

# REVIEW

**by Professor D.Sc. Ivo Mihaylov Mihaylov  
FMI at the University of Shumen "Episkop Konstantin Preslavski"  
in a promotion (competition)  
for the title of the academic position of "Associate Professor"  
in the area of higher education  
4. Natural sciences, mathematics and informatics,  
professional field: 4.5. Mathematics,  
scientific specialty "Algebra and Number Theory",  
announced in DV no. 89 / 16.10.2020**

Based on a decision of the Scientific Council of IMI-BAS (Minutes № 11 / 27.11.2020) and an order of the Director of IMI, № 217 / 14.12.2020, I was elected a member of the scientific jury for the election of candidates for the academic position "Associate Professor", for the needs of the Institute of Mathematics and Informatics - BAS in the area of higher education 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics, scientific specialty Algebra and number theory (Commutative group rings and abelian groups) announced in DV no. 89 / 16.10.2020. By decision of the Scientific Jury (Minutes 11 of 27.11.2020) I was elected a reviewer.

The submitted documents for participation in the competition for the academic position of "Associate Professor" for the needs of IMI of BAS belong to the only candidate Chief Assistant Professor D. Sc. Petar Danchev.

The documentation submitted by the candidate meets the requirements of both the Law for development of the academic staff in the Republic of Bulgaria - ZRASRB (as amended on February 25, 2020), the Regulations for application of the law for development of the academic staff (PP ZRASRB, DV from 19.02.2019 г.), as well as the Regulations for application of the academic staff of BAS and the respective Regulations of IMI - BAS.

## **Brief biographical reference**

Ch. Assistant Professor Petar Vassilev Danchev graduated from high school in 1989 at OMG "Acad. K. Popov", Plovdiv. He graduated from higher education in 1995 in - FMI of PU "P. Hilendarski", Plovdiv with a specialization in "Algebra and Number Theory". His teaching activity is carried out in PU "P. Hilendarski", Plovdiv - 1993-1994, OMG "Acad. K. Popov", Plovdiv - 1995, TsUNT, Plovdiv - 1995-1996, 127 SOU "I. N. Denkoglu", Sofia - 2002-2016, NUTI, Sofia - 2010-2011, Technical University, Sofia - 2017-2018, IMI of BAS, Sofia - 2018-2019. He has a Doctor's degree from 2018 and a Doctor of Science degree from 2020. He speaks Russian and English.

## **General description of the submitted materials in the promotion (competition)**

Petar Danchev has provided an official certificate that he has 2 years and 9 months of experience as an assistant and chief. Assistant Professor at IMI of BAS. This satisfies the requirements of Art. 29. (1) ZRASRB for holding the academic position of associate professor.

In this competition, Ch. Assistant Professor Petar Danchev participated with 23 articles, of which 22 are independent, 4 of them with IF and 4 with SJR, and 1 is joint. The publications, citations and other evidence on the various indicators in the reference for fulfillment of the minimum requirements under this competition have not been used for

obtaining the educational and scientific degree "Ph. D." and for obtaining the scientific degree "Doctor of Sciences" (Art. 24, para. 1, item 3 of ZRASRB).

Data for 24 citations of the articles proposed for the competition are provided. A table is also provided to cover the requirements of a candidate in a competition for the academic position of "Associate Professor" at IMI-BAS, which are the same as in ZRASRB (as amended on February 25, 2020). All points are covered, although the presence of a D. Sc. in section E is omitted.

### **General characteristics of the candidate's works**

The candidate has facilitated the work of the reviewers as much as possible by providing an author's reference in which he has described in detail the contributions of each article and the approbations of the results. My examination confirms the opinion of the candidate, following is the description of the works. The articles submitted for the competition can be divided into the following two groups:

**Group I:** Commutative group rings - these are the articles with numbers [1], [2], [4], [6], [8], [9], [12] - [16], [18] - [23] ].

In commutative [1] and [6], commutative group algebras of summable Abelian periodic groups and their derivative generalizations are considered. The problem of the American mathematician Warren May for the isomorphism of commutative group algebras of Abelian groups with totally projective (= simply represented) periodic components over a field with a simple characteristic is solved.

The article [2] describes to the nearest isomorphism the maximum divisible subgroup of the group of normalized units in a commutative group ring over a field with arbitrary characteristic and in any abelian group serving as a basis. The result is final in this way and cannot be improved.

The article [4] calculates the Warfield invariants of the group consisting only of normalized reversible elements in Abelian group rings of any Abelian group above a ring with some additional conditions introduced on it, such as degree divisibility, characteristic, etc. In this way, its complete characterization is given for some very broad and very important classes of Abelian groups, such as the class of totally projective (or in other words, the class of simply represented) groups.

The articles [8], [9], [12], [13], [14], [15] and [16] are devoted to the description and decomposition of the group of normalized reversible elements in a commutative group ring of an arbitrary Abelian group above a ring. in which the characteristic is either a prime number or the prime powers of its elements are in some way invariant (and / or divisible) in the ring itself. The results obtained in this way are closely related and give a very positive effect on the classical problem of the direct multiplier.

On the other hand, the articles [18], [19], [20], [21], [22] and [23] deal with the complete characterization of different types of commutative group rings, such as the very important for theory and applications classes of nil-pure, soft nil-pure, invo-pure, soft invo-pure, weakly tripotent, periodic and  $\pi$ -regular rings. These results significantly expand some of the classical achievements in this area of group ring theory, as well as further develop the idea, and in some cases even use a different approach, from the article by P.V. Danchev and W.Wm. McGovern, Commutative weakly nil clean unital rings, J. Algebra (5) 425 (2015), 410–422, which article can be considered innovative in this section and has numerous citations. In addition, a new proof of a result of this type for the class of nil-pure rings, originally proved by W.Wm. McGovern co-authored with two other Indian mathematicians in the journal J. Algebra & Appl. (2015).

We will also note that in all the articles in this section innovative methods from the theory of rings and Abelian groups have been used, and significant technical difficulties related to the calculations of the respective cardinal invariants have been overcome.

**Group II:** Abelian groups - these are the articles with numbers [3], [5], [7], [10], [11], [17].

In the article [3] many diverse properties of the broad subgroups of arbitrary Abelian groups are studied in detail. These subgroups play a significant (one might even say very important) role in the general structural theory of Abelian groups. A detailed overview of the results already obtained in this industry has been made.

The article [5] proved a short but very substantial result in which an important classical result of the American mathematician John Irwin concerning the enumerated extensions of separably projective groups was proved in a new way, and the required conditions placed on the Abelian group were significantly reduced. In other words, it has been shown that some of the restrictions on the group are completely unnecessary. The importance of the result will only be noted that it is already known as "Danchev's Lemma" in the newly published monograph *Abelian Groups* (2015) by one of the doyens of this theory Laszlo Fuchs from the University of New Orleans, Louisiana.

The article [7], which is again cited in the above-mentioned monograph of Fuchs, characterizes the completely invariant subgroups of the summable Abelian groups, and the newly obtained results give some significant impact on the general structural theory of Abelian groups.

Articles [10] and [11] deal with the study of the interaction between some sufficiently extensive classes of Abelian groups, showing that the cross section of these classes is already well known and fully characterizable in Zermelo-Frenkel's standard axiomatics together with the Axiom for The choice. These two articles also use methods from set theory and in particular the Continuum Hypothesis in some of its variants.

The paper [17], co-authored with renowned mathematician and former director of the Dublin Institute of Technology Brendan Goldschmidt, examines and examines the behavior of projective-invariant subgroups of Abelian groups and their impact on Abelian groups themselves and some modular structures, such as the ring. from endomorphisms of these Abelian groups. The main result was reported at an international conference in Oberwolfach (Oberwolfach), Germany, in 2012.

Finally, we will emphasize again that in all articles in this section innovative methods from Abelian group theory and set theory are also used, and some specific applications and subsequent non-trivial generalizations in module theory have been found.

### **Notes and recommendations**

The candidate's CV reflects the disciplines he has taught or can lead. The candidate also has experience in developing documents for accreditation of doctoral programs; and experience in developing research projects. In my opinion, it would be appropriate to have a more detailed reference, perhaps as a separate document for these activities (conducted training courses, seminars, workshops, etc.), because the competition is for an academic position and not for a scientific degree. As I have no direct observations on these activities of the candidate, I can only judge from the scarce information provided in the CV and recommend the candidate to achieve an even better balance between his research and organizational, administrative and teaching activities in the IMI of BAS. Regarding the scientific activity, I can only admire the productivity of the candidate, who has 356 publications, 80 of which in impact factor journals and over 500 citations. In this regard, I recommend the candidate to support the development of the scientific staff of the Algebra and

Logic Section of IMI of BAS, including through the guidance of doctoral students on the subject of the competition.

### **Personal impressions**

I have known the scientific activity of the candidate for more than 3 years. I have been a jury in previous competitions in which he has participated. I have worked with him on several university projects. I am impressed by his professionalism and productivity in the field of algebra.

### **CONCLUSION**

Based on what has been said so far about the presented materials, scientific papers, their significance and the scientific contributions contained in them, I believe that Ch. As. D. Sc. Petar Vassilev Danchev **meets** all the requirements of ZRASRB (as amended on February 25, 2020), the Regulations for application of ZRASRB (PP ZRASRB, DV from 19.02.2019 r.), as well as the Regulations for application of the academic staff of BAS and the respective Regulations of IMI – BAS for the academic position of "**Associate Professor**". Therefore, I **strongly recommend** to the esteemed Scientific Jury in the announced competition to propose to the esteemed Scientific Council of the Institute of Mathematics and Informatics at BAS to **elect** Ch. Assistant Professor Petar Vassilev Danchev for the academic position of "**Associate Professor**" in the area of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5, scientific specialty "Algebra and Number Theory", for the needs of the Institute of Mathematics and Informatics - BAS.

19.02. 2021

Reviewer:  
(Prof. D. Sc. Ivo Mihaylov Mihaylov)