ABSTRACTS OF THE PAPERS SUBMITTED FOR THE POSITION OF ASSOCIATE PROFESSOR

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[1] S. Barov, G. Dimov and St. Nedev, On a theorem of H.-J. Schmidt, C. R. Acad. Bulgare Sci., 46, No. 3, (1993), 9-11.

Abstract. *For every topological space* X *let* 2^X *stand for the set of all non-empty closed subsets of* X *and* $cl_X B$*—for the closure of the subset* B *of* X *in the space* X*. The space* 2^X *is endowed with the Tychonoff topology, generated by the base* $\{< U >: U \text{ is open}\}$*, where* $< U >= \{F \in 2^X : F \subset U\}$*. A topological space* X *is called a* HS*-space if, for every subspace* A *of* X*, the map* $i_A : 2^{A,T} \rightarrow: 2^{X,T}$*, defined by the formula* $i_A(B) = cl_X B$*, for every* $B \in 2^A$*, is a continuous map. In this paper we discuss the assertion of H. J. Schmidt, that is, whether every Hausdorff* HS*-space is a* T_3*-space. We give an internal characterization of* HS*-spaces and define a class of spaces, called* $\mathcal{K}^*$*, and show that if a Hausdorff space* X *belongs to that class then* X *is a normal space.*

[2] S. Barov, G. Dimov and St. Nedev, On a question of M. Paoli and E. Ripoli, Bollettino U. M. I. (7), 10-A (1996), 127-141.

Abstract. *For every topological space* X *let* 2^X *stand for the set of all non-empty closed subsets of* X *and* $cl_X B$*—for the closure of the subset* B *of* X *in the space* X*. The space* 2^X *is endowed with the Tychonoff topology, generated by the base* $\{< U >: U \text{ is open}\}$*, where* $< U >= \{F \in 2^X : F \subset U\}$*. A topological space* X *is called a* HS*-space if, for every subspace* A *of* X*, the map* $i_A : 2^{A,T} \rightarrow: 2^{X,T}$*, defined by the formula* $i_A(B) = cl_X B$*, for every* $B \in 2^A$*, is a continuous map. In this paper we discuss the assertion of H. J. Schmidt, that is, is every Hausdorff* HS*-space a* T_3*-space. In the current paper we give a partial solution to this question. More precisely: a) we give an internal characterization of* HS*-spaces and show that the class* HS *is invariant under closed mappings; b) we introduce a large class of spaces, called* $\mathcal{K}^*$*, containing all Haussdorff spaces with a countable character, where we answer in positive to the Schmidt's conjecture; c) we state if-and-only-if statements to the Schmidt's conjecture; d) we show that if* X *is a Hausdorff and* $\mathcal{K}^*$*-space then* X *is even a normal space.*

[3] S. Barov, A note on spaces which are quotient compact-covering s-images of metric spaces, C. R. Acad. Bulgare Sci., 52, No. 5-6, (1999), 11-14.

Abstract. A map $f : X \to Y$ is compact-covering (sequence-covering, countable-compact-covering, respectively) if every compact (convergent sequence, countable and compact, resp.) K in Y is the image of some compact C in X. A map $f : X \to Y$ is called an s-map if each fiber $f^{-1}(y)$ is separable. In this paper we address a question posed by E. Michael and K. Nagami: Is every quotient s-image of a metric space also a compact-covering quotient s-image of a metric space? We characterize spaces that are countable-compact-covering s-images of metric spaces via covers with certain properties.

[4] S. Barov, Some properties of star-countable covers, C. R. Acad. Bulgare Sci., 52, No. 7-8, (1999), 5-8.

Abstract. A family $\mathcal{F}$ of subsets of a space X is called star-countable if for every $V \in \mathcal{F}$ the set $\{U \in \mathcal{F} \colon U \cap V \neq \emptyset\}$ is countable. Our interest in star-countable covers comes from two directions. An open problem of E. Michael and K. Nagami can be restated in terms of point-countable coverings. Here we give a positive answer to this question for a star-countable cover instead of a point-countable one. The second direction is related to various assumptions made on star-countable covers. We show some properties for a space X having certain star-countable covers that are not valid or it is not known whether they are valid for the respective point-countable covers.

[6] S. Barov and J. J. Dijkstra, On boundary avoiding selections and some extension theorems, Pacific J. Math., Vol. 203, No. 1, 2002, 79-87.

Abstract. A theorem of Marc Frantz about controlled continuous extensions of functions inspired us to prove a general result concerning boundary avoiding continuous selections into Banach spaces, which has Frantz' theorem as a corollary. In addition, with relatively simple means we improve upon some other results of Frantz involving extensions of products and of disjoint families of functions.

[8] S. Barov, Covers of topological spaces and compact-covering maps, Topology Proceedings, Vol. 30, No. 1, 2006, 1-10.

Abstract. In this paper, we characterize spaces which are quotient compact-covering s-images of metric spaces. We show that X is a quotient compact-covering s-image of a metric space if and only if X is a quotient countable-compact- covering s-image of a metric space.

[9] S. Barov and Jan J. Dijkstra, On closed sets with convex projections in Hilbert space, Fundamenta Mathematicae, Vol. 197, No. 1, 2007, 17-33.

Abstract. Let k be a fixed natural number. We show that if C is a closed and nonconvex set in Hilbert space such that the closures of the projections onto all

k-hyperplanes (planes with codimension k) are convex and proper, then C must contain a closed copy of Hilbert space. In order to prove this result we introduce for convex closed sets B the set $\mathcal{E}^k(B)$ *consisting of all points of B that are extremal with respect to projections onto k-hyperplanes. We prove that* $\mathcal{E}^k(B)$ *is precisely the intersection of all k-imitations C of B, i.e., closed sets C that have the same projections as B onto all k-hyperplanes. For every closed convex set B in* l^2 *with non-empty interior we construct "minimal" k-imitations C, by which we mean that* $\dim(C \setminus \mathcal{E}^k(B)) \leq 0$. *Finally, we show that whenever a compact set has convex projections onto finite-dimensional planes, then it must be convex.*

[10] S. Barov and Jan J. Dijkstra, On closed sets with convex projections under a narrow set of directions, Transactions of Amer. Math. Soc., Vol. 360, No. 12 (2008), 6525-6543.

Abstract. Dijkstra, Goodsell, and Wright have shown that if a non-convex compactum in $\mathbb{R}^n$ *has the property that its projection onto* all *k-dimensional planes is convex then the compactum contains a topological copy of the* $(k-1)$*-sphere. This theorem was extended over the class of unbounded closed sets by Barov, Cobb, and Dijkstra. We show that the results in these two papers remain valid under the much weaker assumption that the collection of projection directions has a nonempty interior.*

[11] S. Barov, On a characterization of normal and countably paracompact spaces via set-valued selections, Comment. Math. Univ. Carolin. 49, 1 (2008) 45-52.

Abstract. We give a characterization of normal and countably paracompact spaces via continuous set-avoiding selections.

[12] S. Barov and Jan J. Dijkstra, On closed sets with convex projections under somewhere dense sets of directions, Proc. Amer. Math. Soc., 137 (2009), 2425-2435.

Abstract. Let $k, n \in \mathbb{N}$ *with* $k < n$ *and let* $\mathcal{G}_k(\mathbb{R}^n)$ *denote the Grassmann manifold consisting of all k-dimensional linear subspaces in* $\mathbb{R}^n$. *In an earlier paper the authors showed that if the projections of a nonconvex closed set* $C \subset \mathbb{R}^n$ *are convex and proper for projection directions from some nonempty open set* $\mathcal{P} \subset \mathcal{G}_k(\mathbb{R}^n)$, *then C contains a closed copy of an* $(n-k-1)$*-manifold. In this paper we improve on that result by showing that that result remains valid under the weaker assumption that* $\mathcal{P}$ *is somewhere dense in* $\mathcal{G}_k(\mathbb{R}^n)$.

[13] S. Barov and Jan J. Dijkstra, On closed sets in Hilbert space with convex projections under somewhere dense sets of directions, Journal of Topology and Analysis, Vol. 2, No. 1 (2010), 123-143.

Abstract. Let k be a fixed natural number. In an earlier paper the authors show that if C is a closed and nonconvex set in the Hilbert space ℓ^2 such that the closures of the projections onto all *k-hyperplanes (planes with codimension k) are convex and proper then C must contain a closed copy of ℓ^2. In the present paper this theorem is strengthened significantly by making the much weaker assumption that the set of projection directions is somewhere dense. To show the sharpness of the main theorem we construct 'minimal imitations' of closed convex sets in ℓ^2. In addition, we show that closed convex sets with an empty geometric interior cannot be imitated by other closed sets.*

[14] S. Barov, Jan J. Dijkstra and Maurits van der Meer, On Cantor sets with shadows of prescribed dimension, Topology and its Applications, 159 (2012), 2736-2742.

Abstract. In the current manuscript we consider a question raised by John Cobb: given positive integers $m > k > \ell$ is there a Cantor set in $\mathbb{R}^m$ such that all whose projections onto k-planes are exactly ℓ-dimensional? We construct in $\mathbb{R}^m$ a Cantor set such that all its shadows (projections onto hyperplanes) are ℓ-dimensional for every $0 \leq \ell \leq m-1$. Furthermore, we expand Cobb's question in the Hilbert space ℓ^2. We show that for every compactum K in ℓ^2 and every $k \in \mathbb{N}$ there is a dense set of projections of K onto k-hyperplanes such that each projection is a homeomorphism.

[15] S. Barov and Jan J. Dijkstra, On exposed points and extremal points of convex sets in and Hilbert space , Fundamenta Mathematicae, 232 (2016), 117-129.

Abstract. Let $\mathbb{V}$ be a Euclidean space or Hilbert space ℓ^2, let $k \in \mathbb{N}$ with $k <$ $\dim \mathbb{V}$*, and let B be convex and closed in $\mathbb{V}$. Let $\mathcal{P}$ be a collection of linear k-subspaces of $\mathbb{V}$. A set $C \subset \mathbb{V}$ is called a $\mathcal{P}$-*imitation *of B if B and C have identical orthogonal projections along every $P \in \mathcal{P}$. An* extremal *point of B with respect to the projections under $\mathcal{P}$ is a point that all closed subsets of B that are $\mathcal{P}$-imitations of B have in common. A point x of B is called* exposed *by $\mathcal{P}$ if there is a $P \in \mathcal{P}$ such that $(x + P) \cap B = \{x\}$. In the present paper we show that all extremal points are limits of sequences of exposed points whenever $\mathcal{P}$ is open. In addition, we discuss the question whether the exposed points form a G_δ-set.*

[16] S. Barov, Smooth convex bodies in with dense union of facets, Topology Proceedings, 58 (2021), 71-83.

Abstract. Let B be closed and convex in $\mathbb{R}^n$; B is called a convex body if B is compact and has a nonempty interior with respect to $\mathbb{R}^n$. In addition, B is smooth if B has a unique supporting hyperplane at every boundary point. Let $k, n \in \mathbb{N}$ with $k < n$ and let $\mathbb{L}^n_k$ denote the Grassmann manifold consisting of all k-dimensional linear subspaces in $\mathbb{R}^n$. An intersection F of B and a supporting hyperplane is *called a facet if* $\dim F = n-1$. *A point* x *of* B *is called exposed by* $\mathcal{P} \subset \mathbb{L}_k^n$ *if there is a* $P \in \mathcal{P}$ *such that* $(x+P) \cap B = \{x\}$. *In this paper, for every* $n \geq 2$ *we have constructed symmetric smooth convex bodies* $B(n)$ *in* $\mathbb{R}^n$ *whose union of all facets is dense in the boundary of* $B(n)$ *and so that the set of its facets defines a dense set* $\mathcal{P}$ *in* $\mathbb{L}_k^n$ *such that the set of all points in* $B(n)$ *exposed by* $\mathcal{P}$ *is empty.*

[17] S. Barov, More on exposed points and extremal points of convex sets in and Hilbert space, Comment. Math. Univ. Carolin., 64, 1 (2023), 63-72.

Abstract. Let $\mathbb{V}$ *be a separable real Hilbert space,* $k \in \mathbb{N}$ *with* $k < \dim \mathbb{V}$, *and let* B *be convex and closed in* $\mathbb{V}$. *Let* $\mathcal{P}$ *be a collection of linear* k*-subspaces of* $\mathbb{V}$. *A point* $w \in B$ *is called* exposed by $\mathcal{P}$ *if there is a* $P \in \mathcal{P}$ *so that* $(w+P) \cap B = \{w\}$. *We show that, under some natural conditions,* B *can be reconstituted as the convex hull of the closure of all its exposed by* $\mathcal{P}$ *points whenever* $\mathcal{P}$ *is dense and* G_δ. *In addition, we discuss the question when the set of exposed by some* $\mathcal{P}$ *points forms a* G_δ*-set.*