# Extended report

**for the competition for the academic position “Associate Professor” at IMI-BAS, announced in the State Gazette issue No. 28 from 22.12.2020, in the area of science 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics, scientific direction *Probability theory and mathematical statistics (Stochastic models in finance)* prepared by Prof. Dr. Sc. Mladen Svetoslavov Savov, member of sections “Probability, operations research and statistics“, at FMI-SU “St. Kliment Ohridski” and “Operations research, probability and statistics“, at IMI-BAS, appointed member of the scientific panel for the competition by order No. 28/19.02.2021 issued by the Director of IMI-BAS and writing an extended report according to the decision taken by the scientific panel during its first meeting on 02.03.2021.**

This review was prepared in compliance with order no.No.28/19.02.2021 of the Director of IMI-BAS Acad. Veselin Drenski issued on the basis of art. 4 of the Act for the Development of the Academic Staff in the Republic of Bulgaria (Publ. State Gazette, issue.38 from 21 May 2010, changed State Gazette, issue 81 from 15 October 2010, changed State Gazette, issue 101 from 28 December 2010, changed State Gazette, issue 68 from 2 August 2013, changed and extended in State Gazette, issue 30 from 3 April 2018, changed State Gazette, issue 17 from 26 February 2019) following a decision of the Scientific Council of IMI-BAS from 22.01.2021 with Record No.1. The review complies with: Act for the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations of the Institute of Mathematics and Informatics (IMI) at BAS which stipulates the award of scientific titles and the appointment for academic positions and the regulations for the preparation of extended reports and reports for the award of the academic position "Associate Professor"at IMI-BAS.

As a member of the scientific jury I received all the documents submitted by the only applicant to the competition, Assistant Professor Tsvetelin Stefanov Zaevsky (IMI-BAS).

## 1. Biographical data about the candidate

Tsvetelin Zaevsky is born in 1974. He has graduated “Applied Mathematics” from FMI-SU “St. Kliment Ohridski” in 1999 with master thesis title “Models of the financial market. Pricing of European derivatives”. Between 2003 and 2013 Tsvetelin Zaevsky is a PhD student at FMI-SU "St. Kliment Ohridski" under the supervision of Prof. Dr. Sc. Racho Denchev and he has defended his PhD with dissertation titled "Combined processes of Ito and Lévy". At the beginning of 2014 he is appointed as an Assistant Professor in the group "Operations research, probability theory and statistics" at IMI-BAS where he works at the moment. His scientific interests lie in the area of financial mathematics where he has published the overwhelming majority of his papers.

## 2. Fulfillment of the minimal requirements

Tsvetelin Zaevsky has furnished 10 papers for the competition: 5 out of which in Q1, 1 in Q3, 1 in Q4 and 1 indexed in MathScinet. These publications yield respectively 112 points towards section 'B' and 252 points towards section 'Г' with the respective cumulative points exceeding the required minimum according to the regulations of IMI-BAS. The candidate has provided 15 citations, 13 out of which referenced in Web of Science. In total they give 84 points towards section 'Д' which is above the minimum threshold of 70 points. He has applied participation in 3 national science projects which yields 30 points towards section 'E', which is 10 points more than the required minimum of 20 points.

## 3. Research activity and scientific contributions of the candidate as testified by the applied documents

3.1. **Overall evaluation of the scientific achievements of the candidate.** The candidate Ass. Prof. Tsvetelin Zaevsky has applied 10 papers towards the competition 9 out of which are with impact factor. All, but one, deal with and/or are motivated by problems in financial mathematics. I am of the opinion that the level of the papers as a whole is convincing for the position Associate Professor and the applicant is in the process of establishing himself in the international scientific community in financial mathematics.

Reading the papers it is clear that the candidate has mastered the main techniques in financial mathematics, i.e. generators, martingales, stochastic calculus and classical probability. The mathematical tools that are used in the papers are somewhat limited but their application is at a deep level. The key novelty in most of the applicant's publications is the formulation of the price of financial derivatives and the time of their execution in terms of stopping times. In this way one may circumvent some difficulties that stem from the application of Markovian generators, such as smoothness of the functions upon which they are applied, and one may treat a number of derivatives in the same manner.

The journals where Ass. Prof. Tsvetelin Zaevsky has published his work are mainly the journal edited by BAS and a couple of leading journals in the scientific area (with respect to impact factor) such as *Chaos,Solitons and Fractals* and *Communications in Nonlinear*

*Science and Numerical Simulations*. Overall the scientific contributions of Ass. Prof. Tsvetelin Zaevsky are original and deep enough, and in sufficient quantity, so as to merit the award of the academic position "Associate Professor" to the candidate.

3.2. **Discussion of the particular scientific achievements of the candidate.** Paper [1] contains a generalization of the model of Bates. It considers stochastic volatility models of the following type:

$$dS_t = \mu S_t dt + \sqrt{V_t} dB_t + S_t dX_t^S$$
$$dV_t = \xi(\eta - V_t)dt + \theta V_t d\tilde{B}_t,$$

where the Brownian motions $B, \tilde{B}$ are correlated, the Lévy process $X^S$ has jumps strictly larger than $-1$ and the parameters are chosen so that, the stochastic volatility $V$ remains positive. For the process of log-returns $(\log(S_t))$ a class of risk-neutral measures is proposed and these random variables are represented via their characteristic functions with special attention in the case when $X^S$ is a tempered stable process. Although initially in the literature $X^S$ has been a compound Poisson process, the generalization to a Lévy process is not technically difficult. It seems that the main contribution consists of the numerical experiments, which demonstrate that the model based on a tempered stable process, is better at forecasting the market compared to its predecessors. Even more, there is evidence for the case that Lévy processes with infinite activity are more suitable in this setting, since the compound Poisson processes which are used in previous studies turn out to be with very high intensity of the small jumps. Also it is worthwhile noting the contribution regarding the calibration of the model, which involves simultaneously the real-world and the risk neutral measures. The approach with the utilization of the stationary measure of the volatility is also interesting, since it allows for the circumvention of the lack of information about the volatility in the market observations.

Papers [3] and [4], amongst which [3] merely announces [4], consider the price of defaultable derivatives. The default event is incorporated in two distinct ways in models pricing the asset - one way is via dependence of the default on the asset itself and consequently it is a stopping time with respect to the original filtration and the other via an independent-of-the-asset stopping time which is embedded in the model via the enlargement of the filtration. The price of the asset is modelled via a Lévy process with special attention to the particular case when it is a Brownian motion. The case when the default time is distributed according to an exponential law is separately considered and in the first case it is a stopping time in the evolution of the price whereas in the second it is the first jump of an independent counting Poisson process. Through a change of measure the risk neutral measure is discussed via necessary and sufficient conditions on the parameters of the model. Then integro-differential equations are obtained which have to be satisfied by the price of the asset so that the price of the derivative is a martingale. The stochastic differential equations driving the asset in the Brownian motion case are in general of the type:

$$dX_t = \mu X_t dt + \sigma X_t dB_t$$

$$dX_t = \mu X_t dt + \sigma X_t dB_t - \int_0^1 X_{t-} \eta(t, x, X_{t-}) \theta(dt, dx).$$

The first one corresponds to the case when the default time is independent of the filtration of the asset whereas the second incorporates the case when the first jump of a marked point process is the aforementioned default moment. It is worth noting that the models in this paper are rather general and generalize significantly their predecessors. Most of the techniques required for the derivation of the main results are well-established in the literature and the main contribution is a suitable representation of the problem and the utilization of compensators of indicator functions of the type $1_{\tau \leq t}$, where $\tau$ is the default time. Other noteworthy contributions of paper [4] include the representation of the difference between the price of the classical European option and defaultable derivative, i.e.

$$A(t, x) = a(t, x) + D_t, t < \tau$$

and closed form of the price of CoCo bonds, derivatives that have already been studied in the literature. The derived models and results are general enough so to be the stepping stone for further investigations. However, the paper gives more evidence that there are serious technical difficulties when the price of the asset is not driven by a Brownian motion and this probably explains why the Brownian motion is still central in the applications.

Papers [5] and [6] deal with the study of American options. [5] considers a new form for the premium using the fact that the option can be exercised early. The price of the American option is thus represented as the sum of the price of European option and the price of a derivative with stochastic maturity. The price of the latter is then given by the following somewhat lengthy formula

$$\eta_t = \mathbb{E}\left[ e^{-r(\tau - t)} \left( N(\tau, S_\tau) - a(\tau, S_\tau) \right) 1_{\tau \leq T} \middle| \mathcal{F}_t \right].$$

The advantage of this formula compared to its analytic counterpart is the fact that $N$ can be any measurable function, whereas in the equivalent formulation it is required to be smooth enough. Given the required smoothness the candidate shows in [5] that the two representations are indeed equivalent. To furnish some application of the new formula convertible bonds have been considered. In this case $N$ is discontinuous at $T$. In my view this is the original contribution in this paper. Whether it will be important in the future it is hard for me to judge. It seems that the representation above is not easy to be employed apart from cases with simpler structure as discussed in [5] and [6]. Paper [6] studies American put options and offers an approach for the approximation of the region of execution, that is since the option can be exercised at any moment in time the derivative is exercised when the time and price fall into the following well-defined space-time region

$$\Upsilon = \{(t, x) \in [0, T] \times (0, c(t))\} \in \mathbb{R}^2.$$

The execution happens when the price strikes the function $c$. The form of $c$ is not explicit. The candidate proposes a sequential approximation of $c$ with exponents of a piece-wise linear function whereby each line is determined by successively solving cases of pricing special derivatives. I could not understand why given a fixed $N$ and given a set of points in time and values at those times (approximating $c$) there is a financial derivative whose exercise is optimal when the strike to this specially constructed function occurs. For non-specialists in financial mathematics there are also some other unclear points. In any case these papers offer a good contribution to the area.

Paper [7] considers discounted perpetual game call options. These derivatives give right to both the buyer and the seller to exercise the option. However, the seller pays a fine if he decided to cancel the option. A main assumption in this work is that the horizon is infinity but the addition of potentially positive discounting factor means that the option will be exercised in the long run. The main goal of the paper is that given that the price of an asset is driven by a Feller process to study the regions $\Upsilon^b, \Upsilon^s$, that is the set of points such that if the price of the asset is in one of them then either the buyer ($\Upsilon^b$) or the seller($\Upsilon^s$) exercises the option. Since the horizon is infinite then these sets are one-dimensional, i.e. subsets of $\mathbb{R}^+$. The most general assertion is that

$$x \in \Upsilon^b, y > x \implies y \in \Upsilon^b,$$

see Proposition 3.3. In the case of the price being driven by a standard Brownian motion and under some assumptions for $\Upsilon^b$ (I do not know whether they cannot be demonstrated to hold true in general) formulae for $B$, such that $\Upsilon^b = [B, \infty)$ have been deduced and it is discussed when, depending on the parameters, we have any of the following three possible scenarios $\Upsilon^s = \emptyset, \{K\}, (K, A)$. There are some unclear points in the proofs of Proposition 3.2 and Proposition 3.6, but in general this does not take away any of the contributions since these are omissions and minor errors but not irreparable flaws. Paper [8] deals with discounted perpetual game put options. These derivatives give right to both the buyer and the seller to exercises the option. However, the seller pays a fine if he decides to cancel the option. The maturity date of the option is taken to be infinity. The processes that drive the price of the asset are Feller processes but for the majority of results in the paper it is assumed to be driven by a standard Brownian motion. The results are very similar to the ones in paper [7]. The cited literature however differs and I suppose that there are some fine differences between both works. The regions $\Upsilon^b, \Upsilon^s$ are studied in detail the same way as it was done in [7]. Both papers offer numerical experiments. It is worth noting that in this paper there are some unclear points too, e.g. whether $\Upsilon^\cdot$ is assumed to be semi-closed or it is proven so. The paper, however, contains sufficiently original contributions and is in line with the whole train of ideas of the applicant. Paper [9] is the most interesting amongst the three discussed in this paragraph. The main difference is that the penalty of the seller is now proportional to the difference between the strike and the price and this does change the problem sufficiently enough. The candidate has again derived the properties of the regions $\Upsilon^b, \Upsilon^s$ which in all cases are of the type $\Upsilon^s = [0, A), A \geq K$ и $\Upsilon^b = [B, \infty)$. In this work the put and call variants are dealt with together. The case when the price is driven by standard Brownian motion has been discussed separately. The main equation for $A, B$ has been supplied but it is worth noting that it does not have a closed solution. This is the reason for the ensuing numerical experiments and approximations. Overall, papers [7,8,9] are very good contributions of the applicant. I can only recommend that the candidate makes additional efforts when writing his papers since it is sadly evident that in more applied areas even journals with high impact factor do not provide a very good refereeing and this does not help towards the reductions of inaccuracies and flaws. I did not find major flaws in these papers but there are a number of unclear points and inaccuracies.

Paper [10] gives some expressions for the Laplace transforms of the time and the position of Brownian motion as it reaches a piece-wise linear curve. The results are simple and direct application of formulae present in the literature. Corollary 3.1 is known even in the case of spectrally negative Lévy processes. In my opinion the results are suitable for textbooks, tables, or appendix of a research paper but they cannot constitute a paper in themselves.

## 4. Teaching activity

Tsvetelin Zaevsky has a specialized record of teaching. He has taught the seminar classes for "Financial theory 2" during the period 2004-2009 and since 2019 lectures the course "Mathematical theory of the financial market". I expect that he will carry on teaching in the future, thus significantly contributing to the development of the financial mathematics in Bulgaria.

## 5. Personal opinion about the candidate

I have known the candidate since 2014 when we both joined IMI-BAS. I have an excellent opinion about him. He is a reliable and responsible colleague and co-author of mine. I have also witnessed his scientific development in the area of financial mathematics. I am certain that Tsvetelin Zaevsky has the potential to be a key person in the near future development of our section and I hope as an Associate Professor he will attract PhD students at IMI-BAS. The latter is very important for the development of Bulgarian mathematics since we do need active people who not only produce good scientific output but also have the right approach to science, i.e. responsibility, cooperation and can take the initiative.

## 6. Recommendation and overall evaluation

My overall evaluation about the work of Ass. Prof. Tsvetelin Zaevsky is very positive. I think that he can be at the heart of the preservation of our traditions in financial mathematics and its future development. As far as I know he already works with prospective researchers interested in this applied field of mathematics. Since we work in the same department I follow the development of the candidate and I think that Ass. Prof. Tsvetelin Zaevsky is in the process of establishing himself amongst the international scientific community in financial mathematics. This is attested by the increasing number of publications which are published in scientific journals with very high impact factor. Since I have some doubts regarding the refereeing process in the aforementioned journals I have read with additional care the papers of Ass. Prof. Tsvetelin Zaevsky and I am of opinion that overall they are a very good contribution to the area and convincingly demonstrate that he has mastered the mathematical techniques of financial mathematics.

I have two major recommendations. The first one is to broaden his mathematical techniques. One must mention that Ass. Prof. Tsvetelin Zaevsky uses very well specific techniques but in my view the addition of new ones will be very beneficial for his mathematics. This can be achieved for example by participation in some international projects or by working in a new area of probability. The second recommendation is to attempt to establish himself as an author in some of the classical journals, if they accept financial mathematics at all. I think the best international reputation is attained when the papers go through the tough and careful refereeing process of some not very highly ranked journals but with high research standing.

I have some suggestions concerning the writing of journal papers. I think the papers of the candidate are very well written with respect to the English and the general outline, but I think there is room for improvement in the way mathematical details are presented. It is important that the mathematical proofs are accessible to broader specialists. Besides when the proofs are clearly presented there is less room for inaccuracies. Also according to me when Ass. Prof. Tsvetelin Zaevsky is at the stage to progress further in his career it will be good that papers such as [10] and [3] not to be applied for competition. I have commented on [10] in the main text above and therefore I will highlight that [3] is a direct subset of [4] and this is not a result of improvement or generalization.

## 7. Conclusion

According to the applied documents the only candidate Ass. Prof. Tsvetelin Zaevsky satisfies all the minimal requirements set by the Act for the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations of the Institute of Mathematics and Informatics (IMI) at BAS which stipulates the award of scientific titles and academic poistions. My personal opinion concerning his work is fully confirmed by the applied documents which clearly demonstrate that Ass. Prof. Tsvetelin Zaevsky is a very good specialist in the research area of the competition.

I give overall positive evaluation of Ass. Prof. Tsvetelin Zaevsky and I strongly recommend that the scientific panel approves of the candidate and to propose to the Scientific Council of IMI-BAS to elect Ass. Prof. Tsvetelin Zaevsky as an Associate Professor at IMI-BAS in professional field 4.5 Mathematics, scientific direction *Probability theory and mathematical statistics (Stochastic models in finance).*

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