Statement

on the application of Peter Vasilev Danchev
participant in the procedure for the academic position "Associate Professor"
at the Institute of Mathematics and Informatics, BAS.

The procedure was announced in State Gazette no. 89/16.10.2020
Scientific field **4. Natural sciences, mathematics and informatics.**
Professional field **4.5. Mathematics**,
Specialty **Algebra and Number Theory (Commutative group rings and abelian groups)**

Reviewer **Tatiana Gateva-Ivanova, PhD, Professor, American University in Bulgaria**

The procedure is announced in State Gazette (Durzhaven vestnik) no. 89/16.10.2020
There is unique applicant for the position:
Peter Vasilev Danchev, Assistant Professor , PhD, DSci, IMI BAS

1. Personal Data. Dr Peter Vasilev Danchev studied in the Magister programme, "Mathematics and Number Theory" of the University Paisii Hilendarski, Plovdiv 1989-96. In 2018 he received a PhD degree in Mathematics, Algebra and Number Theory from IMI BAS, and in 2020 Peter Danchev was awarded the title Doctor of Sciences in Mathematics. Since 2018 the applicant has been working at IMI BAS. His scientific interest is in Algebra: abelian groups, modules, group algebras, associative algebras and theory of matrices. P. Danchev has published 356 papers in International Journals, among which 80 are with Impact Factor. The number of citations on his works is above 500.
P. Danchev has teaching experience: he has given courses in Linear Algebra and Analytic Geometry, Abstract Algebra, and Real Analysis. He speaks English and Russian fluently.
According the info presented by the candidate he has international experience and adaptes reasonably well in international scientific environment. His experience includes managing and coordination of a team in the area of education and science, preparing documents for accreditation of doctorate programmes, preparing scientific proposals.

2**. General description of the presented materials.** A total of 23 publications have been presented for participation in the procedure: 4 papers published in specialized international journals with impact factor (two with quartile score Q2 and one with score Q3), 4 papers published in scientific editions with SJR (without impact factor), one of the papers is joint with Goldsmith, 23 have unique author, the applicant.

The papers can be naturally split into two groups:
(1) Commutative group rings – these are papers [1],[2], [4], [6], [8], [9], [12]-[16], [18]-[23]
(2) Abelian groups- papers [3], [5], [7], [10], [11], [17].

3. General remarks on the research of the applicant. The research interest of the applicant is in the area of abelian groups, modules, group algebras, associative rings, theory of matrices.

 4. Scientific Contributions.
The papers presented for the procedure can be naturally split into two groups, I shall discuss
each group separately.
(1) Commtative group rings – papers [1],[2], [4], [6], [8], [9], [12]-[16], [18]-[23]
One of the most cited works of Peter is the joint paper: Danchev, Peter V., and W. Wm. McGovern. "Commutative weakly nil clean unital rings." *Journal of Algebra* 425 (2015): 410-422.
Inspite of the fact that the paper is not in the list of works presented for the procedure it deserves a

recognition- the works [18], [19], [20], [21], [22] и [23], are closely related to it, they extend and deepen its results. In these works the author gives a complete characterization of various types of abelian group rings such as classes of feebly nil-clean weakly nil-clean, invo-clean, weakly invo-clean, weakly tripotent, periodic and π-regular rings. The results notably extend some classical results in this area of the theory of group rings, and deepen the ideas of P. Danchev and W.Wm.McCovern. In some cases entirely new original ideas and techniques are presented.

The papers [8], [9], [12], [13], [14], [15] и [16] study the characterization and give a description of the decomposition of normalized inits in abelian group algebras over a commutative ring of prime characteristic, or a ring in which the prime degrees ot its elements are invariant (and/or) divisible in the basic ring. The results are closely related to the classical problem of the direct factors.

The papers [1] and [6] present results on commutative group algebras of summable abelian periodic groups and their various generalizations. P. Danchev partially solves the problem for the isomorphism of commutative group algebras with totally projective primary component of the basis group, posed by Warren May. He gives a complete description (up to isomorphism) of the maximal divisible subgroup of the normalized group of units in a commutative group algebra over a field of arbitrary characteristic and with aritrary abelian group given as a basis. The description is in terms of the basis field and the abelian group. The result is complete.

In papers [4] are found the Warfield invariants of important cases for the normed unit group of an abelian group ring. More precisely, these are the classical Warfield q-invariants $W_{\alpha,q}(VR(G))$ of the group VR(G) of all normalized units in the group ring R(G) . The invariants are given in terms of the commutative unital ring R of prime characteristic p and the abelian p-mixed group G.

(2) Abelian groups are studied in   [3], [5], [7], [10], [11], [17].

The maximal divisible subgroup of the normalized group of units in a commutative group algebra is completely described in terms of the basis field and group.

In paper [3] are studied characteristic properties of large subgroups in primary abelian groups. These subgroups have significant importance in the general theory of abelian groups. The author also gives a referative information on the progress in the area.

An interesting result is proven in [5]. The author presents a new proof of a classical result of J. Irwin. More precisely, Danchev proves that if G is a pure $p^{\omega + n}$-projective subgroup of the separable abelian p-group A for $n\in {N}\cup \lbrace 0\rbrace $ such that $| A/G|\le \aleph _0$, then A is also $p^{\omega +n}$-projective. This is a generalization of results of Irwin-Snabb-Cutler, which shows that the necessary conditions imposed on the abelian group in the classical results are in fact not necessary. This result is now named "Danchev's Lemma" in the monograph "Abelian Groups" by Laszlo Fuchs (one of the leaders in the area) in 2015. The importance of paper [7] is considered in the same monograph. In the paper are characterized fully invariant subgroups of summable abelian groups. These results have an impact on the general structure theory of abelian groups.

 In [10] and [11] are studied the close relations between some significantly large classes of abelian groups. The author shows that the intersection of these classes is in fact completely characterized in the standard axiomatic system of Zermelo–Fraenkel set theory and the Axiom of choice. In particular, it is proven that the every quasy complete Q-group is bounded, cf [10]. Methods from set-theory, more particularly the continuum hypothesis, are used. Joint with Brendan Goldsmith, in [17] Danchev studies the behaviour of projection-invariant subgroups of Abelian p-groups and how

this behaviour affects the abelian group itself, or other module structures such as the ring of endomorphisms of the abelian grpup. The main results are presented on a Conference in Oberwolfach, Germany 2012.

As a conclusion I shall note that the candidate is well informed about the main problems in the area and the literature background of the topics of study. The candidate presents 24 citations of the papers participating in the procedure. He meets all specific requirements of IMI for the academic position Associate Professor.

5. Significance of the contributions. The results of the publications participating in the procedure are interesting and original. They contain new information about objects which appear in a natural way in various areas of mathematics and are actively investigated by various international authors. The results and the new ideas and original methods applied for their verifications have been used by other authors and possibly will be used for future investigations in the area. The significance of some of the results and new ideas of P. Danchev have been noticed by mathematicians with well known names such as Laszlo Fuchs, Brendan Goldsmith, Jason Bell.

6. Critical notes and suggestions.
We kindly recommend the candidate to be concentrated mainly on producing high quality papers and publish them in well respected international journals with quartile score Q1, (or at least Q2) and not so much on the number of publications.

The candidate claims that in the presented works technical difficulties have been resolved and innovation methods presented, and I believe this is true. However, these claims would be better evaluated if he has given some more concrete details about "the difficulties" and has explained briefly what is the escence of his "innovation methods".

CONCLUSION

Dr Peter Danchev has received interesting original results in areas of algebra under active investigation. Most of his results have already been used and can be used for future investigations in the area. Some of the results are published in respected journals with IF. Peter is actively working with a great enthusiasm and motivation.
I am convinced that Dr. Peter Vasilev Danchev has all the merits and professional qualifications required for the position of associate professor of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. He fulfills all the legal requirements plus the specific ones of IMI BAS for the Scientific field 4. Natural sciences, mathematics and informatics, Professional field 4.5. Mathematics, Specialty Algebra and Number Theory (Commutative group rings and abelian groups). I recommend the honorable Scientific Jury to propose the Scientific Council of IMI-BAS to appoint Peter Vasilev Danchev as "Associate Professor.

Sofia 25 February, 2021

Reviewer:

Professor Dr Tatiana Gateva-Ivanova