

REPORT

by Prof. PhD Georgi Venkov, FAMI, TU – Sofia

at the competition for the academic position "Professor"

in direction of higher education 4 Natural sciences, mathematics and informatics

Professional field 4.5. Mathematics, Specialty Mathematical Analysis (Special Functions), for the needs of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Section "Algebra, geometry and topology",

announced in SG No. 52 / 02.07.2019

with candidate: Asso. Prof. D.Sc. Jordanka Paneva-Konovska

I present my report on this competition as a member of the Scientific Jury, determined by the Order No. 321 of 02.09.2019 of the Director of Institute of Mathematics and Informatics (IMI), Bulgarian Academy of Sciences (BAS).

The report was prepared in accordance with the requirements of:

- the Law for the Development of Academic Staff in the Republic of Bulgaria (ZRASRB),
- the Rules for the Implementation of the ZRASRB,
- The Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at BAS and at IMI of BAS.

1. General information for the candidate

According to the documents submitted for participation in the competition, Assoc. Prof. Jordanka Paneva obtained a Master's Degree at the Faculty of Natural Sciences and Mathematics of the Higher Pedagogical Institute (HPI) - Shumen in 1977. In the period 1996-1999 she is a PhD student at the Department of Complex Analysis, IMI -BAS in the field 4.5 Mathematics, specialty "Mathematical analysis" and defends a thesis on the topic "Basicity and Completeness of Enumerated Systems of Bessel Functions and Polynomials". In 2018, Assoc. Prof. Paneva defended her dissertation on "Bessel and Mittag-Leffler Functions and Generalizations" for the acquisition of the Doctor of Sciences degree in IMI-BAS. Assoc. Prof. Paneva's teaching experience began in 1977 as an assistant professor at the Department of Mathematical Analysis, Faculty of Natural Sciences and Mathematics (HPI), and since 1987 she has been an assistant professor at the Department of Mathematical Analysis and Numerical Methods, TU – Sofia. She holds a position of Associate Professor in 2008 at the Faculty of Applied Mathematics and Informatics of the Technical University - Sofia, and in 2014 she was elected to the same academic position in the section "Analysis, Geometry and Topology", IMI-BAS. As a long-time lecturer at TU-Sofia, she teaches lectures and tutorials in the fields of Mathematical Analysis (parts 1 and 2), Complex Analysis, Functional Analysis of the specialties at FAMI, as well as Mathematics (parts 1 – 3) of engineering specialties in other faculties. In addition to being an excellent teacher, Assoc. Prof. Paneva is also distinguished for her active research. She participates in the editorial board of an international scientific journal; she is a reviewer in more than ten international scientific journals; she participates as a member of international Research projects, projects with the National Science Fund, IMI-BAS and NIS of the Technical University - Sofia.

2. General characteristics of the works submitted for the competition

For participation in the competition, Assoc. Prof. Paneva presented a monograph and 25 articles that were not used for the acquisition of the PhD and Doctor of Science degrees, or for the position of the Associate Professor. All articles have been published in international refereed and indexed journals, of which 24 are solo, 3 have impact factor (IF) and 10 have impact rating (SJR). Therefore, the additional IMI – specific requirement to have at least 10 IF or SJR publications has been complied (regarding applicants going to occupy the academic position of Professor in Area 4 - Natural Sciences, Mathematics and Informatics). On the other hand, according to the Annex 1 to Rules of BAS for the Implementation of the ZRASRB, giving the minimal scientific indicators to applicants going to occupy the academic position of Professor in professional field 4.5. Mathematics, the following table is indicative:

	Q1	Q2	Q3	Q4	SJR	sum
public.	1	-	1	1	9	11
points	50		30	24	180	284

It is clear, that the presented publications of the candidate far exceed the minimum requirement of 220 points in Group of indicators G ($G.7 + G.7 = 30 + 284 = 314$) of the Annex 1 to Rules of BAS for the Implementation of the ZRASRB. Moreover, the total IF and SJR of Assoc. Prof. Paneva's articles are respectively 3,937 and 1,646, indicating a high level of results presented in them.

According to the attached list, 13 of the candidate's entries for this competition are cited in 69 scientific publications, 55 of which are in international journals, referenced and indexed in Web of Science and / or Scopus. According to Indicator D.11 of the Annex 1 to Rules of BAS for the Implementation of the ZRASRB, the citation index of Assoc. Prof. Paneva equals 342 points, which far exceeds the minimum requirement of 140 points.

3. General characteristics of the applicant's research and pedagogical activity

Assoc. Prof. Jordanka Paneva is distinguished for her research and publishing activities among the colleges of IMI-BAS and the Faculty of Applied Mathematics and Informatics, TU-Sofia. She has participated in numerous international scientific forums in Bulgaria and abroad, and has participated in editorial boards and research projects.

Furthermore, Assoc. Prof. Paneva has a highly appreciated pedagogical activity. She conducts lectures and exercises in the disciplines Mathematics 1 - 3 part of engineering specialties at TU-Sofia, in the disciplines Mathematical analysis 1 and 2, Complex analysis, Functional analysis for students in the fields of Applied Mathematics and Informatics and Informatics and Software Sciences " at FAMI. As Vice Dean of FAMI, she actively participates in the creation of new curricula and the updating of existing curricula of specialties at the Faculty of Applied Mathematics and Informatics and other faculties of TU-Sofia, collects and prepares the necessary information for accreditation of the professional field 4.5 Mathematics and accreditation of doctoral programs to FAMI.

4. Basic scientific and applied contributions

Assoc. Prof. Jordanka Paneva's main scientific interests are in the field of special functions, and in particular to study the asymptotic behavior of Bessel and Mittag-Leffler types of functions and the convergence of series in such type of functions, as well as differentiation and integration of fractional order.

The main scientific and applied scientific contributions in these fields can be grouped as follows:

4.1. Contributions of the monograph

The monograph "After the Invitation to Bessel Functions" was published in 2019 by Softtraid in English, with reviewer Prof. Virginia Kiryakova. It has a volume of 116 printed pages and consists of an introduction, six chapters and a bibliography citing 68 items. It is dedicated to different families of special functions, namely the Bessel functions and their generalizations with two, three and four indices. It gives basic properties and asymptotic formulas, integration operators and some of their applications for the functions under consideration. The properties discussed allow the author to investigate the convergence in such families of special functions, yielding results such as the Cauchy-Hadamard, Abel and Tauber theorems. Using the Maple computer algebra system, the monograph presents various examples of three-dimensional graphs of generalized Bessel-Maitland functions.

4.2. Contributions from Contest Publications

The contributions of Assoc. Prof. Paneva's publications can be organized in the following directions:

- **Inequalities and asymptotic formulae**

This group includes the results contained in publications [2-6] and [10]. They refer to the different kinds of Bessel (1–4 indices) and Mittag-Leffler (1–3 indices) types functions. The main purpose of these publications is to make more precise estimates of the corresponding residual term, both in the whole complex plane and in its compact subsets. These results are natural generalizations of the well-known formula for Bessel functions of the first kind. According to the parameter values, it is possible that the first few terms of the functions considered are zero. A detailed study on this for Bessel type functions is contained in the publication [6]. More precise estimates of the module of the corresponding residual term in the representation of Bessel type functions are made in publications [2, 3] and [6]. Similar results for the classic Mittag-Leffler functions (with 1 and 2 parameters) were obtained in [4] and [5] and for the 3-parameter functions in the publication [10].

- **Domain of convergence and series behavior of Bessel and Mittag-Leffler functions in the disk of convergence**

These are the results given in [5, 14–16] and [19]. In particular, an investigation of the area of series convergence by Bessel functions is done in articles [14] and [16]. Similar studies for series in classical Mittag-Leffler functions are presented in [5], and for series in 3-parametric Mittag-Leffler functions are contained in [15, 19]. These publications provide statements of the type of the Cauchy-Hadamard classical theorem and the Abel lemma and theorem. It has been found that each of the series under consideration is absolutely convergent in an open disk and diverging outside it. Moreover, inside the disk of convergence, the series are uniformly convergent.

- **Behavior of series by Bessel and Mittag-Leffler functions along the contour of the disk of convergence**

This group combines the candidate's works under numbers [2-5, 7, 9, 12, 14, 16, 19] and [20]. In general, analogues of Tauber and Littlewood's classical theorems for series in Bessel and Mittag-Leffler functions are obtained here. The results are related to Tauber theorems for Mittag-Leffler series in [5, 7, 9] and [19]. The Littlewood theorems for Bessel type series are set out in [2, 3] and [7], and for Mittag-Leffler type series [4, 5] and [19]. Other results that fall in this direction are analogues of the classical Fatou theorem for Bessel and Mittag-Leffler series [12, 14, 16] and [20].

- **Overconvergence**

This direction is related to the results of [17, 18] and [20], which prove Ostrowski's theorem (for overconvergence) and Hadamard's theorem (for gaps) for Bessel and Mittag-Leffler series. Another result in this series of studies is the analogue of Ostrowski's inverse theorem for the overconvergence of the case of 2-index Bessel-Maitland series functions [23].

- **Fractional differential and integral calculus**

The results in this area are related to fractional calculus in the classes of Bessel and Mittag-Leffler multi-index functions considered. In [25] it was proved that the n th derivatives of Bessel-Maitland functions with 2 indices are expressed by the generalized Bessel-Maitland functions with 3 indices. Some special cases of Riemann-Liouville fractional derivatives and integrals of Bessel-Maitland functions are considered here. Similarly, in [24] it is proved that the n th derivatives of Mittag-Leffler's $2m$ -parameter multi-index functions are expressed by the corresponding $3m$ -parameter functions. Some special cases of Riemann-Liouville and Erdélyi-Kober integrals of a fraction of Mittag-Leffler functions are also investigated. On the other hand, in paper [11], the dependences between the $3m$ -parametric functions of Mittag-Leffler and the integrals and fractional derivatives of the $3m$ -parametric functions of Mittag-Leffler multiplied by an appropriate power function are presented. The resulting relationships in [11] and [24] are modified in [26]. In particular cases, the corresponding results for the Prabhakar function (3-parameter Mittag-Leffler function) are also obtained here.

After using the free Crossref iThenticate's database platform (<http://www.ithenticate.com/products/crosscheck>), I can confirm the lack of any plagiarism in the scientific papers submitted by the applicant.

5. Textbooks and teaching materials

For participation in the competition, Assoc. Prof. Jordanka Paneva presented 2 textbooks and 2 teaching aids. The textbook "Complex Numbers. Function of Complex Variable" is in accordance with the program of students at the Faculty of Applied Mathematics and Informatics in TU-Sofia. After each paragraph, a sufficient number of self-study assignments are suggested to students. The textbook "Mathematical Analysis 2 part" is intended for students of the Faculty of Mathematics and Computer Science, Bachelor's Degree, but can also be used by students of all other faculties of TU-Sofia. The textbook

contains the sections "Differential and integral calculus for functions of two or more variables", "Double, triple, curvilinear and surface integral", as well as their applications. Each topic includes the necessary theoretical material. The presented material is supported by many examples that undoubtedly contribute to its fuller and deeper understanding. The Laplace Transformation Tutorial is intended for the students of the Master's degree course in HPE at TU-Sofia, studying the subject "Selected Topics in Higher Mathematics", which includes the presented study material. The Manual for Mathematical Analysis 2 Using MAPLE is intended for students in the specialty of Applied Mathematics and Informatics at FAMI. The manual begins with an introduction to the Maple computer algebra system, but contains the necessary theoretical material and a sufficient amount of problems on the subjects studied in the discipline.

6. Critical notes and recommendations

I have no critical comments on the materials of Assoc. Prof. Jordanka Paneva for participation in this competition. A possible recommendation for the future development of the candidate is that she intensifies her work with graduates and PhD students.

7. Conclusion

In conclusion, I think that the submitted materials of Assoc. Prof. Jordanka Paneva-Konovska on this competition fully meet the requirements of the ZRASRB, the Rules for its implementation and the Regulations on the conditions and procedures for acquiring academic degrees and for occupying academic positions in the Bulgarian Academy of Sciences and at Institute of Mathematics and Informatics (IMI-BAS).

Therefore, I strongly suggest to the Honorable Scientific Jury to positively evaluate the candidature of Assoc. Prof. Jordanka Paneva and to unanimously recommend to the Scientific Council of IMI-BAS her choice for the academic position "Professor" in the field of higher education 4. Natural sciences, mathematics and computer science, professional field 4.5. Mathematics, specialty Mathematical Analysis (Special Functions).

14.10.2019 г.

Assessor:

София

(Prof. PhD Georgi Venkov)