Review

On a competition for an academic position ”professor”, in the field of higher education 4. Natural sciences, mathematics and informatics, professional advancement, 4.5 ”Mathematics”, specialty Mathematical analysis (Special functions).

by Prof. DSc. Tsvatko Rangelov, Member of the Scientific Jury, appointed by order N:321 / 02.09.2018 of the Director of Institute of Mathematics and Informatics, approved by the Scientific Counsil on 19.07.2019 (Protocol N: 8).

1) The competition, with a term of 2 months, was been announced in the State Gazette no. 52 of 02.07.2019 for the needs of the Institute of Mathematics and Informatics (IMI), BAS. Associate Prof. DSc. Yordanka Dobreva Paneva-Konovska has submitted documents for participation in it. She graduated from the Higher Pedagogical Institute (VPI) - Shumen, majoring in mathematics in 1977. In 1999, after postgraduate studies at IMI BAS she defended her PhD thesis "Basicity and completeness of enumerated systems of Bessel functions and polynomials". In 2018 she defended her thesis "Functions of Bessel and Mittag–Leffler and generalizations"for Doctor of Mathematical Sciences at IMI, BAS.

Assoc. Prof. Y. Paneva had been worked as an assistant since 1977 until 1987 in "Nature Faculty of Mathematics"of VPI - Shumen, since 1987 so far, as an assistant and Associate Professor at the Technical University - Sofia, "Faculty of Applied Mathematics and Computer Science". From 2012 to 2014 she worked as an associate, and from 2014 of an additional employment contract, as an Associate Professor at the Institute of Mathematics and Informatics, section "Analysis, geometry and topology".

2) Assoc. Prof. Y. Paneva's scientific activity is in the field of real and complex analysis, special functions: Bessel functions, Mittag–Leffler functions, fractional calculus, convergence of series of special functions, etc. The results of her scientific activities are contemporary publications, some of which are submitted for participation in this competition.

There are presented a total list of 69 publications (including 2 monographs and 4 textbooks), 26 of which (including 1 monograph) for participation in this competition and did not participate in the competition for Associate Professor in 2008. Articles were published since 2008, in renowned mathematics and applied mathematics journals, such as: Fract. Calc. Appl. Analysis - 2; J. Appl. Math. - 1; Int. J. Appl. Math. - 2; Compt. rend. Acad. bulg. Sci - 2; Adv. Math. Sci. - 1;Math. Balk.- 1; Math. Maced. - 1; AIP Conf. Proceed. - 9 and 6 in proceedings of other conferences. With Impact Factor (IF) are 3 publications [2, 4, 24], with SJR are 10 publications [6, 7, 12, 14, 16, 17, 22, 23, 25, 26]. Only 1 of the publications is co-authored - with Y. Nikolova.

It was presented a list of 69 citations (without self-citations) of the publications for the competition. From those citations 45 were in issues with IF or SJR. It is clear that the requirements of Art. 3 (1), 3. of IMI Rules (for at least 10 publications with IF or SJR) are met.

In connection with Art. 2 of the IMI Rules for the "minimum required scores, by set of indicators for the candidate Assoc. Prof. Y. Paneva had been obtained the following: A - 50 points; Б - 100 points; B - 104 points; Г - 314 points; Д - 342 points; E - 321 points, which means that this requirement was fulfilled.

3) The author's report correctly reflects the content and contributions in the works of Assoc. Prof. Y. Paneva.

The submitted works for participation in the competition will be distributed in the following groups:

3a) Asymptotic formulas and representations, here are the works [2 - 6, 10].

3b) Series convergence, here are the works [5, 7, 9, 12 - 20, 23].

3c) Fractional calculus - integrals and derivatives, here are the works [11, 24, 25, 26].

The monograph presented by Assoc. Prof. Y. Paneva [1] deals with Bessel functions and their generalizations, as well as asymptotic formulas for them. It is written in an accessible language. In the beginning there were given concepts and statements for special functions which are necessary for understanding the material related to integral representations, asymptotic formulas and series of Bessel functions, as well as zeros of the Hankel transform. In the last chapter is presented an interesting application for finding non–stationary temperature field, by using the finite Hankel integral transform.

Before I analyze the works, combined in the above groups, I would like to share a few words about the theme of the competition. Special functions are functions that do not express themselves through elementary functions. These are solutions to the equations of mathematical physics, defined by integrals or series. Special functions are included in the monographs, handbooks, and lately they are built in functions in computer programs, like Mathematics, Matlab and more. All this was possible due to the presence of theoretical studies, included into monographs e.g. of Erdelyi, Bateman, Watson, etc., as well as in numerous later publications some of which has been featured in this competition.

3a) In [2, 3] are obtained results for the series

$$\sum_{n=0}^{\infty} a_n J_n(z), \tag{1}$$

where

$$J_n(z) = \sum_{k=0}^{\infty} \frac{(-1)^k (z/2)^{2k+n}}{k!\Gamma(k+n+1)}, \quad z \in C\backslash(-\infty, 0),$$

are Bessel functions as well as Bessel-Maitland series. New results are proved for the convergence of series (1). It was obtained sharp estimate for the representation of Bessel functions

$$J_n(z) = \frac{1}{n!}(z/2)^n \left(1 + \theta_n(z)\right), \quad \theta_n(z) \to 0, \text{ при } \quad n \to \infty, \tag{2}$$

of kind $|\theta_n(z)| \leq \dfrac{C}{n+1}$ and an estimate of a similar kind was obtained for the functions of Bessel–Maitland.

In publications [4, 5, 10] were studied Mittag–Leffler functions

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad \alpha > 0, \quad \beta > 0, \tag{3}$$

and new estimates for the behavior of θ in the representations of $E_n(z)$, $E_{n,\beta}(z)$, $E_{\alpha,n}(z)$ for $n \to \infty$ are obtained. As a note, Mittag–Leffler functions has been intensively studied in the last years in connection with the investigation of solutions of differential equations, with fractional derivatives. Considering this every new result for them is important.

Similar to the publications [2, 3] in [6] for Bessel–type functions (generalized Bessel–Mainland functions with 3 indices) has been obtained new estimates for their asymptotic behavior at large index values.

In this group of results by Assoc. Prof. Paneva, I would like to mention the results, obtained in [10], where for the 3-parameter generalization of (3), obtained by Prabhakar for Mittag-Leffler function, were shown asymptotic formulas, at large parameter values.

3b) Some of these publications [14, 16], [17], [23] are related to convergence of series of Bessel type, while [7-9], [11], [12, 13], [15], [18], [19, 20], [21] are for Mittag–Leffler type series. The main questions, which Assoc. Prof. Y. Paneva considered in these publications, are finding the radius of convergence and estimation of the behavior of the series, on the boundary of the circle of convergence. There has been proved new: Abelian type theorems (if the series are convergent at a point in the boundary z_0, then there is a limit to its sum for $z \to z_0$ in the angular region, with a peak z_0); Tauberian and Littlewood type theorems (provided the order of increase of the coefficients, if there exists a sum of the series for $z \to z_0$ in a radial direction, then the series converges in a point z_0); Fatou type theorems (for series convergence on the arc of the unit circle, provided that the points of this arc are regular for the sum of the series).

In publications [17], [18, 20] Assoc. Prof. Y. Paneva generalize the concept of overconvergence for power series for Bessel type in [17] and for Mittag–Leffler type series in [18, 20]. A power series with a finite radius of convergence R is overconvergence if there exists a subsequence $\{s_{pk}\}$ of the partial sum's sequence and a region G, containing an open disc of radius R and having a nonzero intersection with its boundary, such that $\{s_{pk}\}$ is uniformly convergent in G. In the papers was also generalized the result of Ostrovski for power series, for the series of Bessel functions and series of Mittag-Leffer functions. These generalizations are not trivial, they require precise estimate of the remainders of the series using estimates of the type (2) in 3a) for Bessel series.

In this group of results by Assoc. Prof. Paneva, I will note [11], [15, 19] where new ones were obtained, generalizing those for Mittag–Leffler multi-index functions and for $3n$ parametric functions of Mittag-Leffler. Also, fractional derivatives and integrals of these functions were given there, as well as were proved theorems of Abelian, Tauberian, Littlewood and Fatou type for them.

3c) An area of analysis, investigating fractional integrals and derivatives has been actively developing in the recent years. This is stimulated by the numerous applications of differential equations with fractional derivatives in process modeling in mathematical physics, mechanics, biology and more. Within the papers [11], [24 - 26] Assoc. Prof. Y. Paneva studied fractional derivatives and integrals of functions of Bessel and Mittag-Leffler type, including multi-index ones.

There are different definitions of fractional integrals and fractional derivatives respectively. One of them is a Riemann-Liouville fractional integral:

$$R^{\lambda} f(z) = \frac{z^{\lambda}}{\Gamma(\lambda)} \int_0^1 (1-\tau)^{\lambda-1} f(z\tau) d\tau$$

and respectively a fractional derivative of order $\lambda \in C, Re\lambda > 0$ is $D^{\lambda} f(z) = D^n R^{n-\lambda} f(z)$, n is a natural number.

A number of new results has been obtained, to express the derivative of order n of Bessel–Maitland functions with 2 indices, through those with 3 indices in [25]. Similar dependencies are proven in [11], [24, 26] and for multi-index functions of Mittag-Leffler, as well as for integrals and fractional derivatives.

4) Assoc. Prof. Y. Paneva is actively involved in funded projects from the NSF, as well as in projects between BAS and SANI and BAS and MANI.

5) The teaching activity of Assoc. Prof. Y. Paneva is in the Bbachelor's programs and in the Master's programs of Technical University of Sofia in Mathematical Analysis, Complex Analysis and more. There were presented 4 textbooks, 2 in co-authorship, in the studied mathematical disciplines, at the Faculty of Applied Mathematics and Informatics, Technical University of Sofia. She was a consultant to a defended PhD student on her own preparation.

6) I have no critical remarks. I know Assoc. Prof. Y. Paneva as industrious and actively working in the current field of mathematics.

7) Conclusion: I give a positive assessment of the works of Assoc. Prof. Yordanka Paneva-Konovska and I believe that she fully satisfies the requirements of the ZRASRB for the competitive position and in the submitted articles for the competition there is no plagiarism.

I am recommending the Scientific Jury to propose to the Scientific Council of the Institute of Mathematics and Informatics to select Assoc. Prof. DSc. Yordanka Paneva- Konovska for Professor in the professional field 4.5 "Mathematics specialty Mathematical Analysis (Special Functions).

October 24, 2019 Signature

T. Rangelov