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BOOK OF ABSTRACTS

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FORWORD

Bulgaria is going to host a Congress of the World Federation of National Mathematics Competitions for the second time – the first time it happened in 1994. Initial plan was to accommodate the Congress at the Black See coast. Unfortunately, some global issues forced the Organizing Committee to change the location to Sofia, which is also attractive place in historical and cultural perspective. This interchange could be a reason for the quite modest number of participants. Nevertheless, the thematic range of the talks that are announced is large enough to cover all Topic Groups.

The abstracts of the talks included in the Congress Program were approved by the Program Committee headed by Dr. Robert Geretschlaeger. We publish these abstracts without any changes of the content proposed by the speakers. Some abstracts are comprehensive and give idea about the theme; some are rather short leaving participants to decide whether they attend the presentation at their own risk. Anyway, the purpose of the Book of Abstracts is to help any participant to arrange his personal timetable, but not to replace a potential proceedings book of the Congress. Our hope is that every talk will urge the speakers to publish a corresponding article.

Borislav Lazarov (Editor)

Sofia, June 2022

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Peter Boyvalenkov [1], Stanislav Harizanov [2]

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Abstract. Bulgaria has always been among the pioneers regarding foundation and/or organization of International Mathematical Competitions. Together with Romania, those are the only two countries that have participated in the all 62 editions of the International Mathematical Olympiad (IMO) so far. Up to the moment, Bulgarian students have won 55 IMO golds, 123 IMO silvers, and 114 IMO bronzes, while Bulgaria was twice an organizer - Sofia 1966 and Bourgas 1975.

This talk is devoted to the various stages of the Bulgarian IMO team selection and preparation, as well as the additional activities that the Union of Bulgarian Mathematicians, the Institute of Mathematics and Informatics, and the Leaders of the Bulgarian team became engaged with through the years. For example, Bulgaria was amoung the key initiators of the foundation of the Balkan Mathematical Olympiad in 1984 in order to prepare the Balkan teams for IMO, and (together with Romania and Greece) has not yet missed a single edition. The same with the Junior Balkan Mathematical Olympiad, the European Girls Mathematical Olympiad, etc.

One of the key factors for the traditionally solid results of the Bulgarian students is the active role of the Institute of Mathematics and Informatics (IMI-BAS), which provides a priceless bridge between high-school math competitions and academia, as well as between problems of Mathematical reseach and competitions. For more than 50 years, the Bulgarian IMO leader has always been an employee of IMI-BAS. Most of the authors of national competition problems and lecturers at training camps are renowned Bulgarian mathematicians. And vice versa, the last two Bulgarian IMO leaders are now the director of IMI-BAS, respectively the president of the Union of the Bulgarian Mathematicians. This special collaboration between IMI-BAS and Bulgarian IMO team will be also analysed in detail in this talk.

Keywords: International Mathematical Competitions, Bulgarian participation, IMI-BAS.

MATHEMATICS, ITS HISTORY, AND MATHEMATICAL OLYMPIADS: A GOLDEN BRAID

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Abstract. A beautiful braid of mathematics, history, and mathematical Olympiads will be presented 'in a flesh.' I will convey 4 stories, each featuring a braid of history, old mathematical papers that often contain unnoticed or little noticed treasures that, once dug out, lend themselves to creating new original problems for mathematical Olympiads. Some of these stories have appeared [1, 3, 4]; others are waiting for an appearance in the new edition [2], and so you will be able to preview some pages of my future book.

Keywords: Mathematics, research, History, historical research, Soifer Mathematical Olympiad, Colorado, problem creating.

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PROBLEM-POSING STRATEGIES WITH EXAMPLES

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Abstract. There are many resources for learning problem-solving strategies, but very few for the process of posing or creating problems. The speaker has submitted (and has had accepted) problems to such varied competitions as the MathCounts, AMC-10, AMC-12, and American Invitational Mathematics Exam competitions in the USA, the Kangaruh competition in Europe (and beyond), and the AMC Maths Competition and the Maths Challenge in Australia; this talk will present some problem-posing strategies which the speaker has used to create some of these problems. Strategies include generalization, specialization, implementation, and variation, and the talk will include specific examples of these problem-posing strategies via problems that the speaker has created for some of the competitions listed above.

Keywords: Problem-posing, problem-creating.

HOMOTHETY: ENLARGING OR SHRINKING OF FIGURES MAY HELP

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Abstract. Sometimes in geometry making a figure larger or smaller may help finding a solution to a problem. Such a transformation is called homothety (or dilation). It works quite well for proving collinearity, concurrency, determining ratios, and constructing points. Imagine a geometric problem that seems "not-standard" and/or its geometric construction is complicated. Finding a suitable homothety may transform the construction/problem into "standard" one. The paper considers such problems and presents mathematics competition problems that can be solved using this tool.

Keywords: Homothety, non-standard problems, mathematics competitions.

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SCIENTIFIC AND WORLD AFFAIRS IN THE SOIFER MATHEMATICAL OLYMPIAD

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Abstract. *The Soifer (formerly Colorado) Mathematical Olympiad* differs from other Olympiads in a number of essential ways. We offer the same problems to all Olympians, who usually range from grades 6 through 12. This requires us to lean not on knowledge but instead on ingenuity and originality of our Olympians. Another distinction is our sources of inspiration. We often use old and new research mathematical and historical papers to squeeze out of them Olympiad-style gems of ideas. Then we dress them up to obtain exciting 'stories' that our Olympians enjoy.

In this presentation I will share with you samples of those stories-problems, such as

*) *"In Order to Form a More Perfect Union…"* [Minimizing Disagreements in the United Nations], problem 27.4, 2010 [1];

**) *A Dream for a Peaceful Ukraine*, problem 31.3, 2014;

***) [DNA of] *Love and Death*, problem 22.5 2005 [1]; and

****) *Stopping the Ebola Epidemics*, problem 33.4, 2016.

Time permitting, I will also share with you a problem that proved to be too hard for the use in the Olympiad but has been fruitful in mathematical research [2].

Keywords: Mathematics, research, Soifer Mathematical Olympiad, Colorado, problem creating.

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INDUCTION COMPLETES THE PROOF

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Abstract. The main purpose of the talk is to present some problems that may be solved with the use of the principle of mathematical induction. Many of those problems were given to participants of Mathematical Olympiads, especially international (see, for example, [1] and [3]). In some cases, also alternative solutions, not using mathematical induction, will be presented. I plan to enclose a few more problems of this kind in the article that will be submitted as a chapter for the Congress book.

Other issues concerning mathematical induction will also be discussed. Some typical mistakes that appear where induction is used will be shown and some examples on the incorrect use of induction will be presented (like that described in [2]). A few historical remarks will be made (cf. [4], [5] and [6]). An untypical extra bonus is planned as well.

Keywords: Mathematical induction, mathematics competitions, Peano's axioms, natural numbers.

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N-DIMENSIONAL CUBE, BINOMIAL FORMULAS AND PASCAL'S TRIANGLE

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Abstract. We present a stepwise construction for visualization of a 4-dim cube along with a standard combinatorial description in coordinates. Some illustrations of use of binomial formulas as "generating function" of *n*-dim cube elements should help (high)schoolteachers to make this topic more interesting for the students. We also look at the connection between Pascal's triangle in a lower triangular matrix form, matrix multiplication and (well known) formulas for the number of *k*-dim cubes in a *n*-dim cube.

Keywords: Hypercube, binomial formulas, combinatorics

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HOW TO FIND INFINITELY MANY INTEGERS WHICH FULFILL …

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Abstract. In the last few years we detected a couple of problems where the contestants were asked to prove that there exist infinitely many integers which fulfill certain conditions. We will present several proof techniques which help to tackle problems of this kind, e.g. Vieta Jumping, Pell's equation, a special residue class fulfills the conditions given or a formula appears from nowhere and may be proven directly or by mathematical induction.

The proof techniques will be demonstrated using examples taken mainly from German math competitions. The focus will be on showing how to find possibilities to formulate a certain relationship which can be proven by one of the techniques mentioned above.

Keywords: Infinite integer sets, mathematical induction, Vieta Jumping, Pell's equation.

THE ONLINE COMPETITION "VIVA MATHEMATICS WITH COMPUTER"

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Abstract. We present an online mathematics competition that was started in 2014 with the support of the telecommunication company VIVACOM (this is where the word 'VIVA' in the title of the competition comes from). To participate, the students have to register in advance at the site of the competition https://math.cabinet.bg/. On the preliminary announced date and hour for the beginning of the competition, each participant enters (via the internet) the competition site and gets access to a Worksheet with 10 problems that have to be solved for up to 60 minutes. To explore and solve the problems, participants may use any available help: literature, online resources, computers, advice from more experienced persons, etc. Precisely as it is when someone has to solve a problem in real-life. The answers to the problems are entered as decimal numbers directly in some designed for this purpose fields of the Worksheet. After finishing the process of solving the problems and entering the answers, the participant presses the button "Submit" and the server grades the results.

The goals of the competition, the ideology behind it, the kind of problems used in it as well as the organization and conduction will be briefly described.

Keywords: Online competition, computer algebra system, dynamic geometry software

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MATHEMATICAL OLYMPIAD PROBLEMS AND CODING THEORY: DEVELOPMENT AND PROOF OF A CRITICAL HYPOTHESIS

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Abstract. In a sunny day of May 2019, at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences Prof. Peter Boyvalenkov was teaching us invaluable points concerning spherical designs. At the some part of this lecture, we faced a system of equations of the form $\sum_{i=1}^{s}\beta_i\alpha_i^m=\gamma_i, m=0,\dots,2s-1$ where $\beta_i, i=1,\dots,s$ are positive integers, $\alpha_i, i=1,\dots,s$ are algebraic numbers and $\gamma_i, i=1,\dots,s$ are rational numbers. It was needed to determine $\alpha_i, i=1,\dots,s$.

When Prof. Boyvalenkov was elaborately explained the details of his approaches, I just remembered a problem from the 2016 team selection test of India that had been recently solved by me. It stated: "let a,b be two positive rational numbers such that $a^{\frac{1}{n}}+b^{\frac{1}{m}}$ is a rational number for some positive integers m,n. Prove that $a^{\frac{1}{n}}, b^{\frac{1}{m}}$ are both rational numbers. ". In the course of my proof, I only used that $a^{\frac{1}{n}}, b^{\frac{1}{m}}$ are indeed algebraic numbers. Thus, I came up with an educational guess: in our problem $\alpha_i, i=1,\dots,s$ must be rational numbers. Indeed, this hypothesis was corroborated by some solutions we had through the Banai's conjecture. I then started to work more on our hypothesis. We firstly solved it for the case of $s=3$ and were almost done for the case of $s=4$. We then implemented more advanced techniques and proved it for all s.

In this article, we outline the procedure of the developing the hypothesis, as well as the several steps of searches among mathematical Olympiad resources and research articles. During our works, we also found stronger results for algebraic numbers of prime degree. Our experience shows that how having a prior experience in Olympiad problems solving can facilitate the development of a good hypothesis and can also provide the researcher several interesting resources (i.e., similar problems and articles) to solve and read.

Keywords: Coding theory, spherical design, Banai conjecture, algebraic numbers, mathematical competitions.

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INVESTIGATING NOVEL PROPERTIES OF IRRATIONAL K DESCENDING TREES

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Abstract. This talk will be about my (Sarkar) personal experience with math research as coming from a competitive background. The research is described as follows. For any real valued $k > 1$, we consider the tree rooted at 0, where each positive integer n has parent $\left\lfloor \frac{n}{k} \right\rfloor$. The average number of children per node is easily seen to be k, thus this definition gives a natural way to extend k-ary trees to irrational k. We study the sequence r_d, giving the count of nodes at depth d in this tree. We first prove there exists some constant $c(k)$ such that $r_d \sim c(k) \cdot k^d$. We then study a family of values $k = \frac{a+\sqrt{a+4b}}{2}$ where we prove the sequence satisfies the exact recurrence $r_d = ar_{d-1} + br_{d-2}$. This generalizes a special case when k is the golden ratio and r_d is the Fibonacci sequence.

Keywords: Tree, irrational, constant, recurrence.

FOLDING POLYGONS AND KNOTS

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Abstract. A large number of interesting geometric objects can be folded from a long strip of paper. In our talk, we discuss some known results concerning methods of folding certain regular polygons and consider which knots can be folded in connection with these and what the relation is between them. We show some interesting mathematical activities that can be derived from these results as well as some potential problems as they could be used in mathematics competitions.

Keywords: Knots, origami, polygons.

IT IS (NOT) AS EASY AS IT SEEMS – THE ROLE OF DISTRACTORS OF SPECIFIC TASKS AT THE MATHEMATICAL KANGAROO

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Abstract. In this talk, we will focus on special distractors of tasks of the Mathematical Kangaroo, a popular international multiple-choice competition (e.g. [1]). Previously conducted theoretical considerations of the role of distractors (e.g. [2]) will be expanded by the study of empirical data. Based on the solution frequencies of the Kangaroo of Mathematics in Austria in the years 2015 to 2019, we will point out some surprising findings: Many conspicuously high or low solution rates of tasks can be explained by especially noticeable distractors. The findings can be useful for the creation of tasks for the Mathematical Kangaroo and for mathematics tasks in multiple-choice format in general.

Keywords: Mathematical Kangaroo, multiple-choice competition, distractors.

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WIELKOPOLSKA MATHEMATICAL LEAGUE

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Abstract. The Wielkopolska Mathematical League (WLM) is a competition for primary and secondary school students from Wielkopolska. The organizers are the Faculty of Mathematics and Computer Science of Adam Mickiewicz University in Poznań and the Poznań Division of the Polish Mathematical Society. The first edition took place in 2010. It was a joint idea of prof. Krzysztof Pawałowski and mine. The main task of WLM is to popularize mathematics among students of primary and secondary schools in Wielkopolska. The league is also intended to fill the gap in the difficulty of mathematical problems from school and problems from Polish Mathematical Olympiad. The competition is only by correspondence, so participants work similarly to scholars, with access to all literature and without excessive time pressure.

As the author of most of the WLM problems, I will show a few examples of them. I will also tell how these problems were created and how the ideas of the competition participants may differ from the original solutions.

Keywords: Mathematics competition, solving problems, creating problems.

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PROBLEMS WITH A HOOK AND LONG LEGS

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Abstract. A good problem should possess at least some of the following characteristics: it reveals unexpected connections; possesses intricate ramifications; leads deep enough to unsolved problems in mathematics; it is like a tune which keeps playing in your head over and over teasing and dazzling you.

All the above applies to problems designed for math circles. However, the switch to online math circles brought about by the pandemic has subtly changed which problems are suitable.

This talk will illustrate these ideas using the experience of the Bluebird Math Circle https://aimathcircles.org/bluebird/, an online community which connects mathematicians with Native American students, their teachers, and families.

Keywords: Math circles, Native American student.

THE IBEROAMERICAN MATHEMATICS OLYMPIAD HISTORY AND COMMUNITY

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Abstract. The Iberoamerican Mathematics Olympiad (IbMO) or, as it is fondly called "the Ibero", is one of the most successful and established regional Olympiads in the world with the participation of all Iberoamerican countries. Just recently the author had a chance to collaborate with two experienced and well-known members of the Iberoamerican community, namely Maria Gaspar and Patricia Fauring, to look back and analyze the key to its success and to reminisce on its origins and history. The result is an article that will appear in [1]. In many ways this article is an extension of one written by the author's mother Mary Falk Losada fifteen years ago [2]. The reason is that Mary Falk Losada asked the authors of [1] to continue and expand her analysis of the Iberoamerican Olympiad effort all these years later to be included in a compendium of similar efforts from around the world and, it is relevant that she is the author's mother because the origin of the Ibero was the result of a confluence of influential people and providential circumstances all tied to the efforts of the author's family towards the development of Olympiads, and Education in general, in Colombia and the Iberoamerican region. In fact, Mary Falk de Losada and her husband Ricardo Losada and all their children were part of the organization of the first Ibero in Colombia. The author herself organized another Ibero in 2005 and will organize the Ibero again in 2022.

As we shall exhibit, its continuing success is due to the creation of a real community of collaborating people sharing a common heritage, language, and mathematics competitions interest.

Keywords: Ibero, Iberoamerican Mathematics Olympiad, mathematics competitions.

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SELECTED AUTHOR-CREATED PROBLEMS

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Abstract. In this article is presented a selection of problems created by Nairi Sedrakyan and Hayk Sedrakyan for different national and international math Olympiads from 1985 to 2021. For some of these problems few generalizations are also provided. Few of these problems and/or their generalizations later on were published in [1]-[6].

Keywords: Competition geometry, number theory, algebra, combinatorics.

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THE HISTORY AND METODOLOGY OF KÖMAL COMPETITIONS

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Abstract. In 1893 a high school teacher launched a journal in Hungary for students and for their teachers. This monthly journal is more than a selection of interesting Mathematical and Physical problems: it offers competitions from the beginning to the present day. Mathematical and Physical Journal for High Schools, the so called KöMaL, survived World Wars, different political and educational environments. Inventions, outstanding quality, former results and tradition characterize KöMaL: it is known in Hungary as one of the most important tool – may I say institution – for educating talented students in STEM fields. This talk is about the history, inventions and quests of KöMaL.

Keywords: Point competition, education, talents, Mathematics, Physics, Informatics, high school students, Hungary

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RECENT DEVELOPMENTS ON PROBLEM CREATION AT THE INTERNATIONAL TOURNAMENT OF YOUNG MATHEMATICIANS

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Abstract. We shall introduce the essence of the International Tournament of Young Mathematicians, a competition with research-type partially open problems and we shall discuss the creation process of the problems by the International Organizing Committee, as well as the indirect feedback observed from the contestants. This includes a layout for a good problem, detailed examples of useful sources for approachable open problems, as well as a discussion of an aimed variety of problems in terms of difficulty, concepts and techniques, so that many areas of mathematics are addressed (from combinatorics to abstraction), but also that more countries would find the event attractive and mathematically accessible for participation. We will also include a discussion of possibilities for publishable work from students.

Keywords: Open problems, problem creation, scientific outreach.

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GIVING TRADITION A TRIAL – 30th ANIVERSARY OF CHERNORIZEC HRABAR TOURNAMENT

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Abstract. The first issue of the Chernorizec Hrabar Tournament took place in 1992 [1]. Since then, it become tradition on Nov 1 advanced Bulgarian math students to fete the Day of the Bulgarian Enlighteners by participating this tournament. However, the last two issues of it held in an extraordinary situation, which affects the tradition. The paper gives a retrospective view on the Tournament and speculates on the challenges related to its hybrid organization. Statistics of how the students in intermediate age groups performed in the last edition and two years earlier is provided. A list of test items illustrates the structure and the style of the competition papers. Comments and speculations on the role of some particular math topics in developing students' synthetic type reasoning are done.

Keywords: Chernorizec Hrabar Tournament, multiple-choice competitions, synthetic competence.

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HOW THINGS HAPPEN IN 125th SECONDARY SCHOOL

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Abstract. Two viewpoints on how math competitions serve to the growth of the advanced students in 125th Secondary School – Sofia. On the one hand – the role of the local section of the Union of Bulgarian Mathematicians, which is responsible for the math-competitions-calendar; on the other hand – the school executive body support of the extracurricular activities.

The report traces the almost twenty-year cooperation between the two institutions. The first author, teacher in Math at 125th School and head of the local section of the Union of Bulgarian Mathematicians since its establishment, is also the person to initiate Christmas Math Competition and Easter Math Competition [1] along with the specialized Math education at the cited school. The second author, a Deputy Principal, presents school strategy and project to introduce innovative elements in the organization and content of school curriculum through extracurricular activities, projects, lectures, short-term specialized trainings, etc. tracing students' advancement to independent learning and development of synthetic competence.

Keywords: Math competitions, extracurricular activities, synthetic competence.

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BULGARIAN NATIONAL COMPETITION "DISCOVERY OF YOUNG TALENTS"

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Abstract. Mathematical talents can be discovered at an early age when a child is able to read and to use basic arithmetic operations. Many and varied problems can be constructed in such a way that their solutions do not require much formal knowledge, but mostly thinking, reasoning and logic. Practically we do this in Bulgaria with the competition "Discovery of Young Talents".

Mathematical high schools are specific phenomena in Bulgarian education system. There are about 20 such schools around the country specialized to teach mathematics to talented students. Pupils start its education in mathematical high schools after finishing primary school. To find talented students suitable for mathematical high schools we conduct math competition especially prepared for this aim. The paper describes this competition and presents some examples of problems. The aim is to show that to participate in this competition one needs only the formal knowledge from the standard school syllabi, but to solve the problems the participants must have a lot of logical and non-standard thinking.

Keywords: Mathematical talents, math competition for young students.

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STATISTICAL ANALYSIS OF MATHEMATICAL COMPETITION "IVAN SALABASHEV"

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Abstract. Mathematical competition "Ivan Salabashev" is organized in Bulgaria since 1992. It is aimed to students grades from 2 to 12. Each paper from grades 2 to 9 consists of ten multiple choice questions and five open answer questions. Grades 10 to 12 compete on set of 3 to 5 problems.

The main goal of the organizers is to balance the papers both with respect to difficulty and with respect to different areas of mathematics. In this note we present analyses of distribution of problems with respect to the four main fields: Algebra, Geometry, Number Theory and Combinatorics. For each of the above topics we present set of problems including easy, medium and difficult problems. We also discuss the achievements of the students concerning the three levels of difficulty.

Keywords: Mathematical competition.

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INFORMATION TECHNOLOGIES AND MATHEMATICAL COMPETITIONS

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Abstract. Information technologies are tightly bound to the mathematical contests for decades. TeX was known to the math community long time ago and spreadsheets were used to process the data since early IT days. At the end of previous century the third ingredient (the Internet) become very important one. In my talk I will present how to blend these three ingredients to effectively conduct problem preparation, contestants registration, run the contest, do the grading and finalization operations. The contests to discuss are International mathematical olympiad, Kangaroo contest and National contests (not limited to mathematics).

In the last two years we all have felt a huge impact of Covid-19 to everything in our lives. Online contests, remote/online grading and online meetings were the only feasible options a while ago. Many of these options will stay with us for the future as well.

Keywords: International mathematical Olympiad, Kangaroo contest, TeX, data processing, online services.

MARKING SYSTEM FOR THE ELECTRONIC PAPERS OF THE TOURNAMENT OF TOWNS:

POSSIBILITIES FOR STUDENTS, ORGANIZERS AND JURY

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Abstract. The electronic system for elaboration of the papers of the International Mathematical Tournament of Towns has been existed in various forms since the 80s of the last century, however until recently the Central Jury has marked hardcopies of the papers. On the one side, such a format is simple but on the other side, limits the possibilities of the marking, especially for jury members outside Moscow. It was also uncomfortable for the organizers, who had to print scans.

A few years ago we started the work on creating the system for marking of electronic papers (scans) and three years ago the first beta-version appeared.

The COVID-19 pandemic created new challenges for the organizing committee and accelerated the work of developing the electronic system.

As a result, IS "Tournament of Towns" (https://my.turgor.ru) was created and is currently continuing to develop. It combines the personal account of the participant, the personal account of the Tournament's organizer in the city/town, the system for elaboration of the papers and marking system for electronic papers.

The system also has the features that made it possible to hold the Tournament during the most difficult periods of the pandemic in a mixed online-offline format. Some of them turned out to be very convenient, so we decided to use them in future for a better cooperation between participants and jury, such as for example online appeals etc.

Keywords: Tournament of Towns, electronic marking system

A MASS MATH CIRCLE DURING THE COVID-19 PANDEMIC: NEW EDUCATIONAL AND TECHNOLOGICAL APPROACHES

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Abstract. For many years we hold a math face-to-face circle in Moscow school 179 for several hundreds of students 10 - 14 years old. During the COVID-19 pandemic we had to change the format of our classes in both senses: educational and technical. After a year of distance classes, many students communicated that they liked this new approach, and half of them preferred distance classes next year although face-to-face classes were reopened.

I would like to talk about our new format of math classes and also about some technical details which made the educational process very convenient for students and teachers.

Keywords: Math circles, distance classes, math problems.

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MATHEMATICAL CREATIVITY IN METAVERSE

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Abstract. The metaverse is a network of persistent immersive 3D worlds where users' avatars can explore and interact. Metaverse has already had a significant impact on commerce and entertainment. There are efforts in the industry and research to leverage metaverse for kids learning opportunities. This talk presents a metaverse world for problem solving and creativity. This world allows players to become active partners in discovering and developing their computational thinking and creativity skills. In this world, kids can play, modify or create 3D experiences from games and simulations to storytelling by embedding math expressions attached to any 3D part. We will present an interactive model focused on problem-solving where the math expressions are visually executed, and their impact is simulated in an immersive 3D world. We show how the high level of youth unsupervised engagement through progressively more complex tasks and increasing ownership of their creations accelerates their computational thinking learning and the growth of their creativity. The acquired skills of this cognitive creativity empower learners and enhance their mindset, becoming engaged in creativity and innovative thinking.

Keywords: Computational Thinking, Decentralize Creativity, Metaverse

LESSONS LEARNED FROM NÁBOJ ONLINE COMPETITIONS: TECHNICAL REQUIREMENTS AND PROBLEM SELECTION

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Abstract. Starting in March 2020, the pandemic situation throughout Europe led to the cancellation of many competitions. We would like to share our experience we collected in having three events of the international team competition Náboj [1] in an online format.

On one hand, there are technical issues to cope with because almost every single contestant has his or her personal access to the server system in comparison with few access stations per competition site in a face-to-face event. On the other hand, since contestants need a computer with Internet access to enter the competition, the temptation of using computer power and Internet is enormous. Therefore, creating math problems for an online competition is set under special rules: problems should not be solved faster by using electronic devices than by thinking and computing by hand.

The focus of the talk will be working out the differences in the formulation of math problems for an online competition compared to problem statements for a competition in presence: this will be explained and evaluated with the help of several examples.

Keywords: Online team competition, specific problem posing.

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DESIGNING AND IMPLEMENTING THE WORLD'S FIRST INTERACTIVE ONLINE MATH CONTEST: A ROADMAP FOR NAVIGATING THE NEW NORMAL

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Abstract. When the pandemic of 2020 struck, mathleague.org raced to develop the world's first fully online real time contest delivery system; as a result, mathleague.org did not have to cancel a single contest and was the only major math contest in the US to complete its 2020 season. Many other contests followed suit by the fall of that year, and online contests have now become a regular fixture of the math contest landscape. Within this talk, the challenges every math contest faced starting in March 2020 is recalled, the policy decisions and adaptations mathleague.org implemented in designing its contest delivery system is detailed, the impact our work has had so far and the outlook for online math contests moving forward is explored, and steps other contests can take to implement a fully-online or hybrid math contest are described.

Keywords: Online competition, hybrid competition

SELECTED PROBLEMS FROM THE KÖMAL MAGAZINE

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Abstract. Creating new, non-standard, high-quality contest problems is not easy. Usually it is much harder than actually solving the problems. Problems can come from research questions or results, new mathematical observations, extending other problems, sometimes from accidents. In this talk we present a handful of problems, mainly form the KöMaL Magazine's contest, together with their origins and stories.