria and Regulations of BAS and IMI-BAS for applying the Law. The results obtained allow me to propose to the scientific Jury to award the scientific degree „Doctor of sciences" to prof. assist. Peter Vasilev Danche, PhD, in area of higher education 4. „Natural sciences, mathematics and informatics", professional area 4.5 „Mathematics".

08.06.2020

Signature:

Prof. Dr.Sci Peter Boyvalenkov