R E V I E W

by Prof. D.Sc. Johann Todorov Davidov
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on a competition for occupying the academic position of **Associate Professor**

area of higher education: *4. Natural Sciences, Mathematics and Informatics*,
professional field *4.5 Mathematics*
, scientific speciality *Geometry and Topology*
(convex geometry in topological vector spaces)

for the needs of the Institute of Mathematics and Informatics,
Bulgarian Academy of Scinecs

announced in State Gazette No. 69 of August 11, 2023 and the websites of IMI and BAS

I am a member of the scientific panel for this procedure according to order No. 467/10.10.2023 of the Director of the Institute of Mathematics and Informatics Prof. D.Sc. Peter Boyvalenkov. Documents for participation in the announced competition have been submitted only by Assistant Professor Dr. Stoyu Tzvetkov Barov. As a member of the scientific panel, I have received from Dr. Barov all the administrative and scientific documents required by the Act on the Development of the Academic Staff in the Republic of Bulgaria (ADASRB), the Rules for its implementation and the Rules on the terms and conditions for awarding of academic degrees and occupying of academic positions at Bulgarian Academy of Sciences.

Personal data

Dr. Stoyu Barov was born on March 28, 1964 in Lesichovo, the region of Pazardjik, Bulgaria. In 1982, he graduated from High School of Mathematics in Pazardjik. In the period 1984 - 1988, he was a student at the Faculty of Mathematics and Informatics (FMI) of Sofia University "St. Kliment Ohridski" (SU). Then he has been a programmer at the Institute of Mathematics and Informatics (IMI) of the Bulgaria Academy of Sciences (BAS), Sofia for one year. He received Master's degree in Mathematics (Topology) at FMI, SU in 1992. From 1992 to 1998 he worked as a mathematician at IMI, BAS and during this period he conducted exercise classes at FMI. In the period 1998 - 2001 he was a graduate student and a graduate teaching assistant at the Department of Mathematics, University of Alabama, Tuscaloosa, USA. In 1999, he became a Ph. D. student at the University of Alabama after successfully passing the Qualifying Exams in Mathematics. In 2001, he received Ph.D. degree in Mathematics from the University of Alabama. From 2001 to 2004 he was Assistant Professor at Ball State University, Muncie, Indiana, USA. From 2004 he has working at IMI, BAS.

General characterization of the scientific works of the applicant.

The candidate S. Barov has submitted 15 scientific papers for participation in the competition. These papers meet the minimal requirements according to ADASRB.

Barov is the only author of six of them. He has 1 co-author in seven papers and 2 co-authors in two papers. The 15 papers submitted by S. Barov for the competition have been quoted 13 times by other authors. The list of all publications by the candidate includes 18 papers quoted 17 times (without self-citations).

The papers presented by S. Barov have not been used for awarding the scientific degree "doctor" or occupying an academic position.

I would like also to mention that I have not discovered any plagiarism.

The papers presented by S. Barov are in the field of general topology and geometric tomography, more precisely that part of it which concerns convex geometry. The term geometric tomography was coined by R.J. Gardner at the beginning of this century and denotes a field of mathematics in which properties of geometric objects are established by available information about their projections onto planes or intersections with other objects. The papers of Barov treat interesting and difficult problems in these areas. Most of them are publish in prestigious mathematical journals.

Analysis of the scientific achievements of the applicant

In the review of the candidate's results, we will follow the numbering of the papers in his list of publications for the competition.

The papers numbered 1-4, 6, 8, 11 are in the field of general topology.

In paper No. 1, results that are proved in paper No. 2 are stated. The motivation for the considerations in this paper stems from a claim of H. J. Schmidt. In order to state this claim, we need to define the class of HS-spaces. If X is a topological space, let 2^X be the set of all closed subsets of X endowed with the so-called Tychonoff topology. The space X is called a HS-space if for every subspace A of X the map $i_A : 2^A \to 2^X$ defined by $i_A(B) =$ the closure of B in X is continuous. H. J. Schmidt has claimed that every HS-space is regular (i.e. T_3). However, it has turned out that the Schmidt's proof was incorrect and the natural question of finding classes of topological spaces for which the claim is true arises. In papers No. 1 and No. 2, an interesting class of topological spaces called LF-normal is introduced and it is proved that it coincides with the class of HS-spaces. This result is used to state conditions that are equivalent of the Schmidt's claim. Also, a large class of topological spaces for which this claim is true is introduced.

Before turning to paper No. 3, we will recall the concepts used both in it and in paper No. 8. A compact-covering map is a continuous map $X \to Y$ such that every compact in Y is the image of a compact in X. If every countable compact in Y is the image of a compact in X, the map is called countable- compact-covering. A s-map is defined by the requirement that the pre-image of every point is a countable and dense subset, i.e. a separable subset. A topological space is called compactly generated or a k-space if a subset U of X is open if and only if for every compact space K and every continuous map $f : K \to X$ the set $f^{-1}(U)$ is open in K.

The main result in paper No. 3 states an equivalence between the condition that a Hausdorff topological space X is the image of a metric space via quotient compact-covering s-map and the condition that X is compactly generated (k-space) possessing point-countable cover with certain special properties. Also, a result that is proved in paper No. 8 is announced.

In paper No. 8, a characterization of the topological spaces X that are images of metric spaces via quotient compact-covering s-maps is given. It is shown that this condition on X is equivalent to each of the following ones: X is the image of a quotient countable-compact-covering s-maps; X is a k-space with certain properties that we will not state here.

The motivation for paper No. 4 stems from papers of B. Burke and E. Michaell, and E. Michaell and K. Nagami. In this paper, some properties of topological spaces are characterized by means of the condition for existence of star-countable cover with certain properties. Star-countable cover is a cover such that every set in it intersects at most countable many sets of the cover.

Now, recall that a selection of a function φ that assigns a non-empty subset of a set Y to every point of a set X is a map $f : X \to Y$ such that $f(x) \in \varphi(x)$. In complex analysis, f is called a single-valued branch of the multi-valued function φ. If X and Y are topological spaces and if φ has the property that for every open subset U of Y the set $\varphi^{-1}[U] = \{x : \varphi(x) \cap U \neq \emptyset\}$ is open in X, then φ is said to be lower semi-continuous (LSC for short).

The considerations in paper No. 6 are motivated by two theorems of M. Frantz for extension of continuous functions defined on closed subsets of a normal space with values in the interval $I = [0, 1]$. In paper No. 6, theorems for continuous selections of LSC functions are proved. These set-valued functions are defined on a paracompact or normal countably paracompact Tychonoff space (Tychonoff space= $T_{3\frac{1}{2}}$-space = completely regular space) with values subsets of a convex set in a separable Banach space. As a corollary, the theorems of Frantz are obtained. Also, it is given a relatively simple proof of a theorem of Frantz for which he claims that he has a proof, but it is rather complicated.

In paper No. 11, normal and countably paracompact T_1 topological spaces X are characterized by means of existence of special continuous selections of set-valued LSC-functions on X with values convex sets with finite dimensional affine hull in a separable Banach space.

The papers No. 9, 10 and 12-17 can be attributed to the field of geometric tomography (convex geometry).

In order to review the papers of Stoyu Barov in this field, it is convenient to denote by $\mathcal{G}_k$ the Grassmann space of closed k-dimensional linear subspaces of the separable Hilbert space ℓ^2 or $\mathbb{R}^n$. Grassmannian $\mathcal{G}_k$ is endowed with the topology yielded by the metric defined by means of Hausdorff distance: the distance between two closed k-dimensional linear subspaces is the Hausdorff distance between their intersections with the closed unit ball. For each subset $\mathcal{P}$ of $\mathcal{G}_k$, we can consider the projections of a given subset C of ℓ^2 or $\mathbb{R}^n$ in direction of the spaces $L \in \mathcal{P}$, i.e., the orthogonal projections of C to subspaces $L^\perp$ of codimension k.

In paper No. 9, it is proved that if the projections of a closed non-convex subset C of ℓ^2 to all subspaces of codimension k (i.e. if $\mathcal{P} = \mathcal{G}_k$) are convex and do not coincide with the corresponding subspace, then C contains a copy of ℓ^2, i.e. a closed subset homeomorphic to ℓ^2. A result of similar type for subsets C of $\mathbb{R}^n$ has been obtained by S. Barov, J. Cobb and J. Dijkstra who showed that C contains a copy of $\mathbb{R}^{n-k-1}$. Because of the infinite dimension of ℓ^2, the proof in this case requires a different approach and is more complicated. In fact, the copy of ℓ^2 is the intersection of all closed subsets of ℓ^2 having the same projections to the subspaces of codimension k as C.

In paper No. 10, the aforementioned result of Barov-Cobb-Dijkstra is essentially generalized by showing that it also holds under the weaker assumption that the set $\mathcal{P}$ of projection directions is non-empty and open in $\mathcal{G}_k$. Furthermore, it is shown that for open sets $\mathcal{P}$ a result of J. Dijkstra, T. Goodsell and D. Wright remains valid; according to it if the projections of a compact and non-convex subset C of $\mathbb{R}^n$ to all subspaces of codimension k, i.e. if $\mathcal{P} = \mathcal{G}_k$, are convex and do not coincide with the corresponding subspace, the set C contains a copy of the sphere S^{n-k-1}.

In paper No. 12, the Barov-Cobb-Dijkstra result is further generalized for the case when the set $\mathcal{P}$ is somewhere dense (the closure of $\mathcal{P}$ has a non-empty interior)

In paper No. 13, the main result of paper No. 9 concerning the Hilbert space ℓ^2 mentioned above is generalized for the case of somewhere dense set $\mathcal{P}$ of projection directions.

Paper No. 14 is motivated by a question posed by J. Cobb. In this paper, it is proved that if n and k are integers, $n \geq 2$, $0 \leq k \leq n-1$, then there exists a Cantor set in $\mathbb{R}^n$ all projections of which to $(n-1)$-subspaces are of dimension k. For the Hilbert space ℓ^2, it is shown that for every n there exists a Cantor set whose projections to all n-dimensional subspaces are of dimension $n-1$.

For further exposition, some definitions are necessary.

If B is a closed and convex subset of ℓ^2 or $\mathbb{R}^n$ and $\mathcal{P}$ is a subset of the Grassmannian $\mathcal{G}_k$, a point $x \in B$ is called exposed by $\mathcal{P}$ if there is a k-dimensional subspace $P \in \mathcal{P}$ which, taken parallel through x, intersects B only at the point x, i.e. $\{x + P\} \cap B = \{x\}$. A point $x \in B$ is called extremal with respect to $\mathcal{P}$ if it belongs to the intersection of all closed subsets of B whose projections in the directions of $\mathcal{P}$ coincide with those of B.

Denote the set of points of B exposed by $\mathcal{P}$ by $\chi_p^k(B, \mathcal{P})$. Also, denote the set of extremal points of B with respect to $\mathcal{P}$ by $\chi_t^k(B, \mathcal{P})$.

In paper No. 15, it is proved that if $\mathcal{P}$ is open, then $\chi_p^k(B, \mathcal{P})$ is dense in $\chi_t^k(B, \mathcal{P})$. Furthermore, if B is a subset of $\mathbb{R}^n$ with $n \geq 2$, then $\chi_p^1(B, \mathcal{P})$ is a G_δ-set in $\chi_t^1(B, \mathcal{P})$.

In paper No. 16, a positive answer to a question related to the results in paper No. 15 is given. Namely, are there a compact convex subset B of $\mathbb{R}^n$ and a dense subset $\mathcal{P}$ of $\mathcal{G}_{n-1}$ such that $\chi_p^{n-1}(B, \mathcal{P}) = \emptyset$. In fact, in paper No. 16, for any $n \geq 2$ it is constructed sets B and $\mathcal{P}$ with these properties.

In paper No. 17, it is proved that if B is a closed and convex subset of ℓ^2 or $\mathbb{R}^n$ which in addition satisfies certain geometric conditions and if $\mathcal{P}$ is a dense subset of $\mathcal{G}_k$, then

$$cl < \chi_p^k(B, \mathcal{P} >=< cl\chi_p^k(B, \mathcal{P} >= B,$$

where clA means the closure of a set A in $\mathbb{R}^n$ and $< A >$ stands for the convex hull of A. This result can be considered as an analog of the Krein and Milman theorem (for every compact and convex set A, $cl <$ extreme points of A $>= A$, but the notion of an extreme point in this theorem is different form that used in paper No. 17).

Educational activity

Stoyu Bariov has conducted exercise classes on Applied Differential Equations, Calculus and Intermediate Algebra at the University of Alabama and on Calculus I and II at FMI, SU. He has been a supervisor (jointly with J. Dijkstra) of a student at the masters' program of the Free University, Amsterdam.

Participation in scientific projects

S. Barov has been a member of two scientific projects supported by Bulgarian National Science Fund. He also has had a grant given by Netherlands Organization for Scientific Research and a one year Ball State Research Grant.

CONCLUSION

The documents and materials presented by Dr. Stoyu Barov meet the requirements of the Act on the Development of the Academic Staff in the Republic of Bulgaria , the Rules for its implementation and the Rules on the terms and conditions for awarding of academic degrees and occupying academic positions at Bulgarian Academy of Sciences.

The results obtained by S. Barov are a serious contribution to the field of modern general topology and convex geometry. They go beyond the usual standards for holding the academic position of associate professor.

Based on the comments above, I give a positive assessment of the scientific work of Dr. Stoyu Barov and recommend to the scientific jury to advise the Scientific Council of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences to appoint Assistant Professor Dr. Stoyu Tzvetkov Barov as an Associate Professor in area of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics , scientific speciality Geometry and Topology (convex geometry in topological vector spaces).

16.11.2023 Reviewer:

(Prof. D.Sc. Johann Davidov)