R E V I E W

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about the competition for acquiring the academic position of
“Professor”
at the Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences,

Research area: 4. Natural Sciences, Mathematics and Informatics,
Professional field: 4.5 Mathematics
Scientific specialty: Algebra (Non-Commutative Rings and Algebras).

In the competition for the academic position of "Professor", announced in the State Gazette, issue 69/11.08.2023 and on the website of IMI for the needs of section "Algebra and Logic" at the Institute of Mathematics and Informatics at Bulgarian Academy of Sciences, as a candidate participates Associate Professor DSc Peter Vassilev Danchev.

1. General description of the presented documents.

The presented documents are:

1. Application by Associate Professor Peter Danchev for participation in the competition, 11.09.2023.
2. Curriculum vitae in the common European format. The candidate was careless with the document - the CV is incomplete. In 2019, Peter Danchev defended his dissertation for Doctor of Science, and then he presented a professional CV, which records his entire professional career.
3. Diploma of completed higher education from Plovdiv University.
4. Diploma for the acquired educational and scientific degree "doctor".
5. Copy of diploma for acquired scientific degree "Doctor" from the Institute of Mathematics and Informatics at BAS, 2018.
6. Certificate of obtained scientific degree "Doctor of Sciences" from the Institute of Mathematics and Informatics at BAS, 2020.
7. Complete list of the scientific publications of the candidate.
8. List of scientific papers for participation in the competition.

9. Copies of the papers for participation in the competition.

10. Signed author's reference for the scientific contributions of the works.

11. List of citations.

12. Information on the fulfillment of the minimum requirements for the academic position "Associate Professor" at IMI-BAS.

13. Transcript-extract from the protocol of the Scientific board of IMI-BAS for initiating the procedure.

14. Copy of the State Gazette with the announcement for the competition.

15. Certificate of work experience from IMI - BAS.

16. Two declarations.

2. General characteristics of the candidate's scientific activity.

Peter Danchev participates in the competition with 15 publications. The paper [15] according to the competition publication list is due to be published in 2024, but a check in Springer shows that the real data for this paper are:

Danchev, P.V. A symmetric generalization of π-regular rings. Ricerche mat (2021). https://doi.org/10.1007/s11587-021-00577-1

All papers are in English and have been published in foreign journals. These articles have not been used in the procedures for obtaining the educational and scientific degree "Doctor", the scientific degree "Doctor of Science", and the academic position “Associate Professor”, for which the applicant has applied a declaration.

Seven of the papers are self-contained, six are co-authored (D. D. Anderson in [5], J. Cui in [6] and [14], A. Cimpean in [8], J. P. Bell in [9] and T.- K. Lee in [10]) and two papers in the list have two co-authors - E. Garcia and M. G. Lozano.

The publications are systematized depending on the editions and the corresponding number of points they bring in terms of the requirements for academic positions and scientific degrees, and this systematization is presented in a table in a separate document, and according to some indicators (for publications and for citations) the points exceed the needed minimum. Publications [2], [3], [4], [7], [8], [13] and [14] are included in indicator group B (instead of habilitation work), and they carry 20 points each, a total of 140 points, in the table of scientometric indicators (with a required minimum of 100 points). Publications presented for indicator group D can be grouped according to the points they bring:

- publications [5], [9], [12] and [15] are in quartile Q2 of Web of Science and carry 40 points each;
- articles [6], [10] and [11] are in quartile Q3 of Web of Science and carry 30 points each;
- the paper [1] was published in the Turkish Journal of Mathematics. I checked and found that this journal is also in the Q3 quartile of WoS and also carries 30 points.

The total points in this group are 280 with a required minimum of 220 points.

A reference is presented for 29 citations with which Peter Danchev participates in the competition, with 28 of them carrying 6 points each, and only one carrying 3 points. The total points for this indicator are 171, which exceeds the required minimum of 140 points.

In indicator group E, there are 75 points for the scientific degree "Doctor of Science" and 90 points for participation in projects, a total of 165 points, with a mandatory 150 points.

Associate Professor Peter Danchev has also presented a complete list of 402 publications. They have been published in 195 different journals and proceedings, many of them journals of various universities, academies and mathematical societies around the world. Of these publications, 103 papers have an impact factor, and they are published in 54 different scientific journals.

I also consulted some databases of scientific information. The metric regarding the publication activity of Peter Danchev in Scopus is as follows: 191 documents, 756 citations in 227 documents (including self-citations), h-index 13 with self-citations and 7 without self-citations. It is strange that Peter Danchev appears in Scopus under the name Danchev, Peter Vassilevich.

Peter Danchev has not submitted a reference for lectures and/or exercises led by him. The resume lacks information on where and in what position the candidate worked before 2018.

3. Analysis of the scientific achievements according to the materials submitted for participation in the competition.

I evaluate the contributions from the candidate's research as theoretical. His scientific activity is in the field of algebraic structures, more precisely different types of rings and groups. In his author's reference, Associate Professor Peter Danchev has divided the publications of the competition into seven algebraic areas. In my opinion, the second and seventh areas are better combined, so I divide them into six categories.

1. Nontrivial generalizations of classical regular and π-regular rings (publications [1], [14], [15]). It is proved that π-regular rings are always regular nil clean. However, the opposite is not true, i.e., there exists a regular nil clean ring that is not π-regular. The so-called *D-regularly nil clean rings* are also introduced by showing that these rings give a nontrivial generalization of the classical π-regular rings, their properties are studied in detail and compared with those of classical nil clean rings. Some other close relationships with certain well-known classes of rings such as exchange rings, clean rings, nil-clean rings, etc., are also demonstrated. Recall that a ring R is called strongly π-regular if, for every $a \in R$, there is a positive integer n such that $a^n \in a^{n+1}R \cap Ra^{n+1}$.

2. Representing square matrices as a sum of matrices of special types over different fields (publications [2], [3], [4], [7], [11], [12]). In [3], the author proves that any square matrix over an arbitrary infinite field is the sum of a square-zero matrix and a diagonalizable matrix. In [4], it was proved that any square nilpotent matrix over a field is a difference of two idempotent matrices as well as that any square matrix over an algebraically closed field is a sum of a nilpotent square-zero matrix and a diagonalizable matrix. These two assertions were applied to a variation of π-regular rings. This work finishes with two queries, namely:

Problem 2.4. Extend the considered above property (from Theorem 2.3) for any field F which is not necessarily algebraically closed.

Problem 2.5. Examine those rings R for which, for any $a \in R$, there exists an idempotent $e \in aRa$ such that $a(1-e)a$ is nilpotent.

To find a suitable expression of an arbitrary square matrix over an arbitrary finite commutative ring, Danchev and his coauthors prove in [11] that every such matrix is always representable as a sum of a potent matrix and a nilpotent matrix of order at most two when the Jacobson radical of the ring has zero-square. In [12], the same authors prove that every square matrix over an infinite field is always representable as a sum of a diagonalizable matrix and a nilpotent matrix of order less than or equal to two. In addition, each 2×2 matrix over any field admits such a representation. They show that, for all natural numbers $n \geq 3$ every $n \times n$ matrix over a finite field having no less than $n+1$ elements also admits such a decomposition.

3. Generalization of classical results (publications [5], [9]). These two works summarize classical results such as Jacobson's theorem for commutativity of potent rings and algebras. The Jacobson's theorem states that if a ring R satisfies the property that for each $x \in R$ there exists a natural number $n(x) > 1$ with $x^{n(x)} = x$ (such rings are called *potent*), then R

is commutative. In this case, $x^{n(x)+1} = x^2$ for every $x \in R$. This raises the question: If a ring R satisfies the property that for each $x \in R$, there exists a natural number $n(x) \neq 2$ with $x^{n(x)} = x^2$, must R be potent or even commutative? Both questions have an easy answer "no".

Herstein generalized the Jacobson's assertion by proving that if *R* is a ring with center *Z(R)* such that $x^{n(x)} - x \in Z(r) \forall x \in R$, then R is necessarily commutative. A recent generalization of Jacobson's result is: If R is a ring such that, for any $x \in R$, there are two integers $n(x) > m(x) > 1$ of opposite parity with $x^{n(x)} = x^{m(x)}$, then R is commutative. So, it is quite logical to consider those rings R for which $x^{n(x)} - x^{m(x)} \in Z(r) \forall x \in R$, which is much more complicated and requires additional conditions to obtain a commutativity theorem.

4. New nontrivial characterizations of periodic rings (publications [6], [13]). The paper [6] is devoted to a comprehensive study of the periodicity of arbitrary unital rings. Some new characterizations of periodic rings and their relationship with strongly π-regular rings are provided as well as, furthermore, an application of the obtained main results to a $*$-version of a periodic ring is being considered. In addition, in the last part of the paper the so-called $*$-periodic rings are considered, and these two classes are found to be independent of each other. In [13], Danchev completely described the structure of weakly invo-clean rings possessing weak involution up to equivalence.

5. n-Torsion Clean and Almost n-Torsion Clean Matrix Rings (paper [8]). The authors (coauthor is the Romanian mathematician A. Cîmpean) determine all natural numbers n such that the ring $\mathbb{M}_n(\mathbb{F}_2)$ consisting of all $n \times n$ matrices over the finite field $\mathbb{F}_2$ with two elements and the ring of all triangular matrices over the same field $\mathbb{T}_n(\mathbb{F}_2)$ are either n-torsion clean or almost n-torsion clean. To prove their statements, the authors used some results from Number Theory. Concerning the classical theme of representing matrices as sums (and products of certain elements such as units, idempotents, nilpotents, etc.) one may indicate the following important achievements like these: It was established by de Seguins Pazzisin that if K is a field, then each element in $\mathbb{M}_n(K)$ is a linear combination of 3 idempotents and, in particular, if char(K) is either 2 or 3, then every element of $\mathbb{M}_n(K)$ which is a sum of idempotents is actually a sum of four idempotents; in the case of fields with 2 and 3 elements, then any matrix over these two fields is a sum of three idempotents.

Recall that for some arbitrary fixed positive integer *n*, a ring *R* is said to be *n*-torsion clean if, for each $r \in R$, there exist a unit u with $u^n = 1$ and an idempotent *e* such that *r* = u + *e* and *n* being the smallest possible positive integer having this (decomposable) property. Without the condition for minimality of n, the ring *R* is just called almost n-torsion clean. The our achievements of this work are the following: (1) For an arbitrary natural number n, there exists an integer m, $2 \leq m \leq 4$, such that $\mathbb{M}_n(\mathbb{F}_2)$ is almost m-torsion clean, and (2) $\mathbb{T}_2(\mathbb{F}_2)$ is 2-torsion clean as well as for an arbitrary n ≥ 3, $\mathbb{T}_n(\mathbb{F}_2)$ is almost n-torsion clean if and only if $\mathbb{T}_n(\mathbb{F}_2)$ is n-torsion clean if and only if $n = 2^l$ for an integer $l \geq 2$.

6. **On n-generalized commutators and Lie ideals of rings (paper [10]).** In this work, a significant extension of a theorem of the American mathematician Herstein, proved in 1954, is achieved. This is done by Theorem 2.1, which states that if R is a ring, then $[R, \dots, R]_{2n+1}$ is an ideal of R for $n \geq 1$. Here $[R, \dots, R]_{2n+1}$ is a generalized commutator of the elements of the ring R, defined in the following way: For a given integer $n \geq 2$ and elements $a_1, \dots, a_n \in R$,

$$[a_1, a_2, \dots, a_n]_n \coloneqq a_1 a_2 \cdots a_n - a_n a_{n-1} \cdots a_1.$$

It is also proved that if R is a non-commutative prime ring and $n \geq 3$, then any nonzero n-generalized Lie ideal of R contains a nonzero ideal, where an n-generalized Lie ideal is defined as an additive subgroup A of R, such that $[x_1, \dots, x_r, a, y_1, \dots, y_s]_n \in A$ for all $x_i, y_i \in R$ and $\forall a \in A$. Some generalizations and related questions on n-generalized commutators and their relationship with noncommutative polynomials are also discussed.

4. Conclusion

The above gives me a reason to believe that Peter Danchev is a highly qualified specialist who has proven his ability to conduct research at a high level. According to the presented documents, the candidate Peter Danchev fulfills all the requirements of the law and the Regulations to it and the Regulations for the specific requirements for acquiring academic degrees and occupying academic positions at BAS and IMI-BAS. I strongly recommend to the Honorable Scientific Jury to vote on a proposal to the Scientific Council of IMI - BAS to select **Peter Vassilev Danchev** for the academic position "Professor" in

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20.11.2023 Member of the scientific jury:

/Prof. Stefka Bouyuklieva/