REER REVIEW

For the completion for the academic position "Professor"

for the needs of the Institute of Mathematics and Informatics

at the Bulgarian Academy of Sciences

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Area of Higher Education**: 4. Natural Science, Mathematics and Informatics**

Professional Field: **4.5 Mathematics**

Scientific Specialty: **Algebra (Non-commutative Rings and Algebras)**

1. Biographical data. Assoc. Prof. Peter Vassilev Danchev was born in Plovdiv (Bulgaria) in 1970. He graduated in 2018**.** From then until now, the entire scientific career of Assoc. Prof. Danchev was connected with BAN. In 2020, he completed is Doctor of Science there. He held various academic positions: 2018 until 2020, he was Assistant, 2020 until 2021, he was Assistant Professor and from 2021 until now, he is Associate Professor. Assoc. Prof. Danchev worked at several national projects: 2019 until 2020 in a project leaded by Vesselin Drenski and Ivan Chipchakov, financed by the "Scientific Research Found" of the Bulgarian Ministry of Education and Science; 2020 in a project leaded by I. Mihailov; and 2022 until 2024 in a project leaded by S. Harizanov financed by the European Union. He also worked in three international projects: 2019 until 2021 in a project with Hungaria leaded by Vesselin Drenski; 2022 until 2023 in a project titled by "Junta de Andalusia" financed by the University of Andalusia; 2023 until 2024 in a project "Trends in Ring and Module Theory" financed by the Turkish Ministry of Education and Science. Assoc. Prof. Peter Vassilev Danchev is author of 402 papers are published in referred/non-referred journals, proceedings of scientific conferences and publications of Universities. The papers are in the period from 1993 until 2024. His first publication has 3 pages, In 1995 has three short publications, in 1996 one, in 1997 four, in 1998 three, in 1999 two, in 2000 eight, in 2001 twelve, in 2002 eight, in 20003 six, in 2004 thirteen, in 2005 sixteen, in 2006 he has 23 publications and the first one with a coauthor, namely with K.S. Nedelchev, published in Carpathian Journal of Mathematics. The previous 100 publications were with singleton author. In 2007, he has 23 and in 2008, he has 28 publications, where one of the is with coauthor K.S. Nedelchev, published in Journal of Advances of Mathematical Studies and another one is a note with P.W. Keef in Archivum Mathematicum (Brno). In 2009 he has 25 publication (two of them with coauthor: Keef and Goldsmith, respectively), 2010 thirteen publications (three of them with coauthor), 2011 twenty (4 with coauthor), 2012 twentyone (4 with coauthors), 2013 and 2014 eleven in each year. In 2015 he has 16 publications, where one of them is with coauthor Ivan Chipchakov. In 2016 until 2019 he has 9, 25, 21, and 29, respectively, publications. 2020 he has 13 publications (5 of them with coauthors and in high ranked journals) Also in 2021 he has 13 publications, mainly with coauthor(s) and in high ranked journals. In 2022 there are 12 publications with different coauthors. The list for 2023 and 2024 (accepted papers) is not yet complete. The presence of manly joint papers in the last four years speaks of successful teamwork since his membership at BAN. Moreover, many of the papers are coauthored with different authors. This shows that

Assoc. Prof. Danchev has a wide network. But I would recommend the applicant to find PhD students and lead them to a successful defense, which would increase his scientific authority in the algebraic community. Moreover, Assoc. Prof. Danchev is member of the Editorial Board of the Turkish Mathematical Journal.

2 Description of the present documentation The decimation presented by the applicant Assoc. Prof. Peter Danchev is in accordance with the requirements of the law and accompanying rules of the Institute of Mathematics and Informatics. It contains : the statement that he want to apply for the position Professor; CV; copies of the diplomas for Master in science, PhD, Doctor of Sciences and Associate Professor; list of all publications and the list of the publications presented for the competition; a document confirming the scientific position he has at IMI; the announcement for the competition in the Newspaper of State; proofs that he covers the minimal scientific requirements of the law of the position Professor; the list of citations and the list of citations participate at the competition. Assoc. Prof. Danchev has presented for the competition 15 papers, 7 without coauthor, 6 with one coauthor and 2 with two coauthors, but altogether with seven different coauthors: D. D. Anderson, J. Cui, J. P. Bell, T.-K. Lee, A. Cimpean, E. Garcia, M.G. Lozano. 14 of the papers are published in the period 2019 until 2023. One of the paper is accepted in Ricerche di Mathematica for a publication in 2024. The papers are strong related to the field of (Non-)commutative rings and Algebras and I accept them for reviewing. The papers are published in the following journals: Turkish Journal of Mathematics (1), Chebyshevskii Sbrnik (2), Revista Colombiana de Mathematicas (1), Proceedings of the American Mathematical Society (1), Journal of Algebra and Its Applications (2), Journal of Siberian Federal University (1) Russian Mathematics (1), Israel Journal of Mathematics (1), Journal Algebra & Computation (1), Linear and Multilinear Algebra (1), Vestnik Udmurskogo Universiteta (1), Communications in Algebra (1), and Ricerche di Mathematica (1). Let me note that Proceedings of the American Mathematical Society as well as Communications in Algebra are high ranked journals.

3. General characteristic of the scientific work and achievements of the applicant: The main scientific interests of Assoc. Prof. Peter Danchev are in the field of Ring Theory. In addition, he also has works devoted to problems in classical group theory. In Algebra, Ring Theory is the study of rings , where rings are algebraic structures with two binary operations Addition and Multiplication, where these both operations have similar properties to those defined or integers. Ring Theory originated in algebraic in algebraic number theory, Algebraic Geometry, and in Invariant Theory. The various hypercomplex numbers were indicated with matrix rings. Rings can be used to solve a variety of problems in Number Theory and Algebra. One of the earliest such application was by Fermat to prove the famous two-spare Theorem. There are also examples of rings in Topology and mathematical Analysis. If R is a commutative ring then the ring of matrices over the ring forms an associative algebra over the ring, the so-called matrix-algebra. An essential part of the research of the applicant is devoted the study of matrix algebras. Matrix algebras is one of the fundamental directions in Bulgarian Algebra, for example by Academic Dr. Vesselin Drenski. At the moment, a number of Bulgarian Algebraists in the country and international are working in Ring Theory and topics using Ring Theory, respectively. Here there is a strong opportunity for exchange of ideas and cooperation between Assoc. Prof. Danchev and colleagues from the group. It is well known that there are many problems in Ring Theory, for example given by Cahen, Fontana, Frisch and Glaz in 2013. From this point of view it is natural that the applicant studies matrices and rings, respectively, i.e.

square-zero matrices, diagonal matrices, nilpotent matrices, almost n-torsions clean matrix rings, weakly invo-clean rings, strongly pi-regular rings (with involution) and periodic rings

4. Main scientific and scientific-applied contributions I shall briefly discuss the main results contained in the submitted publications of the applicant, as well as my evaluation of them. I will divide my considerations into particular subjects studied in the publications.

a) Expressions of rings: It is a classical question in Linear Algebra to express any matrix as the sum of two matrices with particular properties. This question is considered in the papers [3, 11, 12] of the list of papers for the competition. In [12], Assoc. Prof. Peter Danchev, together with two co-authors could show that every square matrix over an infinite field can be represented as a sum of a diagonalizable matrix and a nilpotent matrix of order less than or equal to two. Diagonalizable matrices are basically in Linear Algebra. A matrix A is called diagonalizable if there are an invertible matrix P and a diagonal matrix D such that $A=P^{-1}DP$. On the other hand, a square matrix A is called nilpotent if by reaching to a sufficiently high integer power, we get the zero matrix as a result. Nilpotent matrices are of particular interest since they are exactly the matrices with the only eigenvalue zero. This shows that the applicant found a very significant expression. Moreover, in the case of a finite field, such an expression exists for a 2 x 2 matrix and for an n x n matrix (n > 2), whenever the field has less then n+1 elements. The applicant's results for finite fields complete the results by Simon Breaz. In a short note, it is proved that every n x n matrix over a field of odd cardinality is the sum of a periodic and a nilpotent matrix of order 3. This shows that the results of Assoc. Prof. P. Danchev are quite deeper as presented in the mentioned paper. Moreover, the applicant shows that every nilpotent matrix over a field can be expressed as the sum of a potent matrix and a square-zero matrix in [12]. Also these results improve results by A. N. Abyzov, I. I. Mukhametgaliev, and Yaroslav Shitov. In the paper [11] of the applicant, it is proved that each matrix is the sum of a potent matrix and a nilpotent matrix of order at most two, when the Jacobson radical of the ring has zero-square. This is a strict result compared to the existing ones. Moreover, that result improves a previous result of the applicant in [3], which is published in Chebyshevskii Sbornik. In the paper [7], Assoc. Prof. P. Danchev completes his previous results given in [11,12]. In fact, in [12], the Jacobson radical for the ring has zero-square, but in [7], the applicant considers an algebraic closed field of simple a finite field. A complete answer of the question, when each square matrix over an infinite field can be expressed as the sum of a periodic matrix and a nil-potent matrix of order 2 is given by the applicant. In the paper [4], Assoc. Prof. P. Danchev proves that any square nilpotent matrix over a field is the difference of two idempotent matrices. Moreover, the applicant proves that any square matrix over an algebraically closed field is the sum of a nilpotent square-zero matrix and a diagonalizable matrix. This result is related to the applicant's previous result in [12] for infinite fields. In fact, in the paper [4], an algebraic closed field is considered and he can specify the nilpotent matrix.

b) Pi-regular rings: A ring is called pi-regular if a suitable power of any element in the ring is left-regular or right-regular, where an element x is called left (respectively, right) regular if there is an element y in the ring such that $x^2y = y$ (respectively, $yx^2 = x$). With other words, a ring R is pi-regular if for each a in R, there is a positive integer n such that $a^n R$ is generated by an idempotent. If R is left-pi-regular and right-pi-regular then R is called strongly pi-regular. Strongly pi-regular rings were studied since almost 100 years. In the paper [1], of the list of papers for the competition, the applicant introduces a generalization of the concept of pi-regular rings and strongly pi-regular rings, respectively. Assoc. Prof. P. Danchev introduces the concept of a regularly nil clean ring. The motivation of that new concept is given by properties of other ring concepts. In my opinion, for a motivation of a new concept should be more focused to the use of the new concept instead to refer to the introduction of other new concepts. The main results of the paper [4] gives the relationship to other investigated classes of rings. So, I miss a strong justification of the introduction of that new concept. A similar situation, we have in the applicant's paper [15]. Several techniques in the paper [1] are for finite fields. It is well known that finite fields are, surely, not algebraically closed. The purpose of the applicant's paper [2] is to show that improvement is possible by a strengthening of the techniques utilized in [1] in the case of algebraically closed fields. In fact, Peter Danchev´s paper [2] is a note to his paper [1], that improves some of his previous results. The expressions of (nilpotent) square matrices over (algebraic closed) fields in [4] are applied to a variation of pi-regular rings.

c) About Jacobson´s Theorem**:** A ring R is called periodic if for every a in R, there exists two distinct positive integers n and m such that the m power of a and the n power of a coincide. A very recognizable result concerning periodic rings is given in the applicant´s paper [5]. Assoc. Prof. P. Danchev together with D.D. Anderson gives an important extension of the classical Jacobson´s Theorem: Let R be a ring such that for all elements a in the ring, there is an integer n > 1 such that the n power of a coincides with a. Then R is commutative. This theorem was published 1956 by Nathan Jacobson. There are several proofs of that theorem. Several authors have considered the Jacobson´s Theorem under various points of views. So, e.g. Susan Montgomery has generalized that theorem for rings with involution. Assoc. Prof P. Danchev could show that a ring R is commutative if, for each element a in R, there exist two positive integers n and m of different parity such that the m power of a and the n power of a coincide. So, the commutativity for a wide class of a periodic rings was proved, which is an important result in ring theory. One of the most important results by N. Jacobson were theorems which describe the commutativity of a ring. It is a classical problem in Ring Theory to describe a ring is commutative or not. That is the target of the applicant´s paper [9]. Assoc. Prof. P. Danchev together with Janson Bell considered algorithms to decide a ring is commutative or not. As a tool they use polynomial identities on algebras. The main result of the applicant´s paper [9] is a theorem that can be viewed as a kind of machine for producing commutativity theorems. The main theorem of the paper [9] is Theorem 2.5. This theorem makes it able to completely characterize the homogeneous multilinear polynomial identities that force a ring is commutative. Although the main result seams quite technical, its applications show the importance of that theorem. In my opinion, that paper gives an important contribution to the study of (non-) commutative rings.

d) Rings with involution: A ring is called pi uu ring if for each unit u, there is some positive integer n such that the (n^1) - 1 is a nilpotent element. In the applicant´s paper [6], a characterization of pi uu rings is given in terms of other pi uu rings. But the most important result of [6] is the characterization of periodic rings in Theorem 3.4. It is not hard to show that each periodic ring is strong regularly nil clean, using Danchev´s Theorem 3.4. This is an example for the importance of Theorem 3.4 in Danchev´s paper [6]. A ring R is called a ring with involution if there is a binary operation * such that * is an endomorphism on R. In Ring Theory, the version with involution is studied. In the applicant´s paper [6], a characterization of periodic rings with involution is given, which is parallel to the mentioned Theorem 3.4 in the same paper. A characterization of rings in terms of pi uu rings is also given. In [14], Assoc. Prof. Peter Danchev studied (strongly) pi-regular rings with involution. Theorem 2.5 is the main result of that paper. It is shown that a ring with involution is pi-regular if and only if the corresponding ring is pi-regular and for the nil-potent elements or for the Jacobson radical holds an additional condition. The applicant´s paper [14] contributes to the similar study of classes of rings with involution. Another class of rings, for which there are such open problems in the version with involution exist, is the class of weakly invo-clean rings. A ring R is called weakly invo-clean if for each element r in R there are an element v in R, which is self-inverse, and an idempotent e in R such that r = v + e or r + e = v. This concept was introduced by the applicant few years ago. The concept of of a weak involution was also introduced by the applicant in another paper two years later. In fact, we say that a ring R is invo-clean with weak involution if for each r in R there is an element v in R with $v^2 =1$ or $v^2 = -1$ and an idempotent e such that r = v + e. If we have that r = v + e or r + e = v then the ring R is called weakly invo-clean with weak involution. In the paper [13], Assoc. Prof P. Danchev characterizes rings in terms of a direct product of three rings, coming from pairwise different classes of rings. This more or less technical result is the main result of the applicant´s paper [13].

5 Other ring properties: A very important paper by Ass. Prof Dr. P. Danchev is the paper [10]. In that paper, the applicant (together with his coauthor Tsiu-Kwen Lee from Taiwan) studies the n-generalized commutators of rings and prove that if R is a non-commutative prime ring and n > 2 then every n-generalized Lie ideal of R contains a non-zero ideal. The most important result of that paper [10] is that each element in a non-zero n-generalized Lie ideal is of the form $(a_1 \ldots a_n) - (a_n \ldots a_1)$. This generalized a result by Israel Herstein. He has proved that every element of a simple Artinian ring is the sum of tree generalized commutators. In the paper [10], which is a paper of 32 pages that is published in the high ranked journal “Journal of Algebra and Its Applications”, has also several other very important results, for example Theorem 5.1, which shows that every n-generalized Lie ideal of a non-commutative prime ring contains a non-zero ideal, where the concept n-generalized Lie ideal were introduced by the applicant in paper [10] as a generalization of the well-known concept of a Lie ideal. In the paper [8], Assoc. Prof. Dr. Danchev, together with his coauthor A. Cimpean, answers to questions given in his previous papers. In the paper [8], the triangular ring over two element fields and the full matrix ring over a two-element field are described in term of an almost n-torsion clean rings. The latter one is almost m-torsion clean as posted in Theorem 2.1 in [8]. This Theorem 2.1 is not correct presented in [8] since m cannot be fixed. Finally, the applicant could show that the n x n triangular ring is almost clean if and only if n is a power of 2.

5 Significance of contributions to science and practice. The results obtained in the research papers of the applicant are interesting and meaningful. They contain new information about subjects which appear in Ring Theory, Linear Algebra, and also in other areas of mathematics. Assoc. Prof. Dr. Danchev´s results extend often the results of other authors. The results and the methods to obtain them can be successfully used in other research of this kind in the future.

6. Critical remarks and recommendations. I do not have essential critical remarks to the publications of the applicant.

7. Personal impressions for the applicant. I know personally Assoc. Prof. Dr. Peter Danchev from the time he was working at his PhD Theses at IMI-BAN, as a member of the team of the Department of Algebra and Logic, and from his talks at the Algebra and Logic Seminar. I have presented some of his papers in the journal “Asian European Journal of Mathematics” after positive reports form experts. I have excellent impressions of him as a colleague and scientist.

Conclusion

In the present scientific publications, Assoc. Prof Dr. Peter Danchev, has obtained interesting results in current areas of Ring Theory and Linear Algebra. Most of the results have already been used or can be used by other researchers. Several results have been published in respectably journals. I have every reason to confidently propose Assoc. Prof. Dr. Peter Danchev for the position of "Professor" of the Area of Higher Education 4. Natural Science, Mathematics and Informatics, Professional Field: 4.5 Mathematics, Scientific Specialty: Algebra (Non-commutative Rings and Algebras).

Blagoevgrad, November 8, 2023 Referee:

(Prof. Dr. Jörg Koppitz, IMI-BAS)