EULER'S LINE AND EULER'S CURVE DEPENDENT BY A POINT

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Abstract

Let $k(O)$ be a central curve of second degree circumscribed around a triangle ABC and let O be the center of $k(O)$. Further let A_0, B_0, C_0 be the midpoints of BC, CA, AB respectively.

Proposition 1. The lines $h_a(O)$, $h_b(O)$, $h_c(O)$ through A, B, C, and parallel to OA_0, OB_0, OC_0, respectively, concur at a point $H(O)$.

Following the ideas related to the Euler's line, we can consider the point $H(O)$ as an analogue to the orthocenter of the $\triangle ABC$. Let the lines $h_a(O)$, $h_b(O)$, $h_c(O)$ meet BC, CA, AB at A_1, B_1, C_1 and also meet $k(O)$ at A_2, B_2, C_2, respectively. Denote by A', B', C' the symmetric points of $H(O)$ with respect to A_0, B_0, C_0. It holds

Proposition 2. The points A_2, B_2, C_2 are symmetric to $H(O)$ with respect to A_1, B_1, C_1.

Proposition 3. The points A', B', C' are symmetric to $H(O)$ with respect to A_0, B_0, C_0.

So for the centroid G of the $\triangle ABC$ it holds

Proposition 4. Points O, $H(O)$ and G are collinear and $\dfrac{H(O)G}{GO} = \dfrac{1}{2}$.

The line $l(O)$ through O, $H(O)$ and G plays an analogue to the Euler's line of $\triangle ABC$. The curve $\Omega(H(O))$ of second grade defined by the points A_0, B_0, C_0, A_1, B_1, C_1 is an analogue to the Euler's circle and as it should be contains the midpoints A_3, B_3, C_3 of the segments $H(O)A$, $H(O)B$, $H(O)C$, respectively.

The above results give reason to name the line $l(O)$ and curve $\Omega(H(O))$ *Euler's line* and *Euler's curve* for the $\triangle ABC$ *with respect to the point* O. The heuristic part of the results has been done by the software "THE GEOMETER'S SKETCHPAD". A series of properties of $l(O)$ and $\Omega(H(O))$ has been established and proved.