R E P O R T

On the competition

to occupy an academic position "docent"

In professional direction 4.5 Mathematics

Scientific specialty

"Geometry and Topology" (Convex geometry and topological vector spaces)

The competition was announced for the needs of the Institute of Mathematics and Informatics (IMI) of the BAS in SG No. 69/11.08.2023.

This report was prepared by Prof. Dr. Petar Stoyanov Kenderov, retired, in his capacity as a member of a scientific jury, appointed by Order No. 467 dated 10.10.2023. of Prof. Petar Bovalenkov, director of IMI-BAN.

Dr. Stoyu Tsvetkov Barov from the "Analysis, Geometry and Topology" section of IMI-BANS submitted documents for participation in the competition.

General description of the documents submitted in connection with this procedure

The documents presented by the candidate correspond to the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria and the Regulations for its application, as well as the regulations adopted in the BAS and IMI-BAS for the acquisition of scientific degrees and holding scientific positions. To participate in the competition, Stoyu Barov submitted all required materials and documents (17 in total). They are enough to form my final opinion on this contest.

Biographical data for the candidate

Stoyu Barov was born on March 28, 1964, in the village of Lesichevo, Pazardzhik region. In 1982, he graduated from the Mathematics High School in Pazardzhik. From 1984 to 1989, he was a student at the Faculty of Mathematics and Informatics of Sofia University "St. Kliment Ohridski". His marks from academic examinations has an average of “Very Good 4.86” and his state exam marks are “Excellent 6.00.” By decision of the State Examination Commission of February 1992, he was recognized as a "Mathematician" with a specialty in "Topology". His diploma work was developed under the guidance of the Bulgarian topologist Georgi Dimov. From August 1998 to May 2001, he was a doctoral student at the University of Alabama (Tuscaloosa), where he defended his thesis and received a diploma for "Doctor of Philosophy" - a degree equated by the BAS with the Bulgarian educational and scientific degree "Doctor" on 22.06 .2015 (Diploma No. 000035). The topic of the dissertation is "On closed sets with convex shadows" and was developed under the supervision of Professor Jan J. Dijkstra. Stoyu Barov's handwritten summary (biography) omits the word "closed" from the title of his dissertation.

After graduating from higher education in 1989, he worked as a programmer until September 1992 at the Institute of Informatics of the BAS. In the period March 1992 - June 1998, he was a mathematician at IMI-BAN. At the same time, he teached various mathematical disciplines at the Faculty of Mathematics and Informatics.

After defending his doctorate, he worked (from August 2001 to May 2004) as an assistant professor at Ball State University, USA. From June 2004 until now, he has been working at IMI-BAS.

General description of the scientific works and achievements with which he participated in the competition

The list of all published articles contains 17 titles. In five of the articles, Barov is the sole author. The rest of the articles are published in co-authorship. Three of the earliest publications are in Doklady BAS. Among the rest, we see articles in reputable journals such as the Journal of London Math. Society, Transactions of Amer. Math. Society, Proceedings of Amer. Math. Society, Pacific Journal of Mathematics, Topology and its Applications, Fundamenta Mathematicae, Journal of Topology and Analysis and others. To participate in this competition, the candidate submitted 15 articles. The other two (numbered 5 and 7) appear to have been used in the defense of the dissertation for the degree of Doctor.

Two areas of scientific interest are clearly outlined in the candidate's research. The first area is part of General Topology and is represented by the articles numbered 1, 2, 3, 4, 8 and 11 of the list of publications. The second area, where the rest of the papers are, is related to the properties of sets in Hilbert spaces whose orthogonal projections onto different proper subspaces are convex ("cast a convex shadow"). In a sense, these two areas correspond to the chronological evolution of the candidate's scientific interests , but there is also "mixing". To illustrate what these two areas are all about, we'll present a sample of results from each of them.

The first area is from General Topology. An object of frequent research there is the so-called hyperspace 2^X of all closed and nonempty subsets F of the topological space (X, τ). There are a wide variety of ways in which 2^X can be provided with a topology. In the joint articles numbered 1 and 2, written in collaboration with G. Dimov and St. Nedev the earliest appearing topology T for hyperspaces (named for obvious reasons after Tikhonov) is used. In it, all sets of the form $\{F\subset U: F\in 2^X\}$ serve as a basis, where U is an arbitrarily given open set of (X, τ). The hyperspace with this topology is denoted by $2^{(X,T)}$. Although natural, this topology T also has serious drawbacks. It is not Hausdorff (separable). Furthermore, every subset A of X gives rise to the natural mapping $i_A:2^{(A,T)} \to 2^{(X,T)}$, mapping every closed set B in A to the closed envelope of B in X. This natural mapping is not necessarily continuous. In papers 1 and 2, the authors give an internal characterization of the spaces for which this mapping is continuous. They show that the class of such spaces is preserved under closed mappings and clarify claims by other authors.

In the self-authored papers 3 and 8, Barov examines the issue of the so-called compact-covering mappings $f:X \to Y$, where for every compact K in Y there exists a compact K' in X such that $K \subset f(K')$. An interesting question here is, when is a quotient mapping $f:X\to Y$ with separable fibers $f^{-1}(y)$, of a metric space X compact-covering? In paper 8 it is proved that this is true exactly when for every countable compact K there is a compact K' whose image covers K. An interesting point here is the observed connection with the spaces with a point-countable basis and the developed toolkit for working with point-countable covers. This naturally leads to the use of star-countable families of sets, where each element of the family intersects at most countably many other sets of the family. These families are the main toolkit in article 4.

Another focus in this area present papers 6 and 11. They are concerned with finding continuous selections of multivalued mappings whose images avoid some sets. In such terms, a characterization of normal and countably-paracompact spaces (Theorem 4 of article 6) as well as a characterization of paracompact spaces (Theorem 5 of article 6) are obtained. The same type of characterization of countably-paracompact normal spaces is also given in Theorem 1 of article 11, which uses the existence of continuous selection to multivalued mappings with convex finite-dimensional images in a separable Banach space so that the images of the selection are in the relative interior of the images of the multi-valued mapping. As shown in paper 6, this kind of results can be used to proof the existence of continuous extension to the entire space X of a continuous real-valued function defined on a subset of X .

At first glance, these studies in the field of General Topology appear heterogeneous and "scattered". But there is a clear thematic connection between them. Any multivalued image $\phi:X \to Y$ can be regarded as a single-valued mapping of X in the hyperspace 2^Y . If $f:Y \to X$ is a single-valued mapping, then its inverse image $f^{(-1)}:X \to Y$ is multi-valued. If $f^{(-1)}$ admits a continuous single-valued selection, then f is automatically (and trivially!) a compact-covering mapping.

If for the candidate's topological field of research it is difficult to expect any applicability outside mathematics, for the second field this is not the case. Extracting information about the properties of some subset B of a finite-dimensional Euclidean space R^n or the Hilbert space l^2, from the information we have about its orthogonal projections onto a certain class of linear subspaces (or translations of linear subspaces), is a necessity appearing rather often in practice and science. Stoyu Barov's dissertation is dedicated to this topic. After the defense, he continued to develop this topic together with his supervisor. The unit sphere $S^{(n-1)}$ of a finite-dimensional Euclidean space R^n is a good example of what we can expect in this situation. All projections onto affine subspaces (translates of linear subspaces) of $S^{(n-1)}$ are convex but the set $S^{(n-1)}$ itself is not convex. There are a number of studies which aim to show that if the projections of the set B are convex, then B "contains something similar to $S^{(n-1)}$ ". In a paper not belonging to the once submitted for this competition, Barov, Cobb, and Dijkstra prove that if a closed set C from R^n has convex projections onto every k-dimensional affine subspace, then this set contains a closed set that is (k-1)- dimensional manifold without boundary. The aim of paper 9 is to investigate to what extent similar results also hold in the infinite-dimensional Hilbert space l^2. Theorem 1 of this paper states, under certain conditions, that if the projections of C onto all affine subspaces of codimension k are convex, then C contains a topological copy of l^2 itself (here k is an arbitrary natural number). However, there are also serious differences from the finite-dimensional case. According to Theorem 13 of Article 9, if the finite-dimensional projections of a compact subset C of l^2 are convex, then the set C itself is convex (the unit sphere of l^2 is not a compact set).

Another related line of research is followed in Paper 10. Projections only onto hyperplanes in R^n (subspaces of codimension 1) where $n \geq 3$, are considered. In this particular case, the projections are called shadows. It is known, from a result of Dijkstra, Goodsell, and Wright, that if all shadows of a compact C are convex, then C contains a copy of $S^{(n-2)}$. In Barov's paper 10 it is proved that the result holds even under weaker requirements: it suffices to require that the images are convex for some open set of projection directions. This result is greatly strengthened in Paper 12: it suffices to require that the images are convex for a set of project directions that is dense in some open set.

The idea of relaxing the requirements for the set of projection directions has also been pushed into the l^2 Hilbert space. In paper 13 it is shown that the results of paper 9 hold for a poorer but sufficiently rich (ie somewhere dense) family of projections. In article 13, the question of the existence of "minimal imitations" of the set C was investigated. These are such subsets $B \subset C$ for which the closed envelopes of their projections coincide. The goal is to find minimal in some sense imitations of C. Theorem 4 shows that if the geometric interior of C is an empty set, then for C there are no imitations different from itself.

Article 14 deals with the opposite, in a sense, question: How bad can a set be if its projections are good? I.e. how far can our expectations for the properties of a set C stretch if its projections or shadows have some good properties? As early as 1947, Borsuk gave an example of a simple arc in space whose projections on any plane have interior points. There are also Cantor perfect sets with such a property. So, from the fact that projections have interiority, it cannot be concluded that the set itself has interiority. In 1994, Cobb constructed a Cantor set in R^3 whose shadows are all one-dimensional and posed the question: For given numbers $n > l \geq k \geq 0$, is there a Cantor set in R^n for which all projections onto l- dimensional affine subspaces are exactly k-dimensional? Theorem 1 of this paper gives a positive answer to this question for the case when l=n-1. It is also proved (Theorem 2) that for every natural number m there exists a Cantor set in l^2 whose projections on every m-dimensional affine subspace are (m-1)-dimensional.

Article 15 introduces the concept of "extremal point" and the more restrictive concept, "exposed point" of a set B with respect to a family P of k-dimensional linear subspaces of l^2. In spirit and content, these concepts correspond to the concepts with the same names from Functional Analysis and Optimization Theory. It is proved that if P is an open set (in the topology of the Grassmann manifold of all k-dimensional linear subspaces), then the set of exposed points is dense in the set of extremal points. This result is a direct clue to the search for an analogue of the Krein-Millman theorem stating that a given compact convex set can be represented as a convex and closed envelope of its extremal/exposed points. In paper 17 it is shown that there is an analogue of the Krein - Millman theorem if the family P contains a dense G_δ subset of the Grassmann manifold.

What has been written so far, in my opinion, is quite enough to form a positive attitude towards Stoyu Barov's candidacy for this competition.

Personal impressions

I have a vague memory of the time when Stoyu Barov was a student in the topology department and I have heard good words about him from the late Prof. Stoyan Nedev. After his departure abroad, I have not been in contact with him and have not followed his development in a scientific sense. Now, in my capacity as a reviewer, I found that he was engaged in modern and meaningful research and made serious scientific contributions.

Conclusion

From the information provided to me under this procedure, it appears that Barov's articles, citations, and other activities meet the requirements of IMI-BAN for holding the academic position of "docent". I have not detected any plagiarism.

Based on what was written above, I give a positive assessment to this application and recommend to the scientific jury to propose to the Scientific Council of IMI-BAN to elect Dr. Stoyu Tsvetkov Barov for the academic position of "Docent" in professional direction 4.5 Mathematics, Scientific specialty "Geometry and topology" (Convex geometry and topological vector spaces).

20.11.2023 Signature:

Sofia

Prof. PhD Petar Stoyanov Kenderov