DALEST
224269-CP-1-2005-CY-MINERVA-M

STEREOMETRY ACTIVITIES WITH DALEST

UNIVERSITY OF CYPRUS

UNIVERSITY OF ATHENS

UNIVERSITY OF SOFIA

UNIVERSITY OF LISBON

UNIVERSITY OF SOUTHAMPTON

Socrates
Minerva

Education and Culture

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CONTENTS

PREFACE

The work presented in this book results from the collaboration of five universities in five different European countries together with teachers and their pupils from those countries.

This book can be used by teachers in primary and secondary schools, primary and secondary mathematics teachers and teacher educators.

Partners Information

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CHAPTER I

Introduction

The main purpose of the DALEST project was to develop a dynamic three dimensional software suitable for teaching stereometry in middle schools. The software focuses on the development of learners' thinking abilities and on their abilities to model ideas, to analyze and solve problems in their everyday activities. Specifically, the main objectives of this project were: (a) To analyse the existing geometry curricula in European schools and evaluate teachers' requirements related to the teaching of stereometry, (b) To develop a dynamic three dimensional software for teaching and learning stereometry (c) To provide in-service training to a number of teachers on the teaching of stereometry with the use of the software, (d) To develop teaching scenarios for teaching stereometry with the use of the software, (e) To test the software in different schools, (f) To evaluate the effectiveness of the software and the proposed teaching approaches.

In this book the software for teaching stereometry and the activities developed are presented. The project was conducted within the framework of the programme SOCRATES 2005–2007 of the European Community. The partner institutions were – The University of Cyprus (coordinating institution), The University of Southampton, The University of Lisbon, The University of Sofia, The University of Athens, as well as the N.K.M. Netmasters, and the Cyprus Mathematics Teachers Association. The project officially started on the 1st of October, 2005 and ended on the 30th of September, 2007.

The book is divided into three sections. In the first section, we provide some information about the background of the project, while in the second part, we present the functions of the software developed in the framework of this project. In the last section, we present some of the activities that can be used with the software. Some of the activities were found in different websites and were adapted to meet the needs of the project and others have been developed to guide teachers using the software. The software can be downloaded from the project's website (www.ucy.ac.cy/dalest), and can be used by teachers in their classrooms. We welcome any comments and suggestions for future updates of the software and the activities.

Theoretical Background of the Project

With the emergence of dynamic geometry software (DGS), the teaching of geometry in general and geometric theorems in particular, have aroused renewed interest (de Villiers, 1996; Hanna, 2000). Since the 1990s DGS have been increasingly used for teaching plane geometry, mainly in secondary education (Laborde, 1998). Most schools in different countries, and especially in Europe, use DGS to improve curricula in geometry (Jones, 2000). Nowadays there is a number of dynamic two and three dimensional software for teaching geometry; however none of them appears to be suitable for teaching stereometry in middle schools in a way that permits students to explore the interrelationships among figures. Most existing software provide opportunities for students to experiment with objects and shapes as total entities rather than relationships among the component parts of these objects and shapes. Experience shows that representing space geometry facts at the blackboard or in the form of material models is laborious and often unsuccessful even if certain techniques of representation or design are been practiced. The flat representation of a spatial figure does not have any spatial depth; it is static and hardly correction-capable; it can not be manipulated.

The main objective of the project has been to develop a dynamic three-dimensional geometry microworld that enables students to construct, observe and manipulate geometrical figures in space. Students learning 3D geometry need to acquire, and improve, a set of 'abilities' of visualisation to perform the necessary processes with specific mental images for a given 3D problem. Given that there is general agreement that visualisation is a basic component in learning and teaching 3D geometry (Gutiérrez, 1996), a rich concept of visualisation guided the design and the construction of the software. Visualisation, according to Gutiérrez and Jaime (1998) and Presmeg (1986), is an integration of four main elements:

mental images, external representations, visualization processes, and visualization abilities. These four elements of visualisation are used in the account of the software.

Mental Images

A mental image is any kind of cognitive representation of a mathematical concept by means of spatial elements. Thus the design of the software aims, for example, to make it straightforward for students to construct different solids and perceive them in a concrete or pictorial form. The repetition of this process is known to help students to formulate a "picture in their mind's eyes" (Presmeg, 1986) so the software was designed to enable students to see solids in many positions on the screen and consequently gain a rich experience that allows them to form richer mental images than from textbooks or other static resources.

External representations

In cognitive theory, a distinction is often made between internal and external representation in that, while an internal representation is a hypothesised mental construct, an external representation is a material notation of some kind, such as a graph, an equation, or a geometrical figure (Ainley, Barton, Jones, Pfannkuch & Thomas, 2002). In this context, the environment of the software is designed to be a rich environment for manipulating and transforming representations of solids. Given that most middle school students are known to find it difficult to understand that Figure 1, for example, could be the representation of either a pyramid or an octahedron, the software is designed so that students can rotate such a representation of a pyramid and see that Figure 1 is a special position of it.

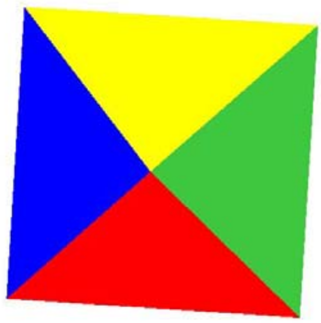
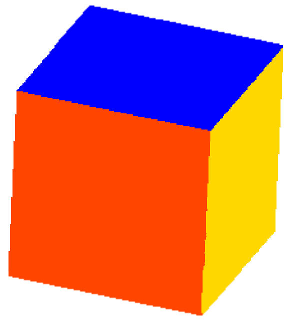


Figure 1. A Special position of a pyramid or octahedron

Visualisation Processes

According to Presmeg (1986), a visualisation process is a mental or physical action where mental images are involved. Bishop (1980) identified two relevant processes of visualisation: firstly, interpreting figural information and the visual processing of abstract information; and secondly, the translation of abstract relationships and non-figural data into visual terms, the manipulation and extrapolation of visual imagery, and the transformation of one visual image into another. The design of the software incorporates Bishop's ideas by focusing on the processes of observation, construction and exploration in the ways described below.

Observing: observation allows students to see and understand the third dimension by changing the spatial system of reference (axes), choosing perspectives and displaying visual feedback on objects. The software was designed so that students can rotate a geometric object with reference to the three axes and thus gain a holistic view of the object. Features designed into the software include easy variation of the speed and the direction of the rotation of any object, directly controlled by the user of the software. What is more, the software is designed such that the drawing style of any object can be in a 'solid colour' view or in a 'transparent line' view, as illustrated in Figure 2, and students can select, label and colour the edges and faces of the objects.



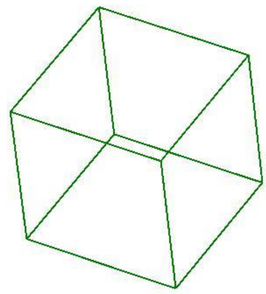


Figure 2. A solid and transparent view of a Cube

Constructing: construction allows a dynamic construction of geometrical figures from elementary objects (points, lines, planes) and construction primitives (intersection, parallel, etc.). The software was designed so that students can use such elementary objects to create 3D shapes, including being able to select the appropriate 2D figures and then forming the solids by dynamic animations, as illustrated in Figure 3.

Exploring: exploration allows students to explore and discover the geometrical properties of a figure. This is the main procedure adopted in most of the teaching scenarios that are available to accompany the software.

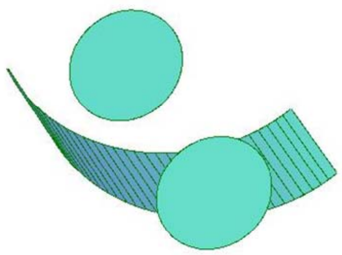


Figure 3. The construction of a cylinder

Visualisation Abilities

Informed by relevant research (see Gutiérrez, 1996, for a review), the software is designed in such a way as to accommodate the development of the following visualisation abilities: (a) the figure-ground perception, (b) perceptual constancy, (c) mental rotation, (d) perception of spatial positions, (e) perception of spatial relationships, and (f) visual discrimination. In order to contribute to the development of these abilities, the design of the software encompasses the following features:

"Dragging": the dragging capability of the software enables students to rotate, move and resize 3D objects in much the same way as the commonly available 2D dynamic geometry software environments. The design approach focused on enabling rotation to be executed in all directions through the provision of an on-screen rotation cursor that could also be used to determine the speed of the rotation. In addition, the design was made such that students are able to resize, proportionally, all the dimensions of the object or resize it only in one dimension, according to the requirements of the problem. For example, students can resize a cylinder by shrinking or enlarging the diameter *KL* of its base or by enlarging the height *LM*.

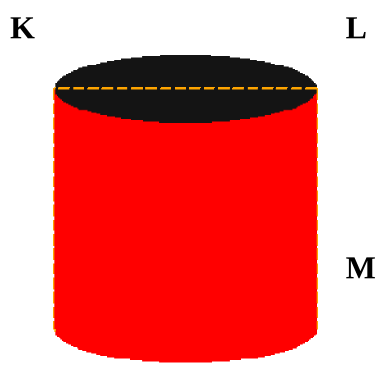


Figure 4. Resizing a cuboid and a cylinder

"Tracing": tracing is a particular instance of the design of the interface where only parts of the figure are displayed. The purpose of this feature is to provide learners with a way of performing a visual filtering of the main construction represented on the screen; that is, to allow them to extract and observe parts of the construction in an independent view.

"Measuring": the software was designed, as with commonly available 2D dynamic geometry software, so that students can measure the length of edges and the area of faces. In the software, students can also obtain the measure the volume of a solid. In providing these features, the software was designed such that all measurements are dynamic as the solids are being resized. The dynamic characteristic of the measurement facility allows students dynamically to explore the properties within and amongst figures; for example, students can obtain the measure of the volume of a cone and then double its height and see how its volume is altered.

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CHAPTER II

Introduction

In this chapter we describe the main functions and menus of the two interrelated softwares. First, we provide some information about the Dalest Applications and then we describe the main functions of the Dalest Stereometry. Both softwares can be downloaded from the project's website: http://www.ucy.ac.cy/dalest. On the website there also are a few demos showing Dalest Stereometry menus and some main 3D constructions.

Dalest Applications

Cubix

This application represents a 3D cubic structure made of unit-sized cubes (see Figure 1). By examining it from different perspectives the students can calculate (or guess) the number of cubes used to construct the shape and its external area.

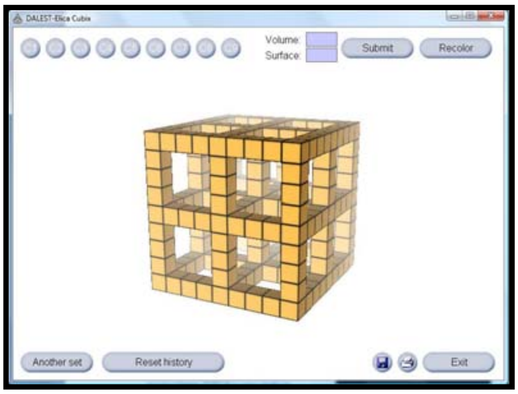


Figure 1: Cubix

Slider

A plane cuts an invisible 3D solid (a cube, sphere, cone, etc., see Figure2). Only the intersection becomes visible to the viewer. By moving and rotating the cutting plane students are expected to examine and analyze the changes on the intersection and discover the

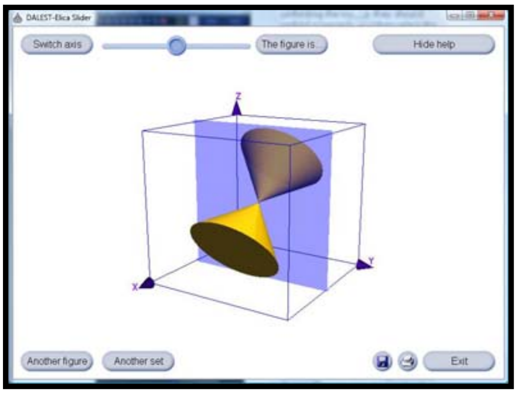


Figure 2: Slider

solid. Solids may have various orientations that can be revealed. The student can switch the axis of view so as to have different representations of the intersection of the solid and be able to combine them to a unified visual image.

Stuffed Toys

A set of toys all made of six faces are cut off in a random way (see Figure 3). When the toy unfolds, it is possible to have 11 different cube nets. The students must guess which net will occur by observing, rotating and examining the stuffed toy without unfolding it. Students are expected to mentally unfold the stuffed toy and select the appropriate net from the dialog windows.

There are two sets of toys – simple white cubes and coloured toys of various shapes – cubes, round balls, rugby balls, sharp and soft octahedrons, eggs, pebbles, etc.

Cubix Editor

This is an editor which can be used to create 3D constructions (see Figure 4). These constructions are built with unit-sized cubes which constructions can be used by the Cubix and the Cubix Shadow applications. The Cubix Editor allows the student to save their constructions and reload them, thus making a library of various figures. The application comes with a small set of predefined figures.

The student can also color the cubes and get the measurements for the volume and surface area of the solids constructed.

A very useful function of this application is the possibility to rotate the whole platform of the construction. This procedure results to the creation of dynamic images and allows the student to view the construction from the front, side and top. It is assumed that this function enhances the student's ability to visualize an object from

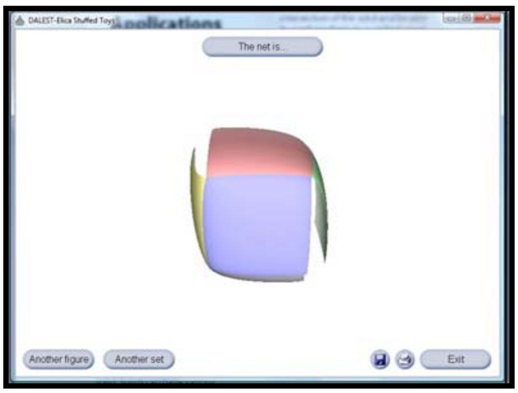


Figure 3: Stuffed Toys

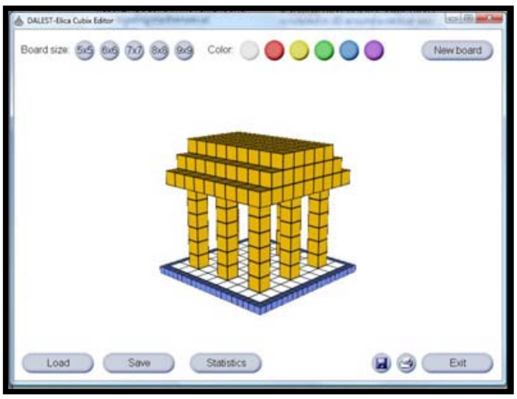


Figure 4: Cubix Editor

different perspectives. This ability can also be enhanced by constructing an object based only on the image of its top, side or front view. The dragging capability of the application is a tool that can be used to develop didactical situations enhancing visual abilities, such as “Perceptual constancy” and “Mental rotation” by producing dynamic mental images, visualizing a solid in movement, and investigating its mathematical properties.

Cubix Shadow

A cubic composition made of unit-sized cubes casts shadows on the OXY, OYZ and OZX planes (see Figure 5). The students are asked to recover the composition by examining its three silhouettes (the projections on the three coordinate planes). Some activities in this application provide extra challenge – they are a 3D variant of the “8-Queens” chess problem. Students can generate Cubix Shadow problems for other students to solve by using the Cubix Editor.

Potter’s Wheel

When a simple 2D object (e.g. a segment, a circle, a square, a triangle, or a free shape curve) is rotated in 3D-space around, a vertical axis, it generates a number of 3D rotational objects (see Figure 6). This application provides a tool where students can experiment with various cases. By using only the five simple 2D objects, and by varying their position and orientation relative to the axis, it is possible to generate different 3D shapes. Students can also split the constructed solid and observe its interior design. The software presents to the students various solids to be constructed.

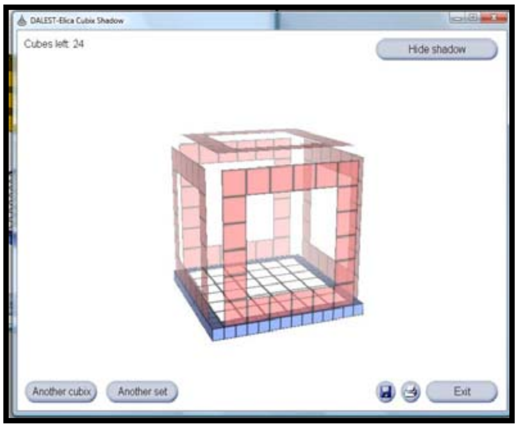


Figure 5: Cubix Shadow

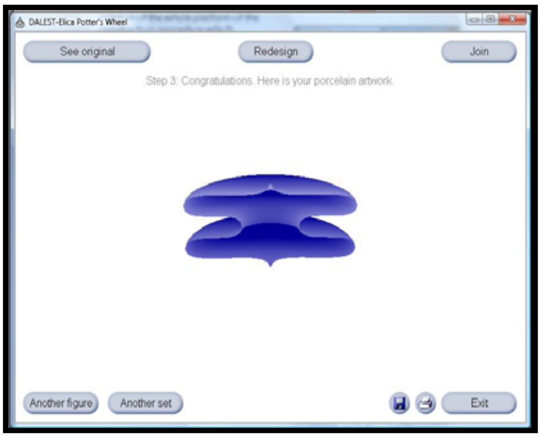


Figure 6: Potter’s Wheel

Math Wheel

This application is similar to Potter's Wheel in the sense that students create rotational solids from 2D shapes – triangles, quadrangles and circles (see Figure 7). The vertices can be freely moved, thus students can make various types of these shapes. For each shape the system can calculate the surface and the volume of the constructed solid. This is provided as a series of calculations starting from the symbolic formula and ending with a concrete number.

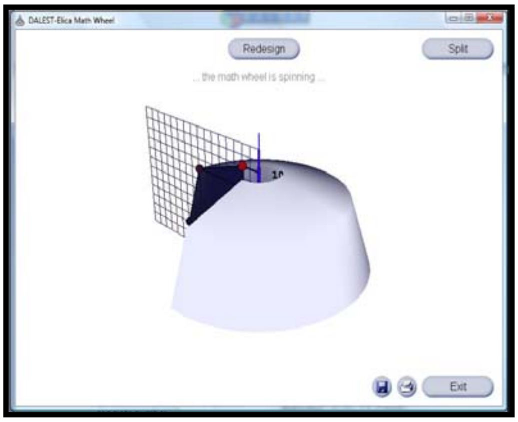


Figure 7: Math Wheel

Scissors

Students have to solve the reverse problem to net folding (see Figure 8). More specifically, they have a cube and must decide which edges to cut, so that the remaining figure can be unfolded into a predefined net. The level of difficulty in this application varies according to the object that the student has to cut. In the first level the object is a cube, in the second some more advanced figures are presented and, finally, at the third level some more complex ribbons. During this activity the student gets a feedback. If the student cuts more than the required number of seams, the ripped cube cannot be unfolded completely, because some faces will be entirely detached from the main body of the shape. On the other hand, if the student cuts fewer seams than needed, the resulting figure will not unfold into a *planar* net. And finally, even if the number of cut off seams is the correct one, the faces unfold into another net, not the targeted one.

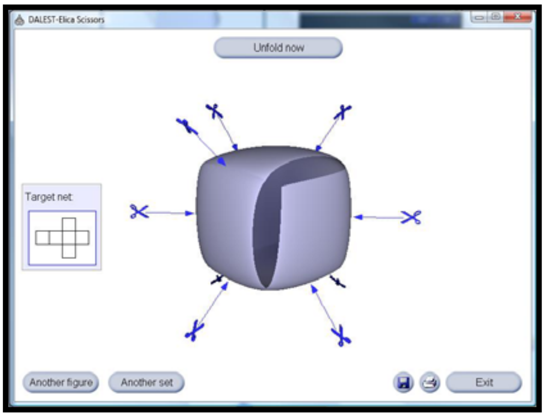


Figure 8: Scissors

Origami Nets

In this application the student has the opportunity to build nets with triangles, squares, rectangles, and regular polygon (see Figures 9 &

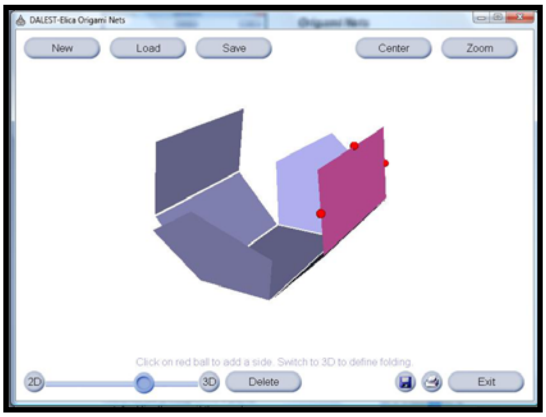


Figure 9: Origami Nets (1)

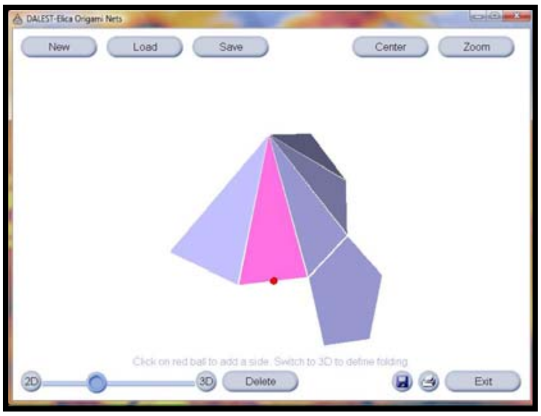


Figure 10: Origami Nets (2)

10). Once this is done then the student has to define the folding angles. In this way, it is possible for students to experiment with different nets of various 3D solids. The application allows the smooth transition from 2D flat net to 3D folded net. Adding new elements to the net is done interactively. It is also possible to create chain of rectangles or triangles which fold into a cylinder or a cone.

The application is extremely powerful because it builds up an open, dynamic and interactive environment where the student can construct a net without any restrictions. The construction of the net is not a static procedure. The building net can be rotated helping the student to visualize how the solid will change if a side is added on an edge of the net. Another dynamic feature of the application is that the student can move from the 2D to the 3D view of the net and vice versa. As a result the student can fold the net before it is completely constructed and get an immediate feedback.

Dalest Stereometry

General Characteristics of Stereometry Learning Environment

The distinguished feature implemented in the software is the ability to construct geometrical objects in successive steps that correspond to their mathematical properties. For example, a cuboid is constructed in three successive movements, defining in each movement the length of one of the bone edges of the solid. The manual construction of solids gives to students an intuitive conception of the 3D nature of the solid.

The geometrical objects created on the screen can be manipulated, moved and reshaped interactively by means of the mouse. The software allows students to see a geometric solid represented in several possible ways on the screen and to transform it, helping students to acquire and develop abilities of visualization in the context of 3D geometry. The design of the software facilitates the informed manipulation of directions of rotation. In the following sections, we will describe the main features of the software and provide some tutorials.

Drawing Mode

With Dalest, the student has the choice to select to work in **dissection** or **normal** mode. The student can define the drawing mode by selecting the "Drawing Mode" in the Settings toolbar (see Figure 11).

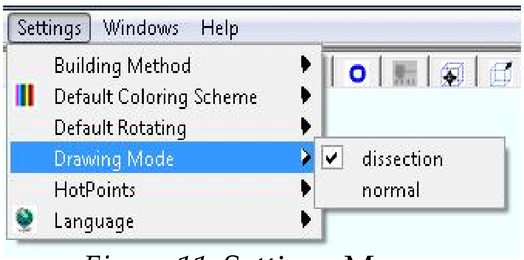


Figure 11: Settings Menu

If the student chooses to work in **normal** mode, s/he can only see the objects from an external perspective.

If the student selects the dissection mode (see Figure 12), then s/he can view and draw segments, planes

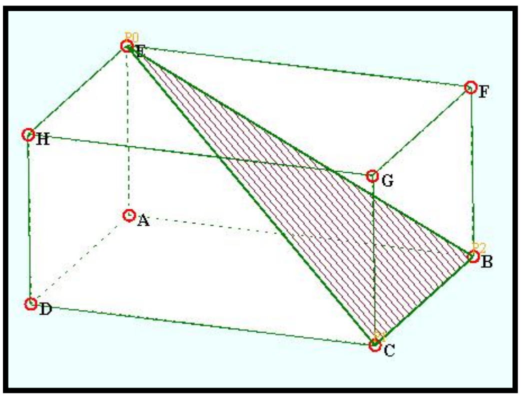


Figure 12: A cuboid in dissection mode

and other objects inside the object.

Drawing Style

To access the drawing style option (see Figure 13), the student has to click on view, then select "drawing style" and finally choose from the four options (Transparent lines, Transparent dots , Filled colors + lines, Filled colors).

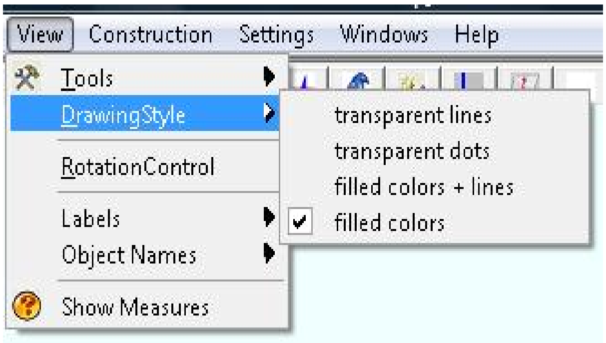


Figure 13: Drawing Style

The four options can be activated when the student is in the normal drawing mode. The student can view the boundaries of the object in four different ways: Transparent lines (see Figure 14), Transparent dots (see Figure 15), Filled colours plus lines (see Figure 16) and Filled colours (see Figure 17).

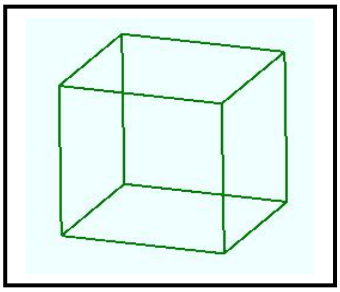


Figure 14: Transparent lines

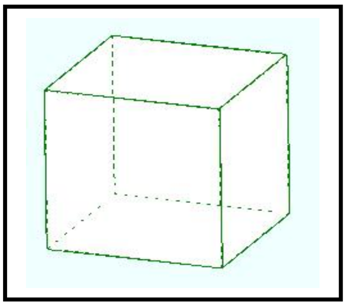


Figure 15: Transparent dots

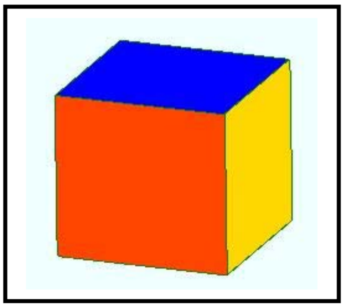


Figure 16: Filled colours plus lines

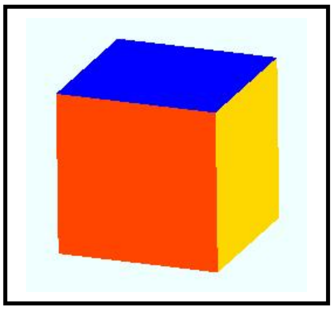
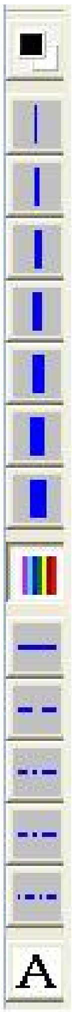


Figure 17: Filled Colours

Pallete

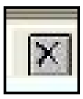


The toolbar for line width, line style, fonts and colours of the fonts and lines is located on the right side of the screen.

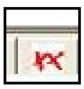
The student has to select the width, the line style and the colour of a line **before** constructing a segment. The student cannot change the above properties once a segment has been constructed or a text box has been created.

ToolBar

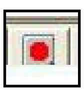
Tools for deleting one or more objects and rotation control.



This tool can be used to delete a selected object.



This tool can be used to clear all the objects from the screen.



This tool can be used to stop/continue the rotation of objects.

ToolBar Drawing Menu

The objects in DALEST can also be drawn by selecting the Object from the toolbar. The toolbar is located on the bottom left of the screen.

To draw an object, the student has to click on the object s/he wishes to draw and then move on the screen and define a virtual rectangle where the object will appear. To define a rectangle, the student should click and drag from a starting point to an end point (see Figure 18). The selected object is also displayed as the mouse cursor.

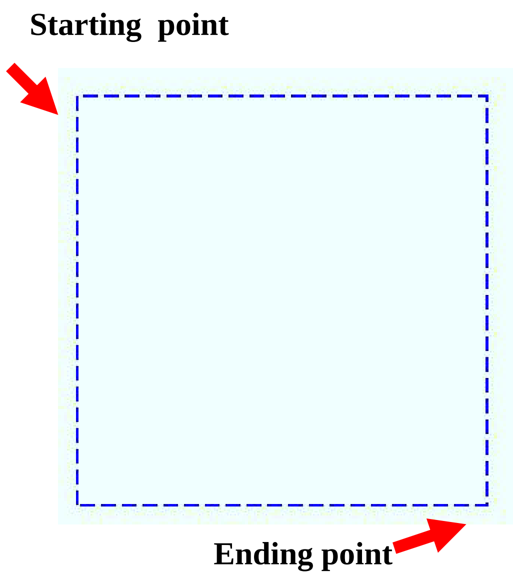


Figure 18: Drawing an object

The Objects that can be drawn from the icons on the toolbar menu are:

1. Cube

2. Cuboid

3. Pyramid

4. Frustum of pyramid

5. Prism

6. Parallelogram

7. Cone

8. Frustum of cone

9. Cylinder

10. Sphere

Quick Start Menu

Upon Starting the program, the user is presented with the Quick Start Menu (see Figure 19) which allows the user to select one of the following options:

1. Object Wizard --> Construct New Object Wizard

2. Previous Construction --> Continue Previous Construction

3. Open File --> Open Saved Construction

4. Draw Manually --> Construct 3D Objects Manually

5. Animation --> Show Animation

6. Object Grouping --> Grouping Wizard

The above options will activate the appropriate dialog. If none of the dialogs is selected, the user can still use all the wizards by selecting the appropriate wizard either through the toolbar or the menu.

Figure 19: Quick Start Menu

The Draw Manually Wizard

The object wizard dialog can be activated in two ways:

(a) By clicking on the  icon located on the toolbar.

(b) By selecting "Object Wizard" in the Construction toolbar.

The student may choose to draw objects manually and take advantage of DALEST's capability in drawing 3D objects with this mechanism. DALEST's Draw Manually Wizard allows the user to define the dimensions for any required object. The Objects that can be drawn manually are shown in Figure 20.

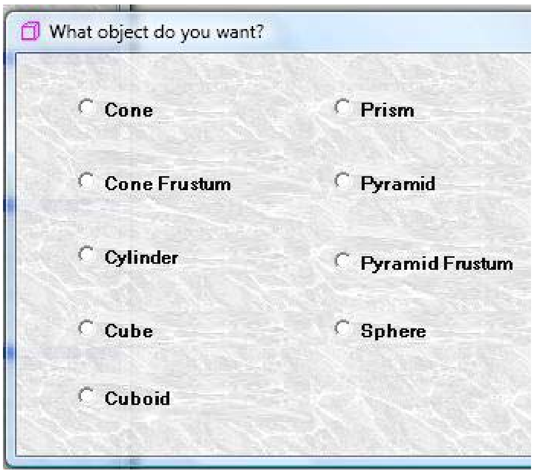


Figure 20: Manually drawn objects

1. Cone
2. Cone Frustrum
3. Cylinder
4. Cube
5. Cuboid
6. Prism
7. Pyramid
8. Pyramid Frustrum
9. Sphere

Example: How to draw a Cylinder manually.

1. Select the "Draw Manually" Wizard. This can be done either from the Quick Start Menu or by clicking on the Draw Manually Wizard Button.

2. Select the Object Cylinder.

3. Upon selecting the Object Cylinder, the 3D-axis grid will be displayed on the screen (see Figure 21).

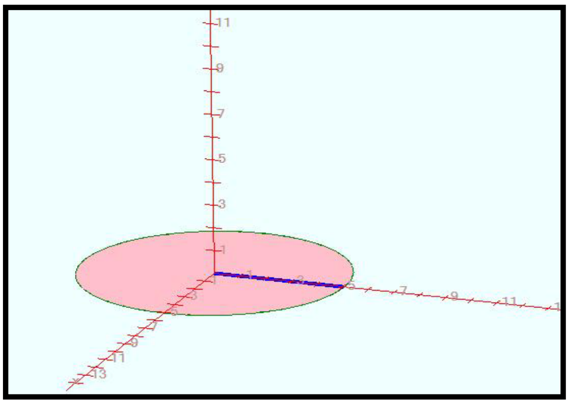


Figure 21: 3D axis grid

4. A Cylinder has two dimensions, radius of circle (bottom and upper base of the cylinder) and the height of the Cylinder.

5. Drag the mouse over the respective axis and click the left button mouse button indicating the length of the radius.

6. Drag the mouse over the z-axis and click the left button mouse button indicating the length of the height (see Figure 22).

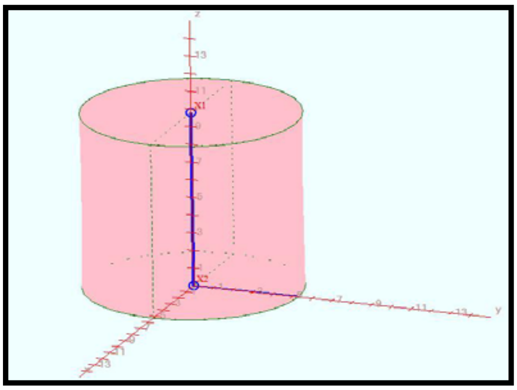


Figure 22: Defining the height

7. Once the dimensions of the object are determined the object will appear (see Figure 23).

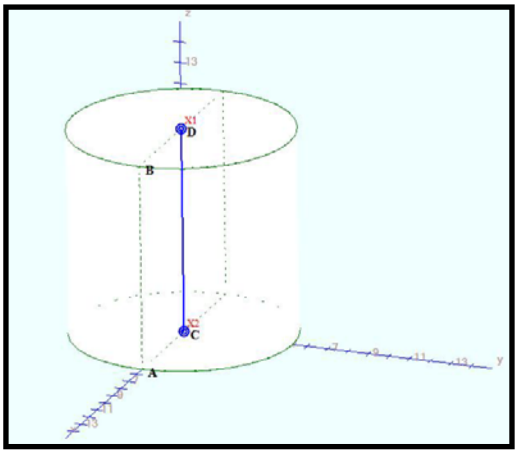
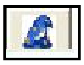


Figure 23: Constructed Cylinder

Object Wizard

The object wizard dialogue can be activated in two ways:

(a) By clicking on the  icon located on the toolbar.

(b) By selecting "Object Wizard" in the Construction menu (see Figure 24).

Figure 24: Construction Menu

The Object Wizard is a useful wizard which can be used in two ways:

1. To draw objects with specific dimensions that students already know. It allows students to create a New Object on the grid by choosing the type of objects they wish to draw from a predetermined set of options. The Object Wizard provides the opportunity to the student to select the type of Object to be drawn (see Figure 25) and then allows them to set the dimensions of the Object. After selecting the Object the student has to press the NEXT button. A new dialogue box then appears which allows students to adjust the parameters of the Object accordingly. The following Objects can be constructed by using the Object Wizard:

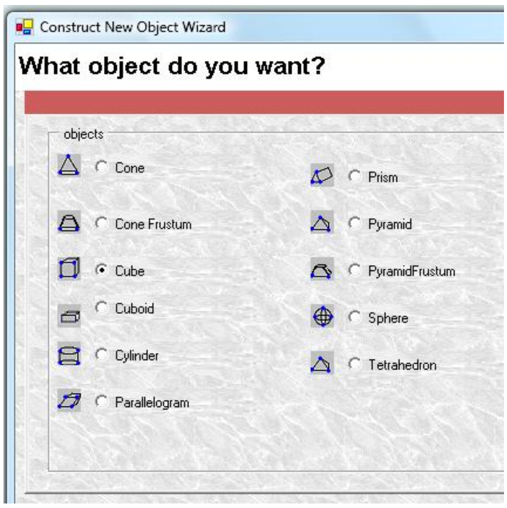


Figure 25: Type of object

1. Cone
2. Cone Frustrum

3. Cube
4. Cuboid
5. Cylinder
6. Parallelogram
7. Prism
8. Pyramid
9. Pyramid Frustrum
10. Sphere
11. Tetrahedron

When the user selects the type of Object that s/he wishes to draw a dialogue box appears where the dimensions of the object can be specified. The dimensions that can be adjusted are: Name of Object Name of Dimensions (Height/ Width/ Length).

Example: Defining the dimensions of a cuboid.

In the dialogue box presented in Figure 26, the user can rename the Object and its parameters. The new names appear in the Preview Box and in the Measurements Window. In the example, the three dimensions of a cuboid "length", "width" and "height" have been renamed and the dimensions have been set. In the Preview Box, students can see the area of the object's faces and its volume and the way in which these measures are calculated in terms of the three parameters. To construct the Object, the user must click on the Construct button.

2. The wizard can be used to resize an object to specific dimensions. i.e., the user can provide DALEST with precise directions regarding the specific parameters of the Object. To redefine the dimensions of an object

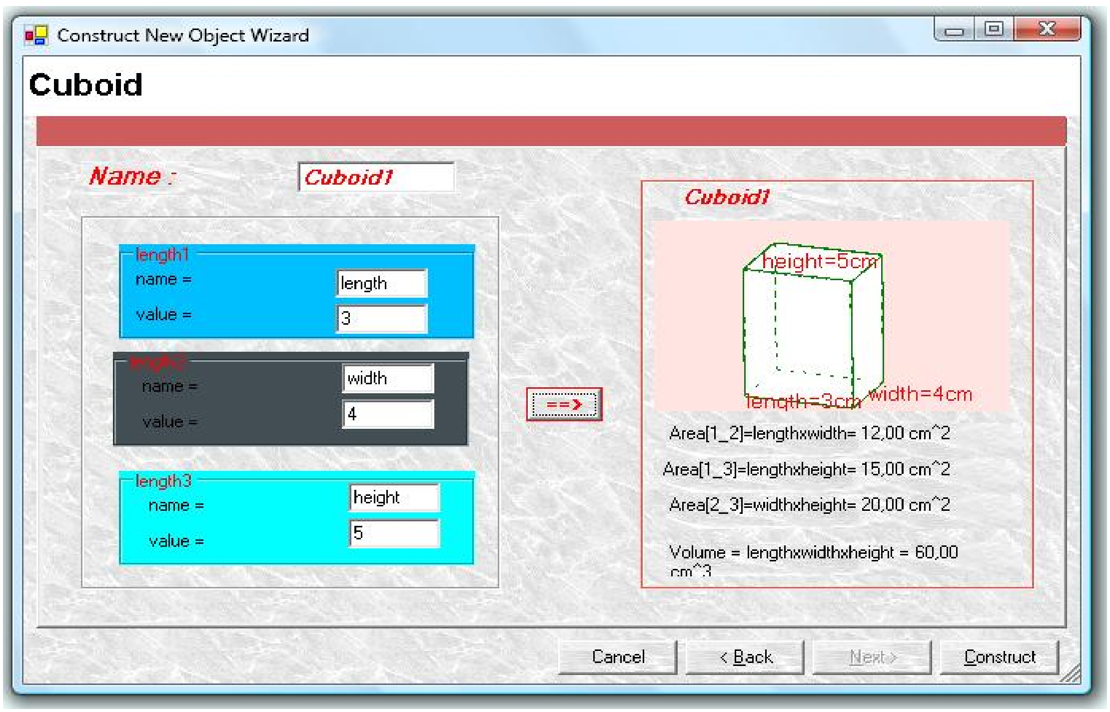


Figure 26: Setting the dimensions

the students has to right-click on it and select "Object Wizard". When the "Object Wizard" is selected, the setting parameters dialogue box will appear. In this wizard, the user can redefine and rename the parameters of the object.

Measures

Measures window is used to display the measurements of the 3D objects appearing on the screen. The measurement dialogue can be activated in three ways.

(a) The first is by clicking  on the icon with the question mark, located on the toolbar.

(b) The second way is by selecting "Show Measures" under the View toolbar.

(c) The third is by right clicking on the 3D object and selecting the measures option.

The "Measures Window" will show all the available measures for all objects. When the student clicks on an object, its measures in the "Measures Window" appear with blue background (see Figure 27).

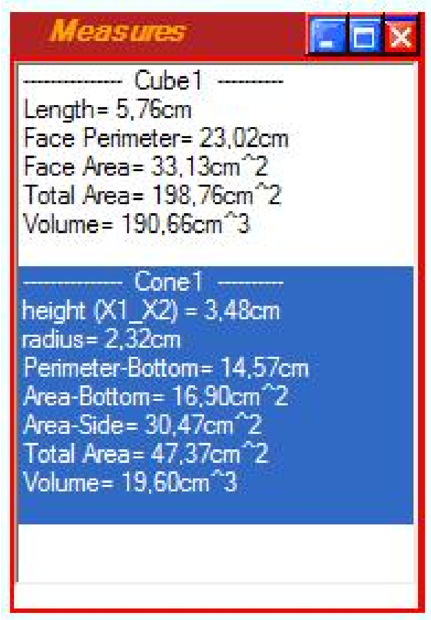
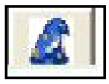
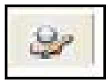


Figure 27: Measures Box

Drawing 3D objects

The student can draw 3D objects in three different ways:

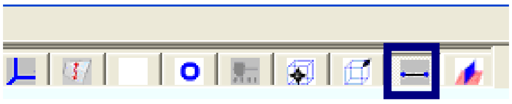
1. By using the Object Wizard. 

2. By using the Draw Manually Wizard. 

3. By using the toolbar icons.

Drawing Segments

DALEST allows students to draw segments. To draw a segment, the student must click on the line icon that is located on the menu bar.



Once the segment button is highlighted, the student is ready to draw a segment. To draw the segment, the student has to click on an edge of the 3D object to select a point and then drag to a second point to define the segment. When moving the mouse cursor on top of an edge of the object, a hint "this point?" is displayed.

The starting and end points of a segment can be dragged on the screen or along the edge they belong to. For example, in the following cuboid, the starting point of the segment can be dragged along DC and its ending point can be dragged along HG.

Every segment that is constructed is accompanied by (see Figure 28):

1. An arbitrary label assigned to each point i.e., P0 and P1.

2. DALEST always displays the length of the object when the mouse cursor is placed on top of the segment line.

3. The precise dimension of the segment is also indicated in the measurements box.

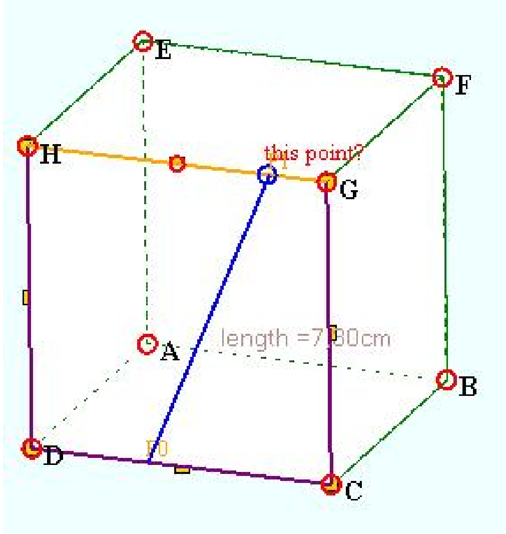


Figure 28: Construction of segment

Renaming Points

To rename a point on an object, the student must double click on the point. A dialogue window will appear asking for a new name (see Figure 29).

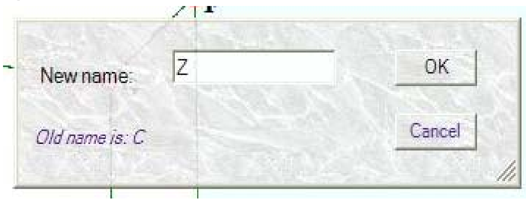


Figure 29: Renaming a point

Drawing Triangular Planes

The student can construct a three point defined plane by  clicking on the main toolbar or by selecting the "Triangular Plane" in the Construction toolbar (see Figure 30).

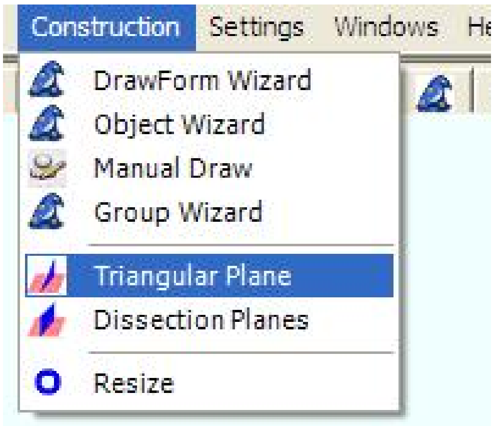


Figure 30: Construction of segment

The "Triangular Plane" is drawn just like a segment with the exception that the student must select three points on a given 3D object. The process of selecting the points is the same with the Drawing Segments procedure. The student can construct a triangular plane by performing the following three steps.

Step 1: Click on an edge to define the first point. Note that by moving the cursor over an edge, a tooltip "this point?" is presented to help the student select the point (see Figure 31).

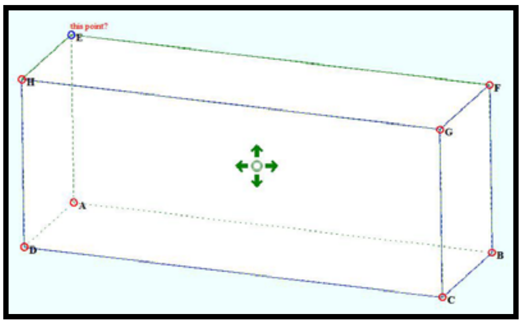


Figure 31: First step

Step 2: The student repeats the same procedure by clicking at "this point?" on an edge and by dragging to a second point. Once the second step is completed a segment will appear that represents the base of the triangular plane.

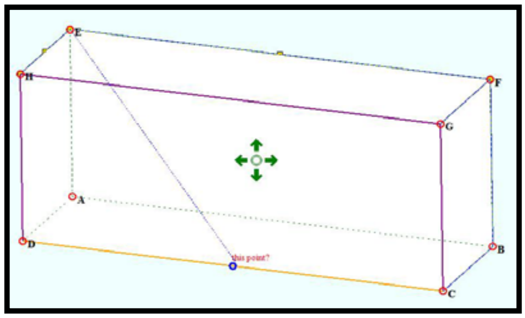


Figure 32: Second step

Step 3: The user can then click on a third point to define the triangular plane.

The student can click and drag on the three points to change the dimensions of the triangular plane. Dalest presents the area of the defined triangular plane in the "Measures Windows" or by approaching the mouse cursor in one of the three points, as presented in Figure 33).

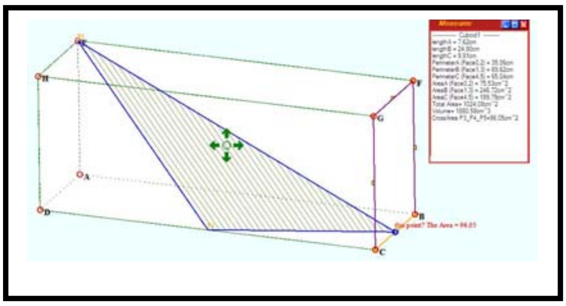
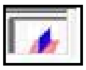


Figure 33: Area of defined triangle

Dissection Planes

The student can construct a dissection plane by selecting the "Dissection Planes" in the Construction toolbar or by clicking  on the main toolbar (see Figure 34).

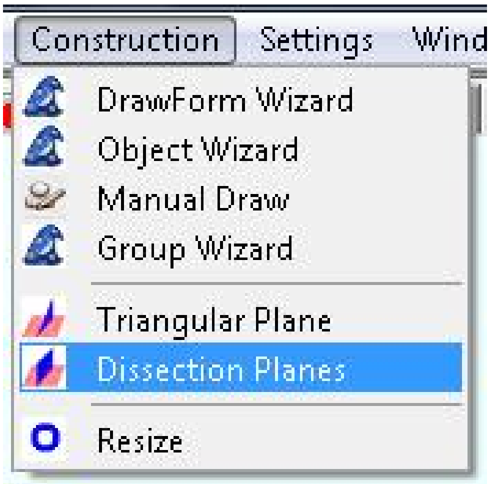


Figure 34: Construction menu

The student can construct a plane parallel to the x, y or z axis by clicking on the appropriate field in the dissection control panel (see Figure 35).

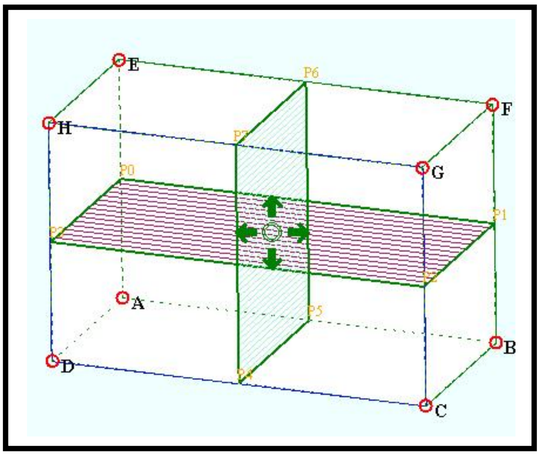


Figure 35: Dissection planes

Rotating 3D Objects

In Dalest an object can be rotated in two ways:

1. With every construction of a 3D object, a green rotation control appears. Four green arrows are embedded in the rotation control. By clicking on an arrow the user selects to rotate the object around a vertical or a horizontal direction. However, the user can click on two arrows (for example the top and the right arrows) to define another direction.

2. Students have the option to rotate a solid around a specific dimension, by right clicking on an edge of the solid. To stop the rotation, click on the object so the rotation control will appear and then click on the circle of the cross (see Figure 36).

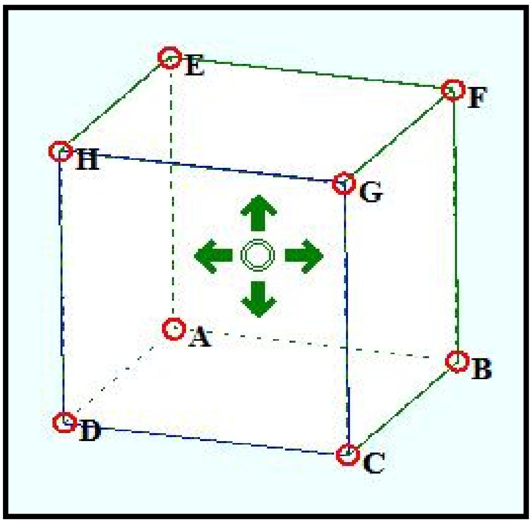
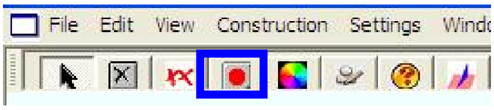


Figure 36: Rotation control

The rotation can also be stopped by clicking on the stop icon on the toolbar (the red circle).



A parameter in using the rotation control is that the number of clicks on an arrow defines the speed of the rotation. If the cursor is at a point on the object which does not allow the user to view the rotation easily, the cursor can be moved away from the object. To do this, stop the rotation and then drag the rotation control to the position where you would like it to appear.

Object Resizing

Students can resize 3D objects in two ways:

1. By clicking on the "Resize proportionally all the dimensions of the object" button in the main toolbar (see Figure 37). After activating this button, students must click on a point inside the object and drag the mouse to resize the object.

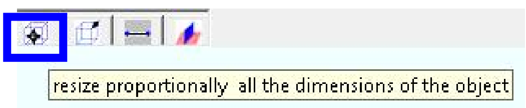


Figure 37: Resize proportionally

2. By selecting the resize option, from the edit menu (see Figure 38).

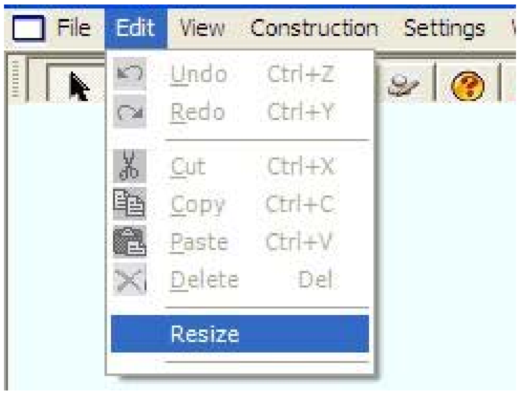


Figure 38: Edit Menu

After selecting the "Resize" option, a slider is presented (see Figure 39). Dragging the slider to the left will resize the object accordingly thus decreasing the size. When the resize slider is on the far left, the 3D object is at its smaller possible representation. Like-wise, dragging the slider to the right will increase the size of the object.

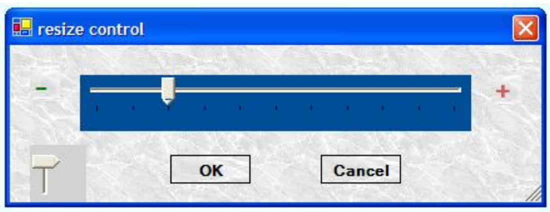


Figure 39: Resize slider

Stretching the object. Stretching is different from resizing since resizing implies increasing and decreasing the size of an object proportionally (with respect to all object's dimensions). Stretching allows the student to increase or decrease the size by changing one dimension. For example stretching occurs by increasing the height of the cylinder while keeping the radius constant. Students must click on the blue circle icon on the toolbar. It represents the hot-points.

As soon as the hot-point icon is activated, the 3D object will be displayed with small blue circles (see Figure 40). These small circles are the hot-points. For example, in the figure below, the user can adjust the height and the radius of the base of the cylinder by dragging its two hot points.

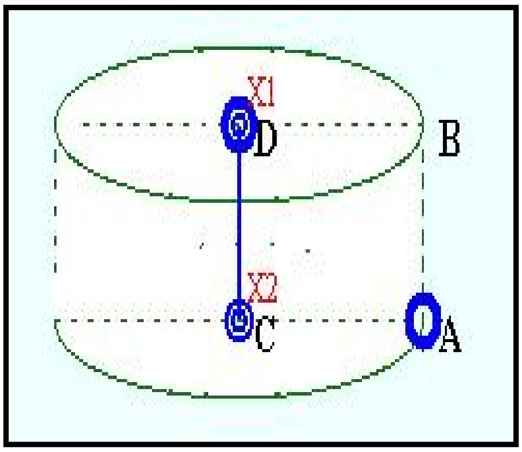


Figure 40: Hot points in a cylinder

Object Grouping

Dalest software provides the student with the capability to group two same or different 3D objects together. The student can activate the grouping wizard by clicking on the main toolbar or by selecting the "Grouping Wizard" in the Construction toolbar (see Figure 41).

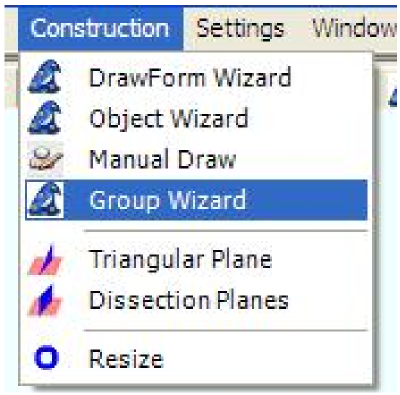


Figure 41: Group Wizard

The procedure for grouping two objects is the following:

Step 1: Click on the Grouping Wizard Icon.

Step 2: Following the pop up dialogue window, click on a face of the first cube that will be joined with one face of the second cube (see the figure below).

Step 3: Click on the corresponding face of the second cube.

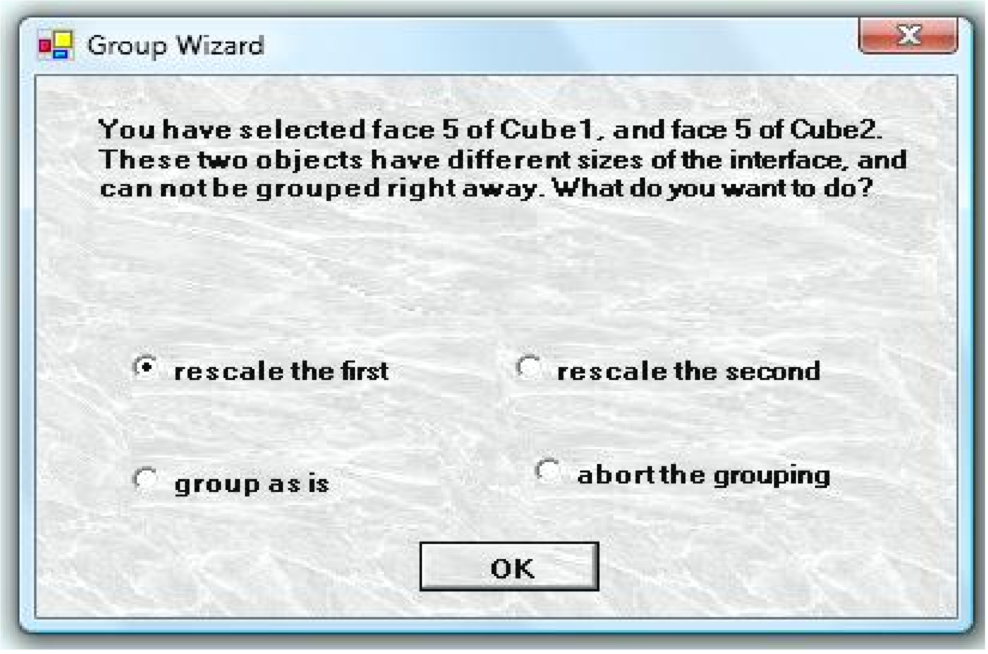


Figure 42: Group wizard dialogue

Note: A new pop up dialogue window appears. In this window, the user has the following options (Remember that the two cubes and therefore the two faces are of different size), as presented in Figure 42:

a. Rescale the first
b. Rescale the second
c. Group as is.
d. Abort the grouping

In Figure 43, the results of selecting the third option are presented. When Dalest encounters objects that are of different type, only "Grouping as is" functionality appears. With this option, Dalest groups the objects based on a common point ignoring the fact that the two objects are different and have different size.

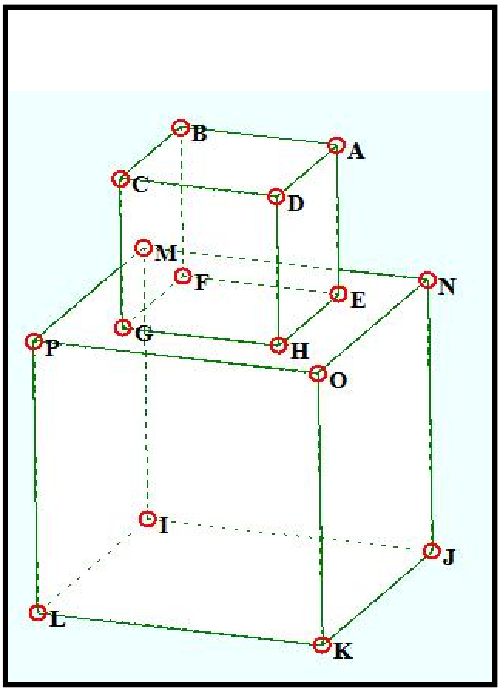


Figure 43: Grouping as is

CHAPTER III

Introduction

The development of the Dalest Stereometry and Applications aimed to promote constructivist teaching approaches in geometry, to change the role of the teacher in the teacher in the classroom, to make the students more responsible for their learning, to enable the students to understand the underlying concepts and to make mathematics more meaningful and realistic and thus give purpose and enjoyment in mathematics for the students. This chapter presents most of the activities that were designed to connect stereometry to the real world of students.

The activities were developed and trialled in all countries involved in the project and all participants met together to discuss the value of the activities and different approaches to the same task, thus ensuring that the final activities met the needs of all schools in the European Union. We hope the out-come of all approaches will be that the students have a more positive attitude towards stereo-metry.

The age range for which the activities are aimed is from 9 to 15 years. The activities follow the same format. First, in each activity the aim is articulated in a descriptive way. Second, in the introduction section of each activity, a real situation is presented as well as relevant questions. In this phase students are asked to explore the problem situation using the Dalest Stereometry or Applications software. Third, there is an extension part, in which students seek to generalize their solutions to similar situations. Finally, the last section of the activity requires students to communicate their solutions or generalizations by writing a letter to a friend or to an interested person.

In Table 1 each activity is linked to school syllabi content and stereometry goals. Table 1 is useful for teachers who may try these activities in their classrooms. The fourth column of the Table indicates the source of the original activity.

Table 1: The Objectives of the Activities, Connections to the Geometry Content and Software used

	Activity	Objectives	Geometry Content	Source	Software
1	Exploring Pyramids	Exploration of the relation between the number of vertices, edges and faces in pyramids	Euler's rule Properties of solid shapes	New	Dalest Stereometry
2	All Tied Up	Distance Explorations	Properties of solid shapes	http://nrich.maths.org/public/index.php	Dalest Stereometry
3	Candles	Volume calculation	Volume, spatial structuring	New	Dalest Stereometry
4	Joining Vertices	Triangles in a cube	Properties of solid shapes	New	Dalest Stereometry
5	Making Parcels	Surface area calculation of a cuboid Relation between volume and area	Surface Area and Volume	http://nrich.maths.org/public/index.php	Dalest Stereometry
6	Plutarch's Boxes	Cuboid surface area and volume	Surface Area and Volume	http://nrich.maths.org/public/index.php	Dalest Stereometry
7	Pop Corn	Relations between the dimensions of a cuboid and its volume	Volume of cube and cuboids	http://nrich.maths.org/public/index.php	Dalest Stereometry

8	Sculptures	Grouping of solids	Properties of solid shapes	New	Dalest Stereometry
9	Cuboid Nets	Recognition and construction of cuboid nets	Nets	New	Dalest Origami Nets
10	Prism Nets	Recognition and construction of prism nets	Nets	New	Dalest Origami Nets
11	Pyramid Nets	Recognition and construction of pyramid nets	Nets	New	Dalest Origami Nets
12	Designing Tetrahedral	Construction of tetrahedral by designing and folding their nets	Properties of solids, Nets	New	Dalest Origami Nets
13	Imagination Cubes and Nets	Visualization, recognition and construction of cube nets Development of visualization skills	Nets, visualization of nets	New	Dalest Stuffed Toys Scissors
14	Cube arrangements	Patterns and properties in cubes Visualization skills	Spatial structuring	New	Dalest Cubix Editor
15	Cube	Completion of constructions to	Properties of solid	New	Dalest

	Constructions	turn them into cubes or cuboids	shapes, spatial structuring		Cubix Editor
16	Modular Houses	Relation between volume and surface area	Surface area and Volume (without using formulas) Volume applications	New	Dalest Cubix Editor
17	Sums	Exploration and representation of cubic numbers	Volume concept	New	Dalest Cubix Editor
18	Cuboid Volume	Finding the formula for calculating the volume of a cube	Volume calculation	New	Dalest Cubix Editor
19	Relations in a cone	Relation between the dimensions of a cone and its volume	Cylinders and Cone	New	Dalest Math Wheel
20	Relations in a cylinder	Relation between the dimensions of a cylinder and its surface and volume	Cylinders and Cone	New	Dalest Math Wheel

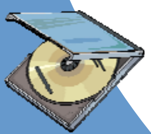
DALEST

Exploring Pyramids

In this activity you will explore the relation among the number of vertices, edges and faces in 3D shapes.
For your explorations you will use the Dalest Stereometry software.

Introduction

1 Construct the following pyramids and complete the table:



P1: a pyramid with a triangular base
P2: a pyramid with a square base
P3: a pyramid with a pentagonal base

	Base	Number of vertices	Number of faces	Number of edges	Vertices + Faces
P1	Triangle				
P2	Square				
P3	Pentagon				

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DALEST

Exploration

(2) What is the relation between the number of vertices and the number of faces in the above pyramids?

(3) What is the relation among the number of vertices, faces and edges in the above pyramids?

Extension

4 Is your conjecture valid for a pyramid with an heptagonal base?

5 How many vertices, edges and faces are there in a pyramid with octagonal base?

Extension

6 Examine whether the relation you have discovered for pyramids is also valid for prisms.

DALEST

All Tied Up

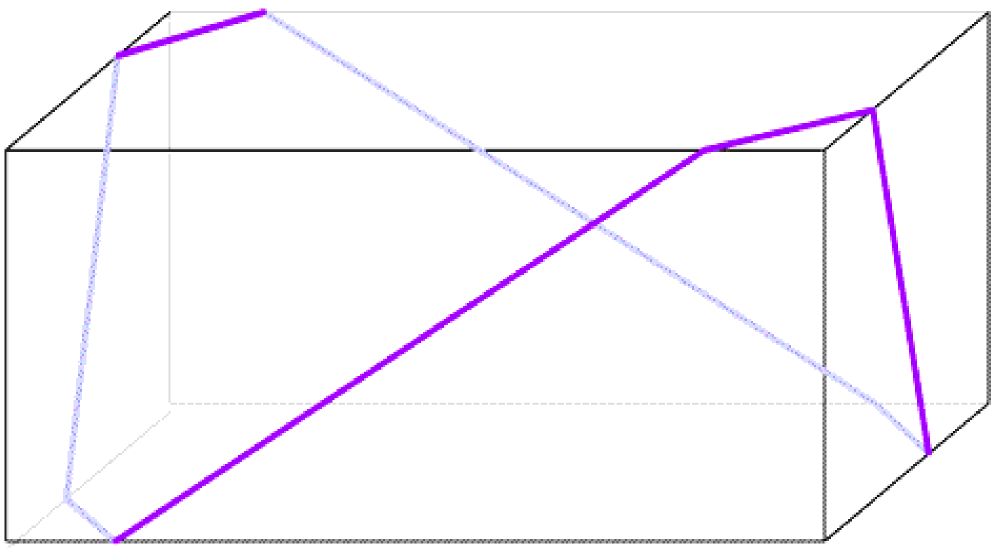
In this activity you will explore the way in which you can run one piece of ribbon around a box.
For your explorations you will use the Dalest Stereometry software.

Introduction

1 Alex enjoys wrapping presents - pieces of ribbon, bows and pretty paper - trying to make the present as attractive as possible.

Alex likes to run a ribbon around the box so that it makes a complete loop with two parallel pieces of ribbon on the top (and on the bottom) of the box.

The ribbon crosses every face once, except the top and bottom, which it crosses twice. The ribbon rests tightly against the box all the way round.



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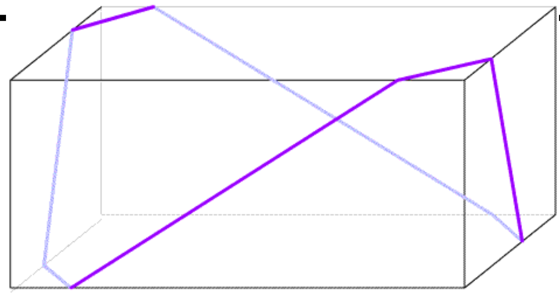
Exploration

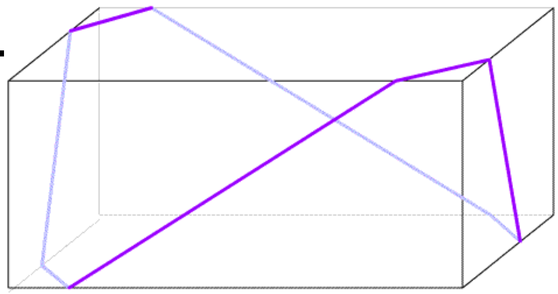
Alex can cut the ribbon before placing it around the box and can slide the ribbon around a little to position it.
If the box is 20 cm by 10cm by 5cm how long should the ribbon be?

Try your ideas in the Dalest Stereometry software (You can move points to make your ribbon look nicer!).



Extension

2 What will the length of ribbon be for a box with **height h, width w and length l**? (length **l** and width **w** are the longer distances and constitute the dimensions of the top of the box)



Write a letter to a friend, explaining how you solved the problem.

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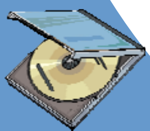
Candles

In this activity you will explore how much is the empty space if a cylindrical candle is placed in a rectangular box.
For your explorations you will use the Dalest Stereometry software.

Introduction

1 Large wax candles are made in the shape of a cylinder. The length of the candle is 20 cm and the diameter of the candle is 8 cm. They are packed neatly into individual rectangular boxes in which they just fit.

Construct in the Dalest Stereometry software the rectangular box and find its dimensions.



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Exploration

2 What percentage of the space in each box is occupied by air?

3 If the dimensions of the candle and its box are doubled, what effect will these changes have on the percentage of air space?

Extension

(4) If we want to reduce as much as possible the percentage of the space in each box which is occupied by air, what should the dimensions of the candle and the box be?

Write a letter to a friend, explaining how you solved the problem.

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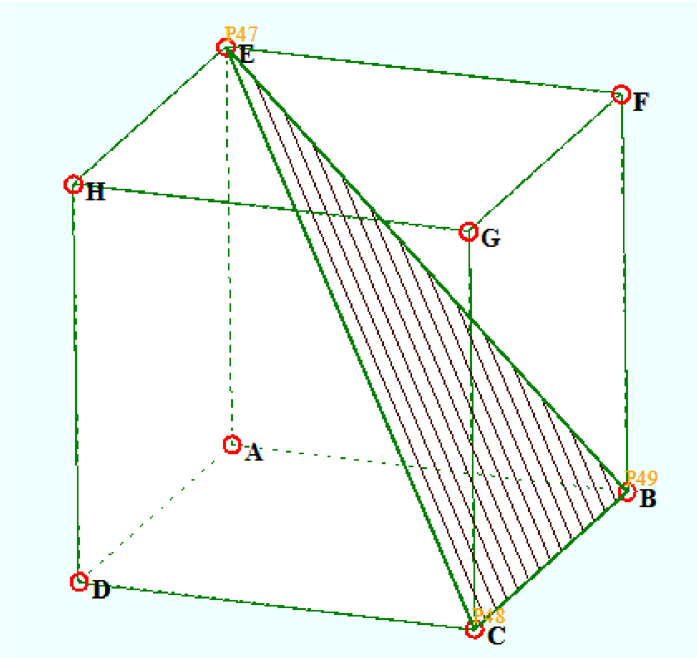
DALEST

Joining Vertices

In this activity you will explore the number of triangles you can make by joining vertices or other edge points in a cube.
For your explorations you will use the Dalest Stereometry software.

Introduction

1 Triangles can be constructed by joining three vertices of a cube, as shown in the example below. Construct as many different triangles as possible, using the *Triangular Plane* tool.



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Exploration

2 How many different triangles can you make if two of the triangle's vertices belong to the same edge?

Copy your constructions here.

Extension

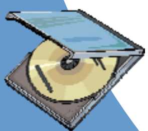
3 Triangles can also be constructed by joining three points that belong to different edges.
Can you find which one of these triangles has the maximum surface area? Explain your answer.

DALEST

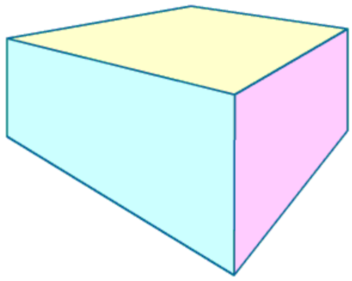
Making Parcels

In this activity you will explore cuboids that have the same surface area. For your explorations you will use the Dalest Stereometry software.

Introduction



1 A parcel making factory wants to construct a cuboid parcel (with edges of integer values) that has a surface area of exactly 100 square units. Can you construct the parcel with Dalest Stereometry software and find its dimensions?



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Exploration

2 Are there more than one cuboids with surface area of 100 square units? Can you find them all? (*You can use a systematic way!*). Use the space below to draw them.

Complete the table with your findings.
Note: You might need to add or remove rows, since we DO NOT know yet the possible number of different cuboids!

Cuboid	Side 1	Side 2	Side 3
1			
2			

DALEST

Extension

(3) If the factory owners want to construct the parcel with the maximum volume, which one of the parcels would you suggest and why?

Write a letter to the company, explaining how you solved the problem.

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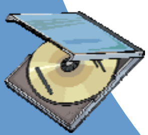
DALEST

Plutarch's Boxes

In this activity you will explore rectangles whose area is equal to their perimeter and cuboids whose surface area is equal to their volume.
For your explorations you will use the Dalest Stereometry software.

Introduction-Exploration

1 According to Plutarch, the Greeks found all the rectangles with integer sides, whose areas are equal to their perimeters. Can you find them as well?



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Extension

2

Which rectangular boxes, with integer dimensions, have their surface areas equal to their volumes? One example is the box with dimensions 4 by 6 by 12.
Find at least five different examples.

Write a letter to a friend, explaining how you solved the problem.

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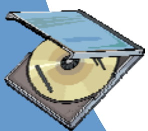
DALEST

Enjoy your Popcorn

In this activity you will explore the capacity of different sizes of popcorn bags and you will examine the relation between them. For your explorations you will use the *Dalest Stereometry* software.

Introduction

1 I went to the cinema with some friends last week and we decided to buy some bags of popcorn during the movie.



My friend, Nicholas, asked about the prices.
"One euro for the small size and four euros for the large size", the popcorn vendor replied.
"Can you tell us about the two boxes?", Nicholas asked her.
"Sure", the vendor replied. "The large bag's dimensions are two times the small bag's dimensions. See, it's taller, it's wider and it's deeper."
We estimated that the size of the small bag was 20cm by 10cm by 5cm.

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Exploration

Nicholas thought that the large size was a bargain!
He suggested to his friends that it was better to buy and share one large size bag, instead of buying three small bags.

Is Nicholas right or wrong?

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Construct one model for the small and one for the large sized bag to help you explain your ideas, using the *Dalest Stereometry* software.

DALEST

Extension

2 The popcorn vendor told them that: "The large size is double the size of the small bag, of course".
"Do you mean each side is two times bigger?" Nicholas asked.
"Naturally!" the vendor answered.

Is the vendor's comment correct?

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How big would the large size bag be?

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If the small bag contains one serving of popcorn, how many servings does the large bag contain?

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Write a letter to a friend, explaining how you solved the problem. Can you generalize your findings?

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DALEST

Sculptures

In this activity you will explore the number of vertices, faces and edges of 3D shapes.

For your explorations you will use Dalest Stereometry software

Introduction

1 A very well known sculptor works with geometric solids to create his sculptures. He created a cube and then put a square based pyramid on it. The area of the base of the pyramid is equal to the area of the side of the cube. Make a rough sketch in the space below of the artist's sculpture and make a prediction of its number of edges, vertices and faces.

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Exploration

Verify your prediction by constructing the sculpture with DALEST software (use GROUP wizard).

2 A second sculpture consists of two cubes and a square based pyramid (the solids are arranged in a vertical order: cube, cube and pyramid). The area of the base of the pyramid is equal to the area of the side of the two cubes. Construct the sculpture with the software and find the number of its vertices, edges and faces.

Extension

2 The artist continued to make sculptures in this pattern. Find the number of vertices, edges and faces of the sculpture consisting of 10 cubes and a pyramid. Which is the ratio between the volume of this sculpture and the volume of the sculpture with a cube and a pyramid?

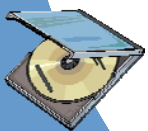
DALEST

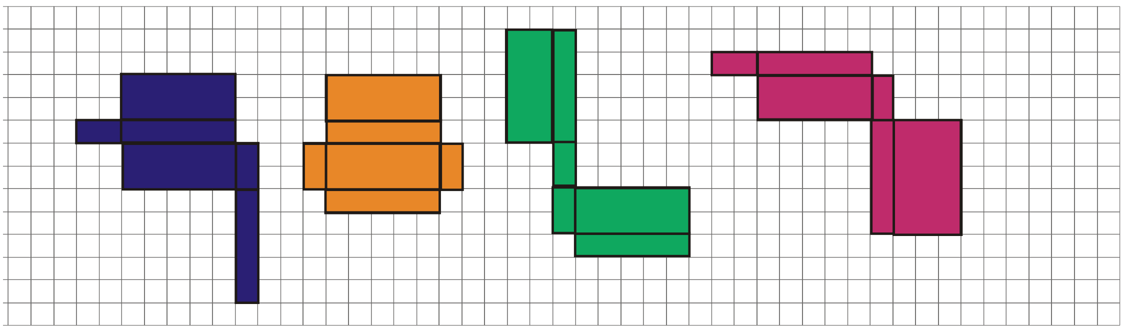
Cuboid Nets

In this activity you will identify and construct different cuboid nets.
For your explorations you will use the Dalest Origami Nets application.

Introduction

1 Which one of the figures below can not be a net of the given cuboid-box (Only folding is allowed, no cutting should take place)?





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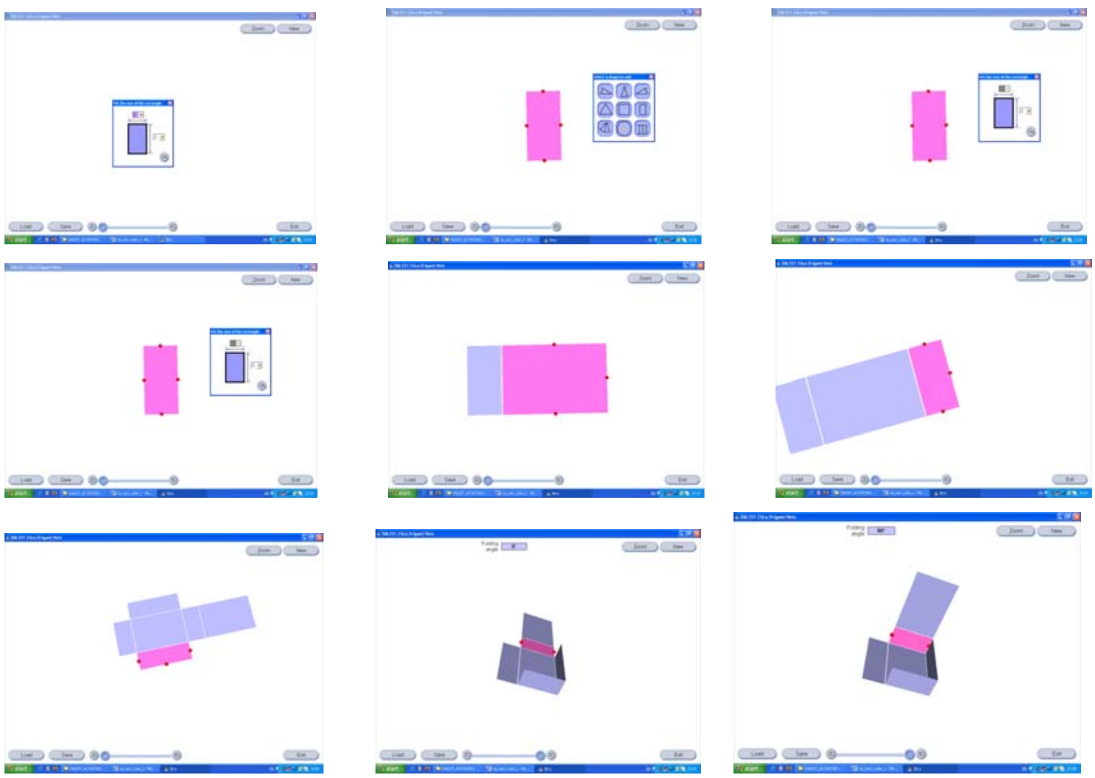
Explorations

2 Construct a model in Dalest Origami Nets.
To do this:

- Select a rectangle and its size
- Click on the side to which you want to add a second rectangle
- Select a rectangle from the figure menu and define its length (the other side is equal to the side you clicked earlier)
- Complete your model net
- If necessary use the *Zoom* button to see the whole figure
- Select the 3D mode and determine the folding angle of the corresponding rectangles

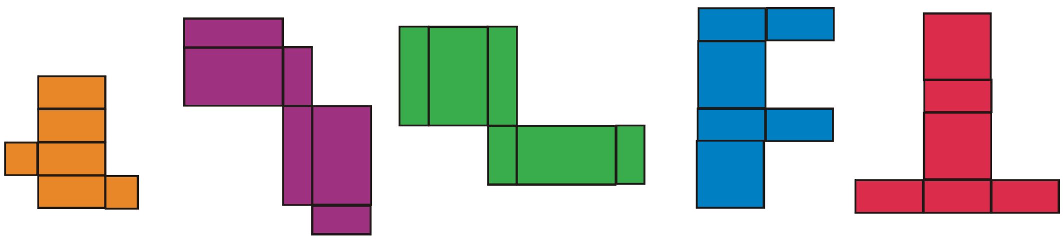


(3) Construct several possible nets of a 1 x 2 x 3 cuboid.

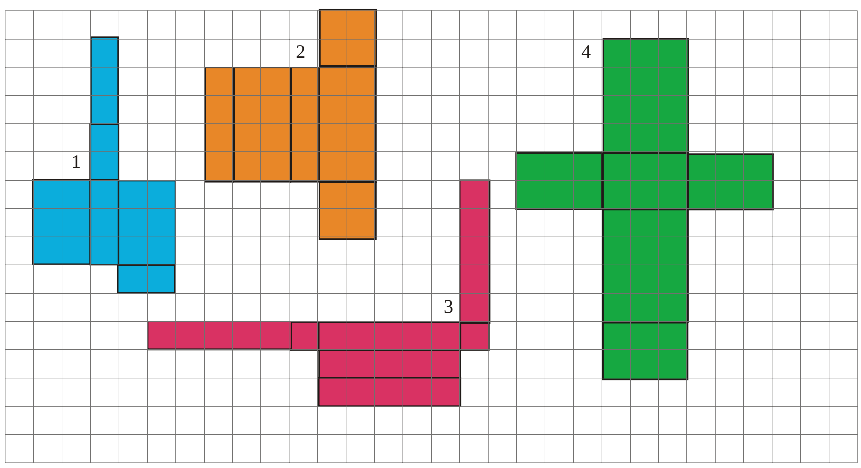
Check your solution in Dalest Origami Nets application.

4

Which one of the figures below is NOT a cuboid net? Check your answer with the *Dalest Origami Nets* application.

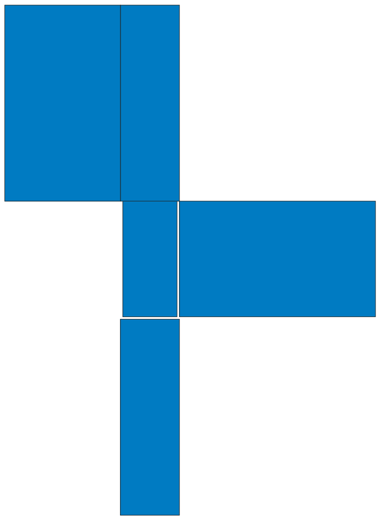


Which one of the figures below IS a cuboid net? Check your answer with the *Dalest Origami Nets* application.



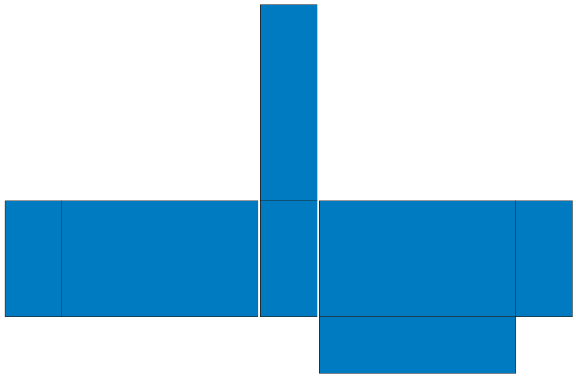
5

Add a rectangle to the figure below, to turn it into a cuboid net (there are more than one solutions).
Use the *Dalest Origami Nets* application to check your solutions.



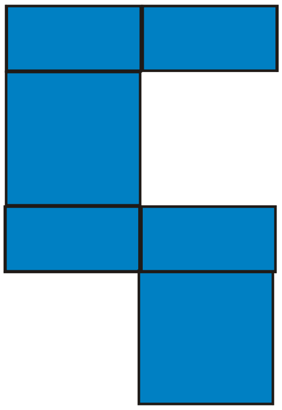
Draw your figures here.

6 Remove a rectangle from the figure below, to turn it into a cuboid net.
Use the *Dalest Origami Nets* application to check your solution.



Draw your solution here

7 The following figure in NOT a cuboid net. Turn it into a cuboid net by moving one rectangle in a different place.
Use the *Dalest Origami Nets* application to check your solution.



Draw your solution here

Write a letter to a friend to tell her what you liked the most and what difficulties you faced while working on this activity.

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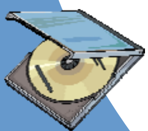
DALEST

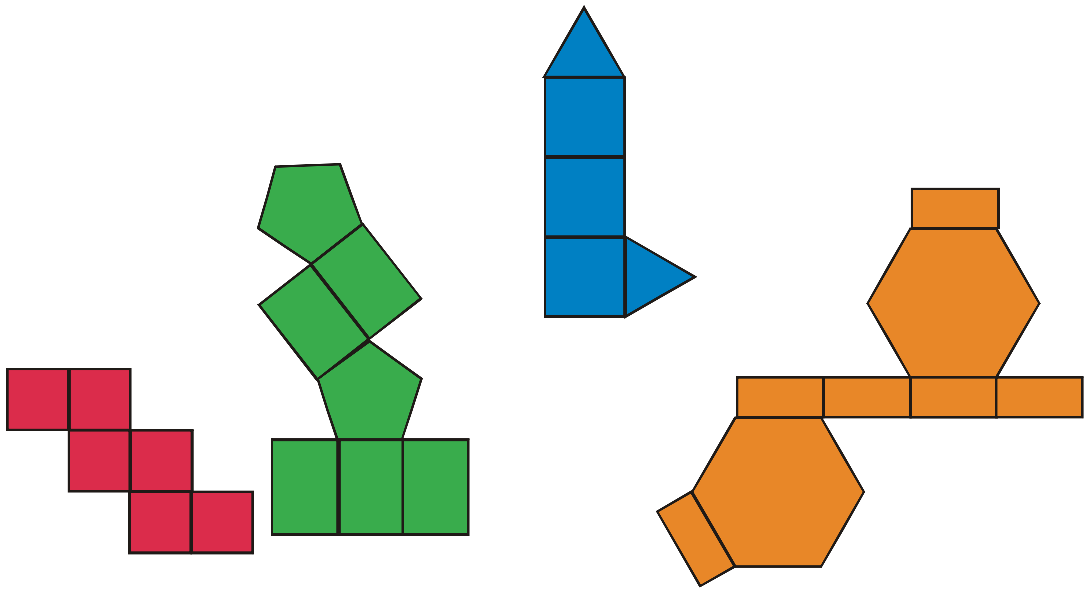
Prism Nets

In this activity you will find and construct nets of regular prisms. For your explorations you will use Dalest Origami Nets software.

Introduction

1 Which one of the following figures is NOT a prism net?





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Activity 10

Exploration

(2) With the use of the *Dalest Origami Nets* application, construct a model for each net, to explain your answer.

Copy your constructions here.

Extension

3 Construct seven different nets of prism with a triangular base.

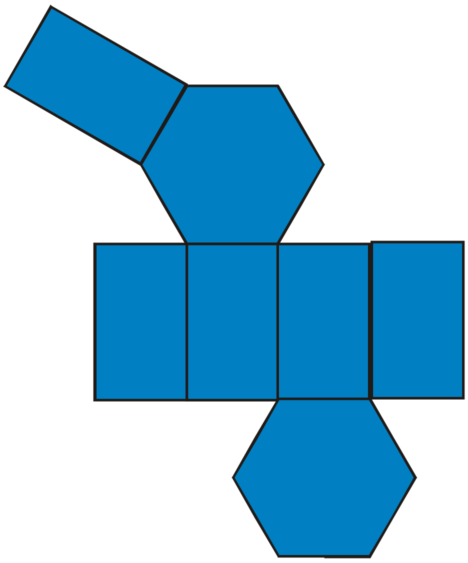
Check your solution using the *Dalest Origami Nets* application.

Copy your constructions here.

Extension

4 Add a polygon to the following figure to get a prism with an hexagonal base net.
Use the *Dalest Origami Nets* application to check your solution.

Create a similar problem for a prism with an octagonal base net.



Prism with an octagonal base net.

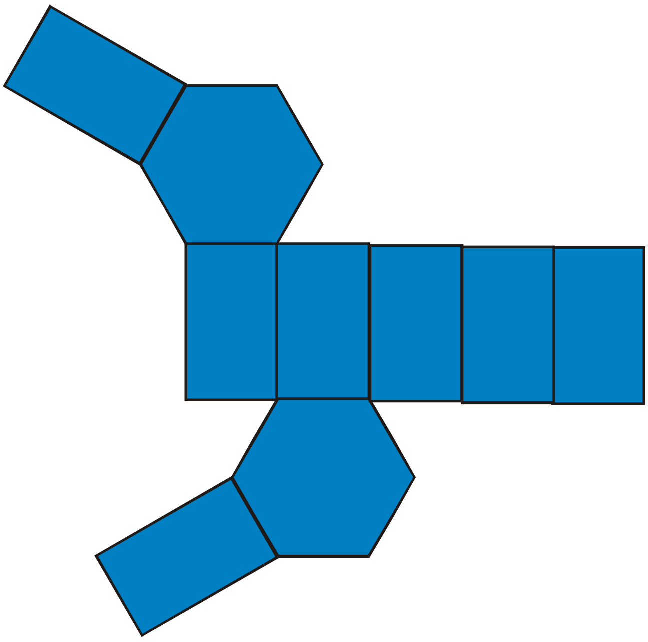
DALEST

Extension

5 Remove a polygon from the following figure to get a prism with an hexagonal base net.

Use the *Dalest Origami Nets* application to check your solution.

Create a similar problem for a prism with an octagonal base net.

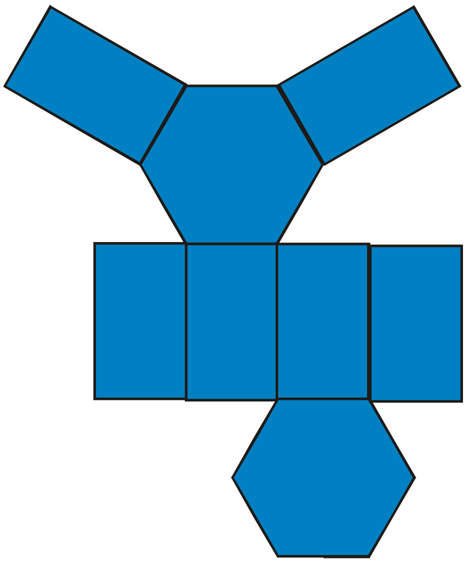


Prism with an octagonal base net.

Extension

(6) Move one polygon from one place to another in the following figure to get a prism with an hexagonal base net. Use Dalest Origami Nets application to check your solution.

Create a similar problem for a prism with a pentagonal base net.



Prism with a pentagonal base net.

Write a letter to a friend, explaining how you solved the problems. Can you generalize your findings?

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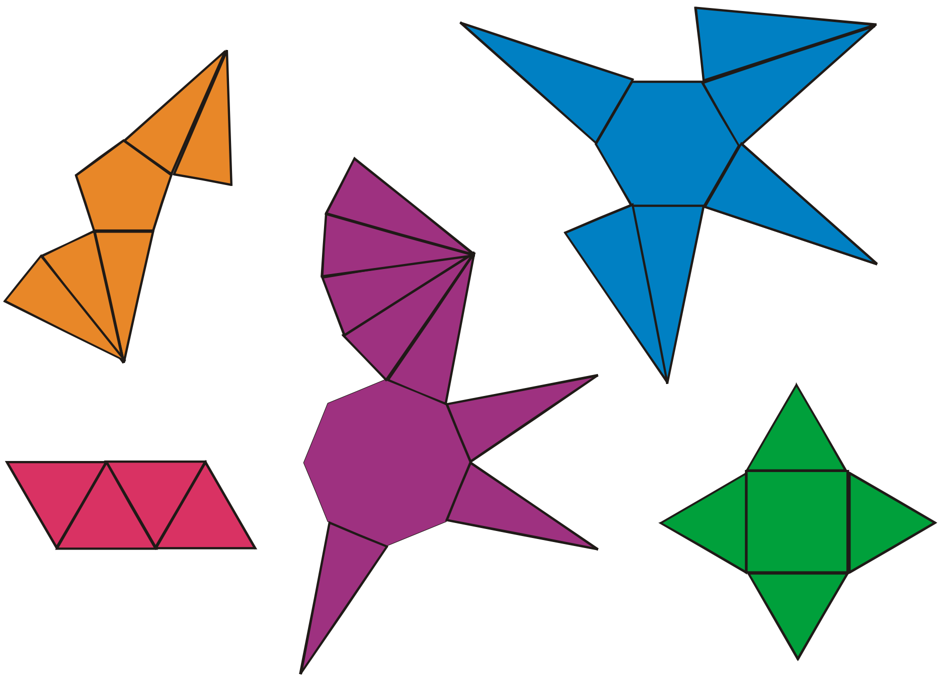
DALEST

Pyramid Nets

In this activity you will find and construct nets of regular pyramids.
For your explorations you will use the *Dalest Origami Nets* application.

Introduction

1 Which of the figure below is NOT a pyramid net?



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DALEST

Exploration

2 Construct a model to explain your answer using the *Dalest Origami Nets* application.

Draw your construction here.

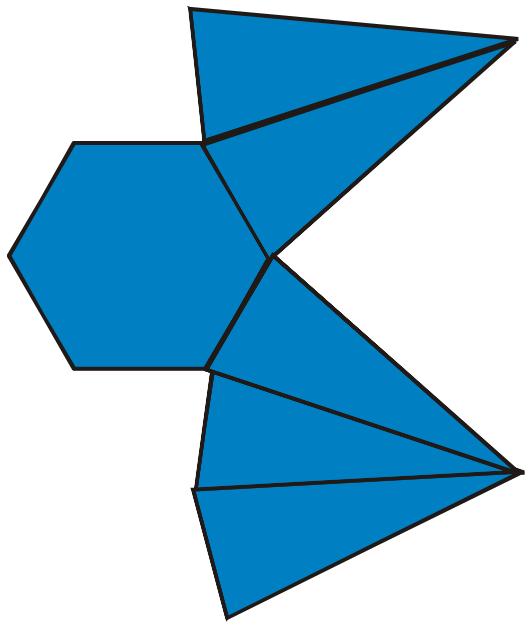
Extension

3 Construct three different nets of a square pyramid. Check your solution with the *Dalest Origami Nets* application.

Draw your nets here.

Extension

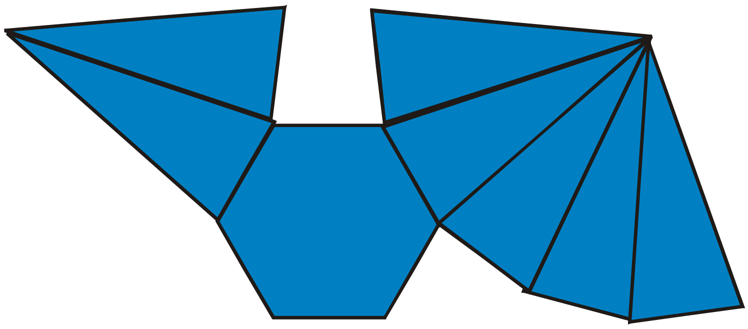
4 Add a polygon to the figure below to turn it into a pyramid with an hexagonal base net.
Check your solution in the *Dalest Origami Nets* application.



5 Pose a similar problem for a pyramid with an octagonal base net.

Extension

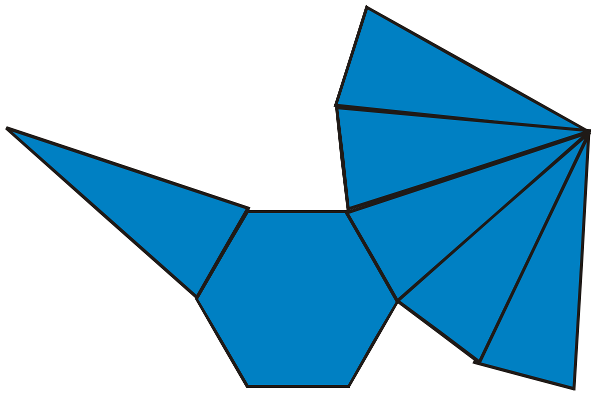
6 Remove a polygon from the figure below to turn it into a pyramid with an hexagonal base net.
Check your solution in the *Dalest Origami Nets* application.



7 Pose a similar problem for a pyramid with a pentagonal base net.

Extension

8 Move one polygon in the following figure to turn it into a pyramid with an hexagonal base net.
Check your solution in the *Dalest Origami Nets* application.



9 Pose a similar problem for a pyramid with a pentagonal base net.

Write a letter to a friend, explaining how you solved the problem. Can you generalize your findings?

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DALEST

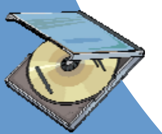
Designing Tetrahedral

In this activity you will construct tetrahedral by constructing and folding their nets.

For your explorations you will use Dalest Origami Nets application

Introduction

1 Use Origami Nets to build one tetrahedron net. Copy your net below.



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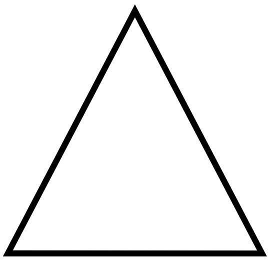
Exploration

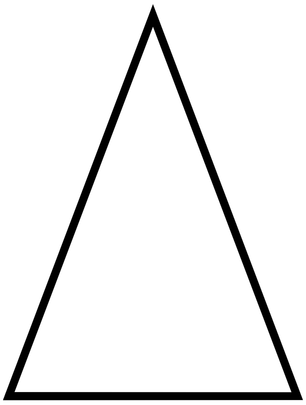
2

Here are four different triangles:

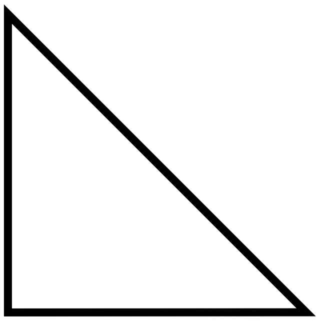
equilateral with size length 1 unit



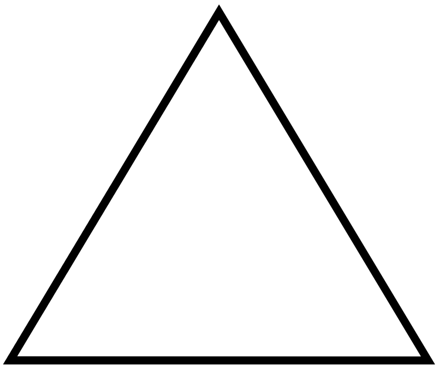
isosceles



Right-angled



equilateral with size length 2 units

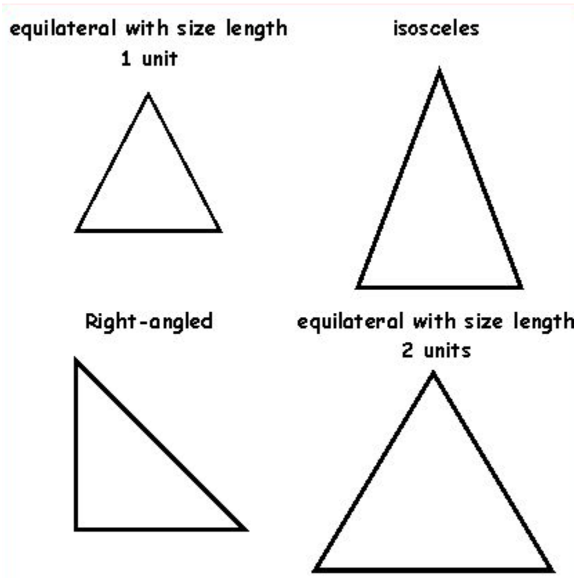


The sides of the small equilateral triangle have the same length as the short side of the isosceles triangle and the short sides of the right-angled triangle. The sides of the large equilateral triangle have the same length as the long sides of the isosceles triangle.

Continued...

(A) How many different tetrahedral can you make if you use each time only one type of triangles?

(B) How many different tetrahedral can you make if you use each time more than one triangles of each type?



Extension

3 Convince your classmates that you have found all the possible tetrahedral !

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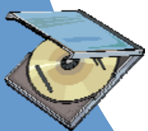
DALEST

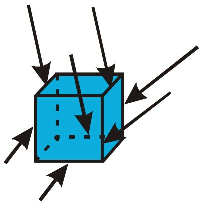
Imagination, Cubes and Nets

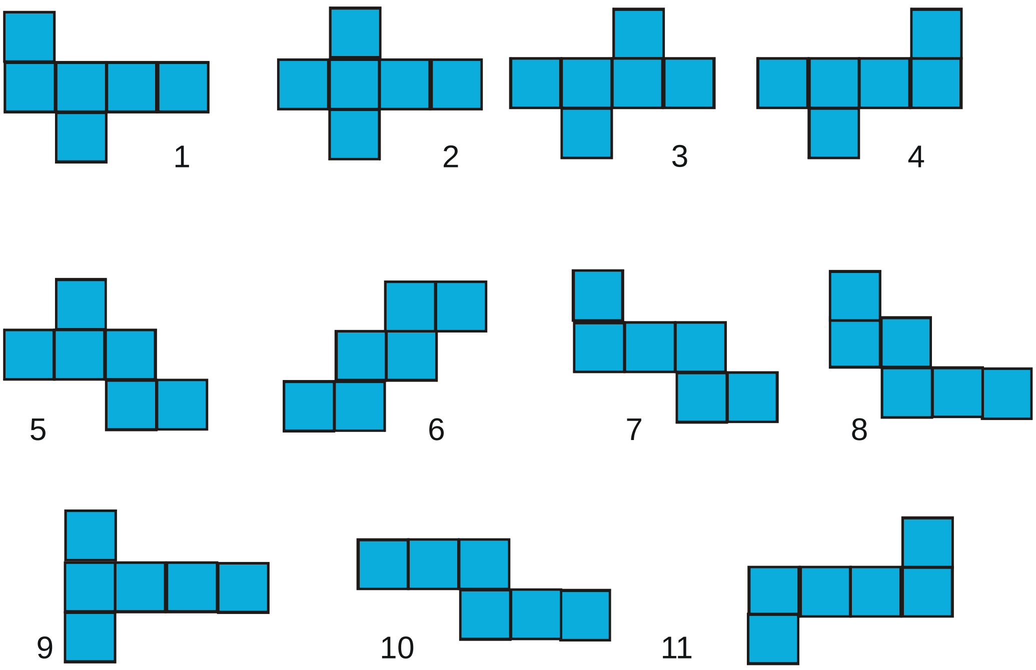
In this activity you will imaginary cut the edges of a cube in order to get its net.
For your explorations you will use the Dalest Stuffed toys and the Dalest Scissors applications.

Introduction



1 Which net will you get after cutting the edges indicated by arrows in the figure below?





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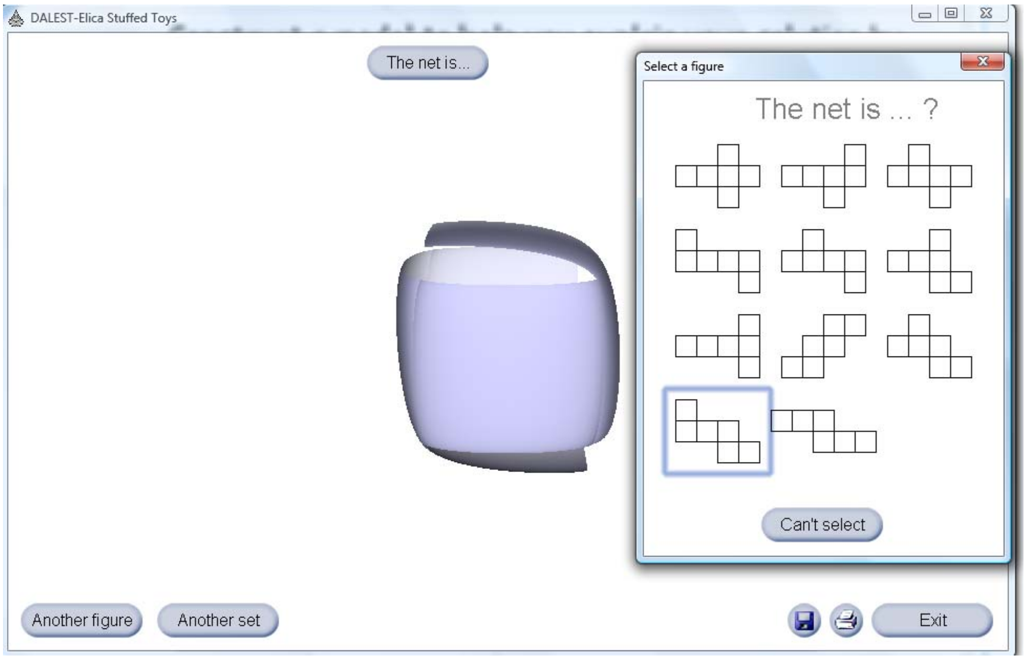
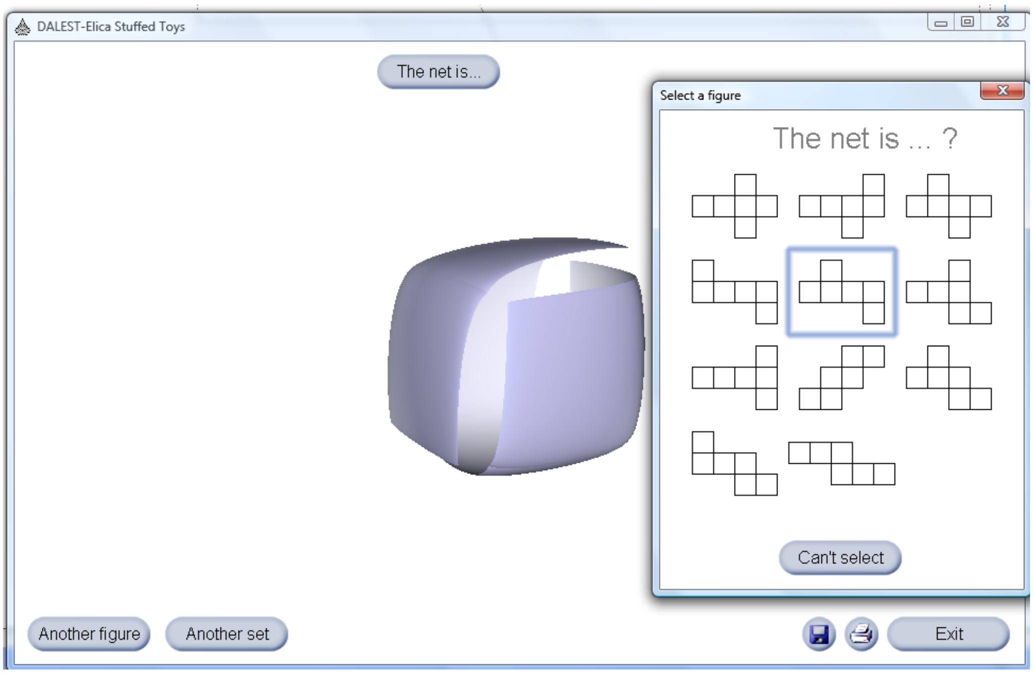
Exploration

2 Construct a model to help you explain your solution by means of Dalest Stuffed Toys software.



Extension

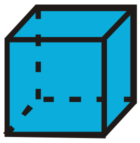
3 Try to find the net of 5 different cubes in *Dalest Stuffed Toys*.

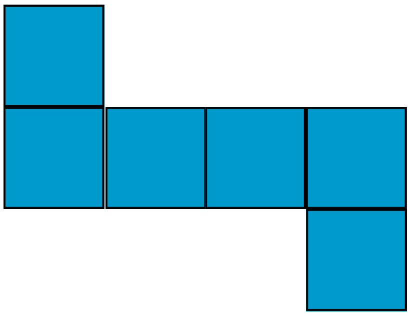


DALEST

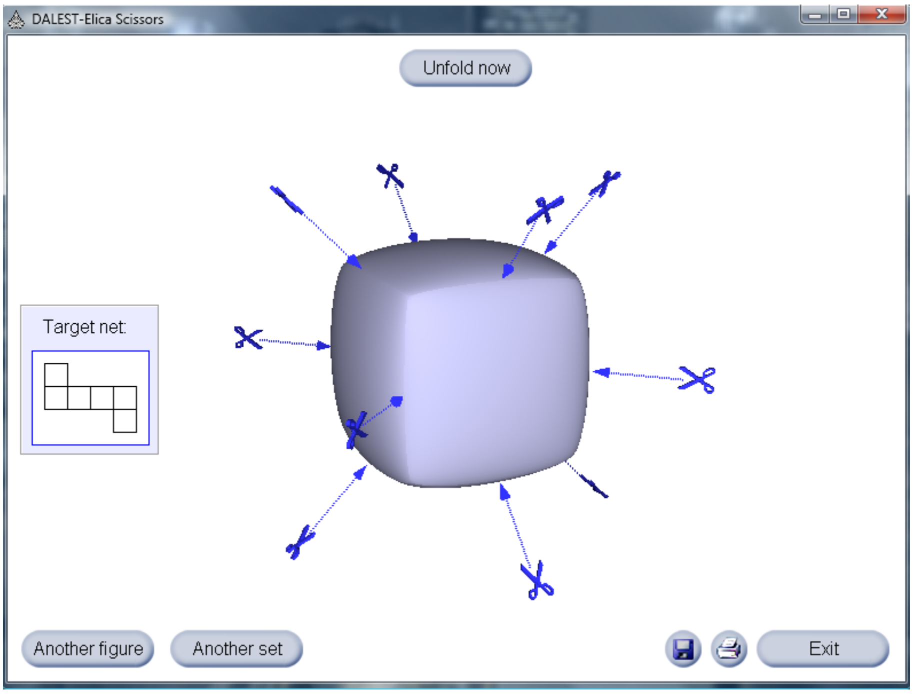
Exploration

4 Point at 7 edges of the cube to get the following net.



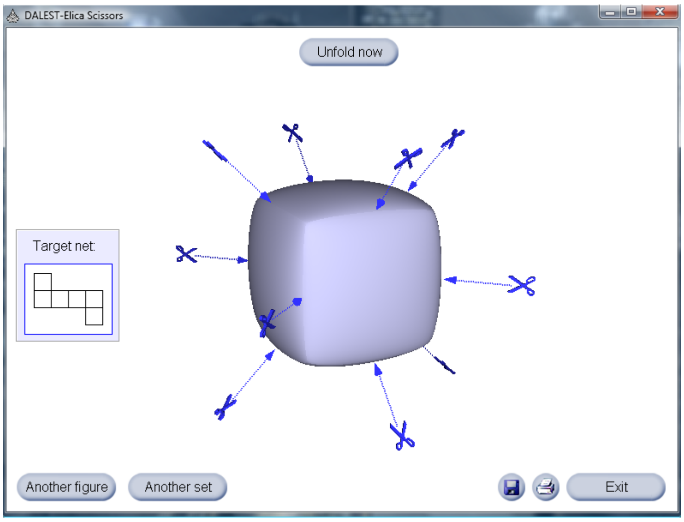


Construct a model to explain your solution in Dalest Scissors application



Extension

5 Play with 5 different target nets in Dalest Scissors software.



Write a letter to a friend explaining how you worked to find the nets in the previous activities.

Can you generalize your findings?

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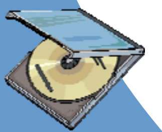
DALEST

Cube Arrangements

In this activity you will arrange different colour cubes, to design patterns and to explore properties.
For this activity you will use the Dalest Cubix Editor application.

Introduction

1 Use the Cubix editor application to create an arrangement of 9 cubes. Use different colours to make an attractive arrangement.



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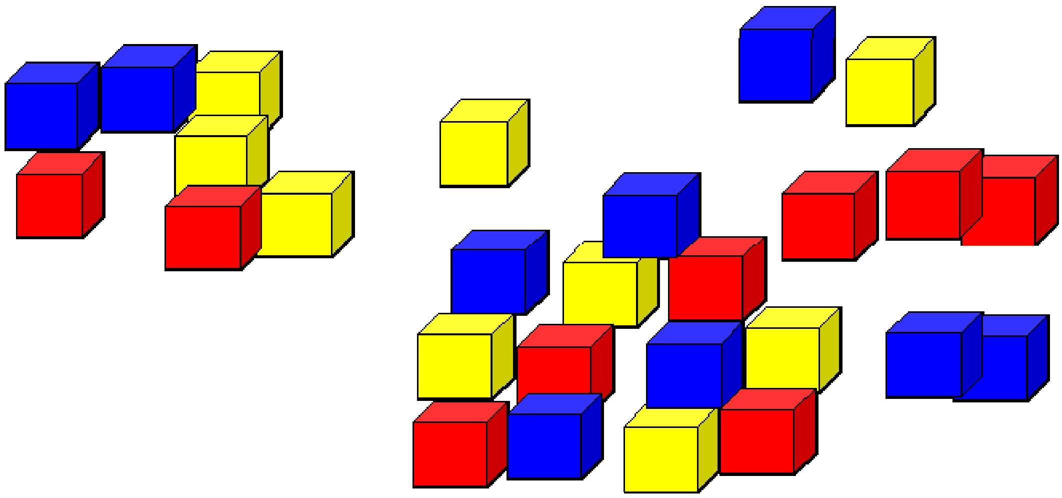
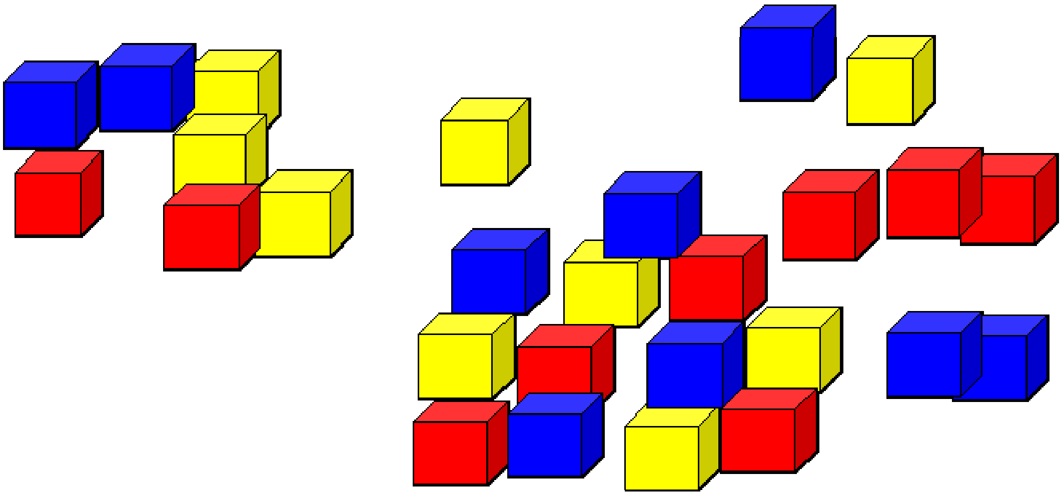
Activity 14

Exploration

2 Arrange 9 red cubes, 9 blue cubes and 9 yellow cubes to construct a large 3 by 3 cube. No row or column of cubes must contain two cubes of the same colour.



3 Rearrange the same cubes into a large 3 by 3 by 3 cube creating your own pattern and draw it in the space below.



Extension

4 You have 8 small cubes, 2 of each of the four colours. Use the small cubes to make a 2 by 2 by 2 cube so that each face of the constructed cube contains one of every colour.

Try to solve the same problem as the one above, using 27 small cubes, 3 of each of the nine colours.

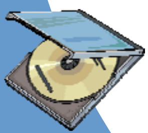
DALEST

Cube Constructions

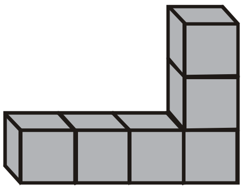
In this activity you will construct solids and consider their volume.
For your explorations you will use the Dalest Cubix Editor application.

Introduction

1 How many unit cubes you need to add to the construction below to get a cube.



Is it possible to do this with a smaller number of cubes?



Draw your constructions here.

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Exploration

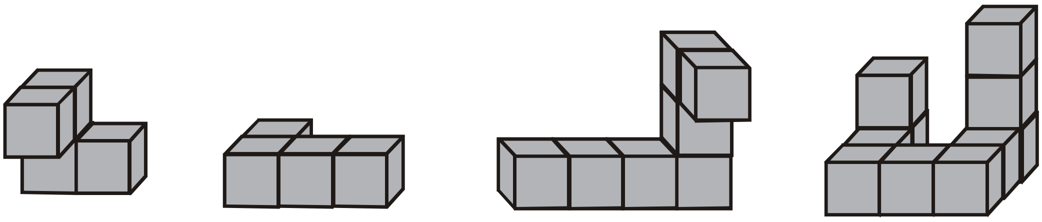
Construct a model to help you explain your answers, using the Dalest Cubix Editor application.

Copy your models here

DALEST

Extension

2 Add the least number of unit cubes to each of the constructions below to turn them into bigger cubes.



a) b) c) d)

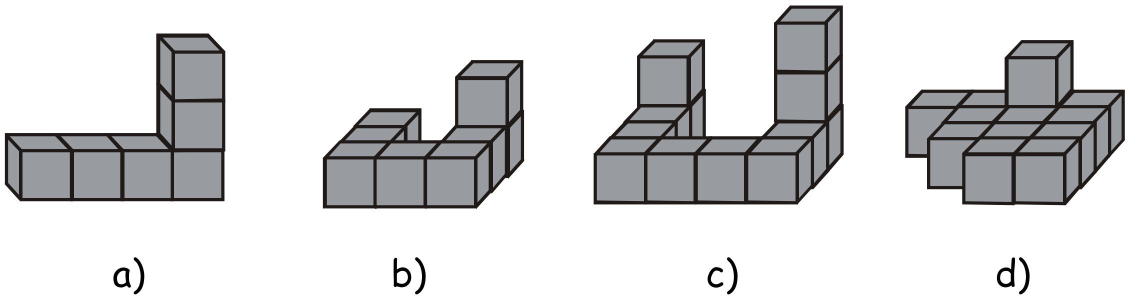
Draw the completed cubes here

Fill the table with your results.

	(a)	(b)	(c)	(d)
Volume of the given construction				
Volume of the completed cube				
Number of the added cubes				

Extension

2 Add the least number of unit cubes to each of the constructions below to turn them into cuboids.



Draw the completed cuboids here

Fill the table with your results..

	(a)	(b)	(c)	(d)
Volume of the given construction				
Volume of the completed cuboid				
Number of the added unit cubes				

Write a letter to a friend, explaining how you solved the problem. Can you generalize your findings?

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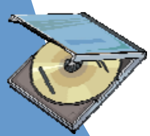
DALEST

Modular Houses

In this activity you will explore the possibilities of building small modular houses with unit-sized cubes.
For your explorations you will use the Dalest Cubix Editor application.

Introduction

1 Our company (Modular Houses Ltd) is about to build small modular houses. Each house is to be made from four cubic modules, all the same size. Cubic modules will touch each other along complete faces.



Design, with the help of the application, the biggest possible number of different four-cube houses that you can.

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Exploration

2

Calculate the construction costs of each design based on the following:

- € 10000 for each (unit) square of land covered
- € 4000 per square of external wall
- € 6000 per square of roof

Which is the most expensive design?

Which is the cheapest design?

Extension

3 The company is expanding its product range. The company director wants to include some 5-module houses.

As the head company architect, your job is to prepare a brochure. The brochure should contain three unique designs for 5-module houses.

The brochure should also contain the following information of each design:

(a)A description of the layout of the house.
(b) Drawing of the finished house.
(c) Costing for land, walls, roof and total cost of the house.
(d) Summary of the best features that will help to sell the house.

DALEST

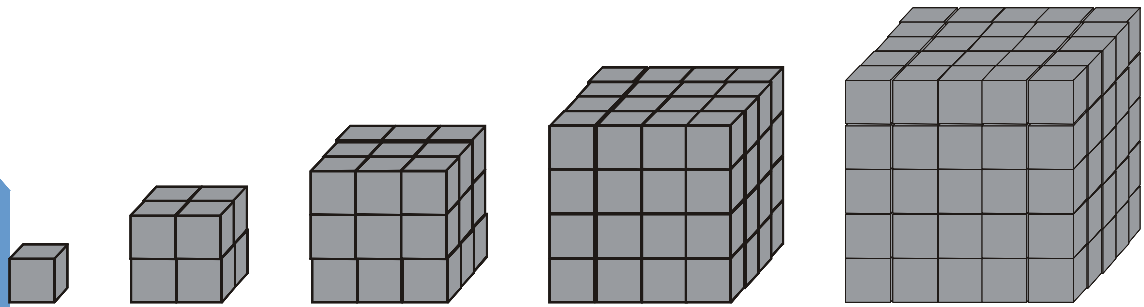
Sums

In this activity you will learn about **cubic numbers** and how to represent them with unit cubes.
For your explorations you will use the Dalest Cubix Editor application.

Introduction - Exploration

1

Find the volume of each of the cubes below.



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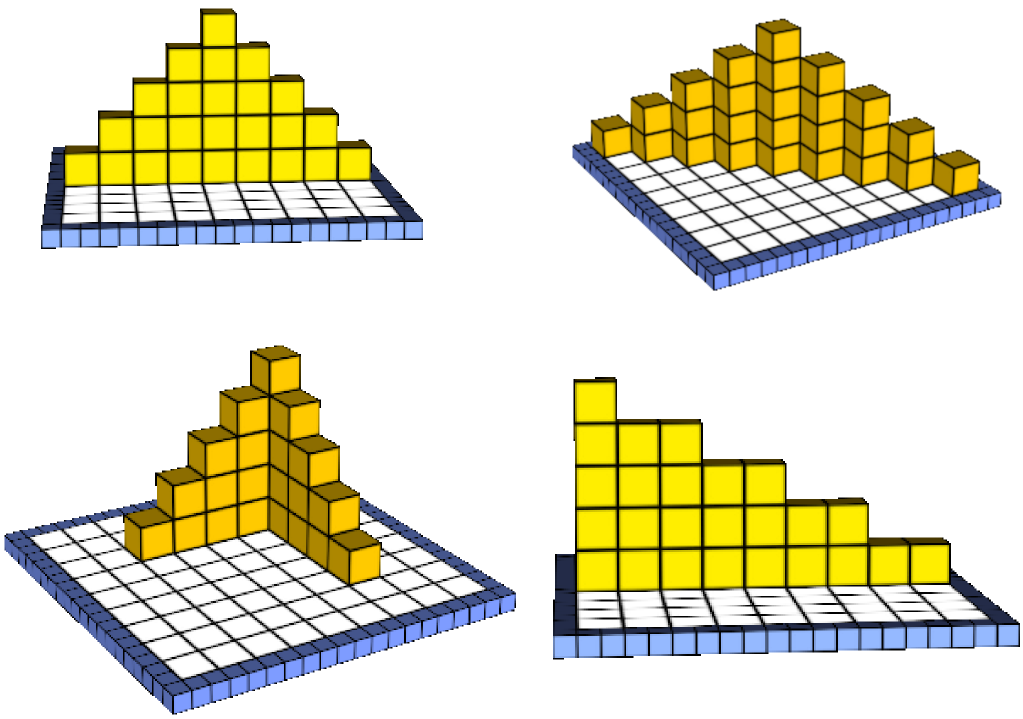
Extension

2 The sum 1 + 3 + 5 + 7 + 9 can be represented by constructions of unit cubes as follows:



Use the the *Dalest Cubix Editor* application to model the following sums:

(a) 1 + 2 + 3 + 4 + 5 + 6 + 7

(b) 2 + 4 + 6 + 8

(c) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$

(d) $1^3 + 2^3 + 3^3$

Write a letter to a friend, explaining how you solved the problem by providing a convincing argument.

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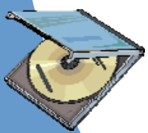
DALEST

Cuboid Volume

In this activity you will discover the formula for finding the volume of a cuboid.
For your explorations you will use DALEST Cubix Editor software.

Introduction



1 Alex tries to put 28 unit-sided cubes (1 cm edge) in a rectangular box with dimensions 2 cm x 5 cm x 3 cm. Is this possible?

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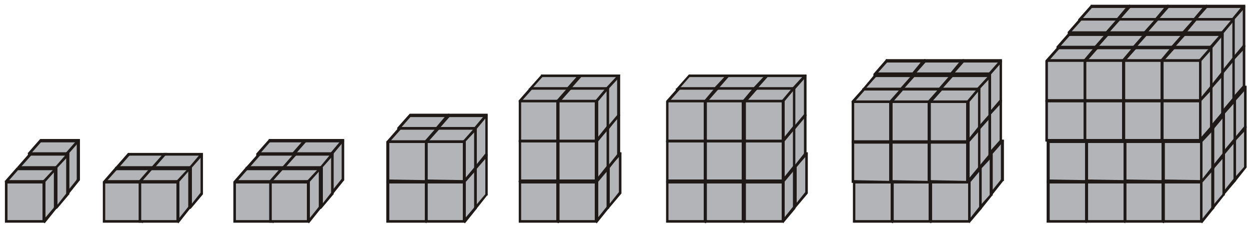
Exploration

(2) Construct a model to help you explain, using DALEST Cubix Editor.

DALEST

Extension

3 Find the **volume (V)** of the following cuboids:.



(a) (b) (c) (d) (e) (f) (g) (h)

Write the results in the table below:.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
V								

What happens when the length or width or height of the cuboid increases one unit?

Write a letter to a friend, explaining how you solved the problem. Can you generalize your findings?

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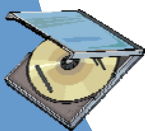
DALEST

Relations in a Cone

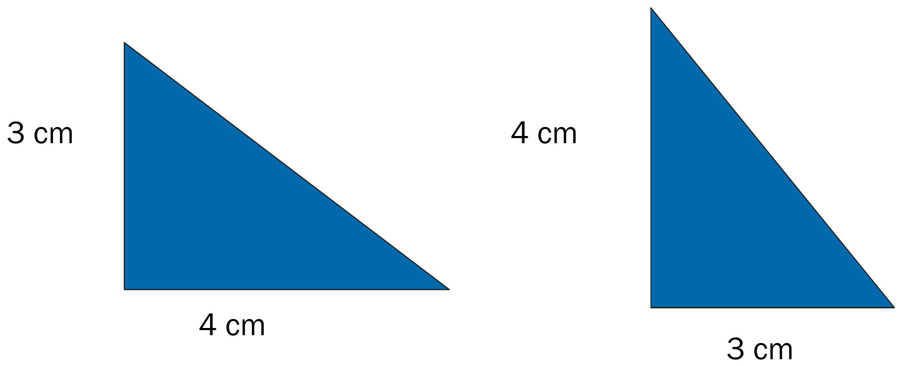
In this activity you will explore the relation between the dimensions and the volume of a cone.
For your explorations you will use the Dalest Math Wheel application

Introduction

1 Compare the volumes of two cones generated by revolution of a right-angled triangle.



The dimensions of the two vertical sides are 3 cm and 4 cm respectively. The triangle is rotated around each one of its two vertical sides for constructing the two cones.



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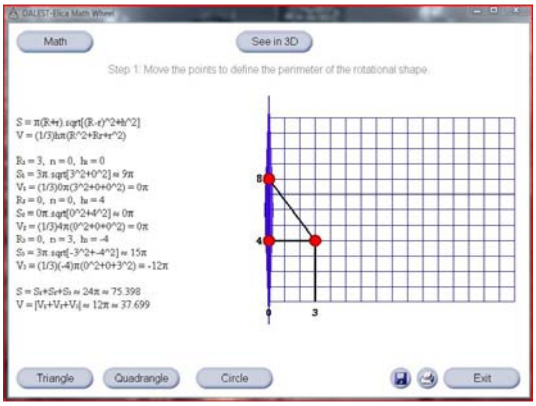
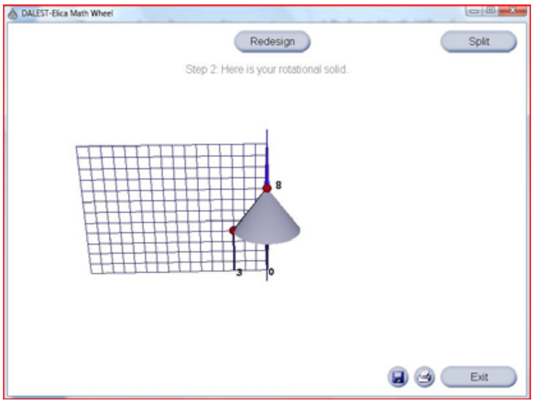
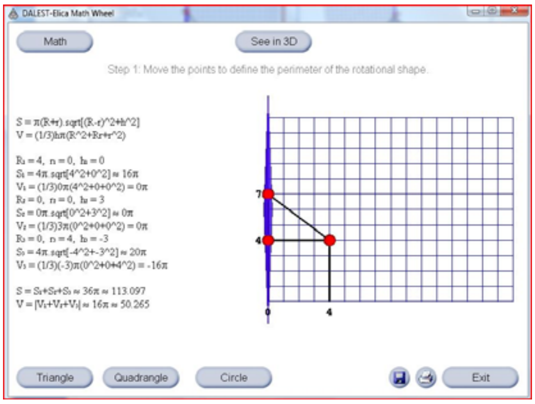
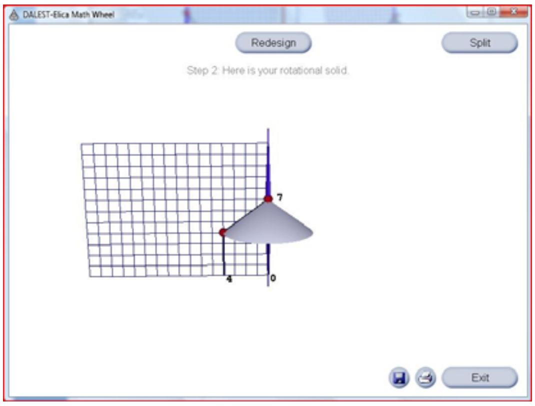
Exploration

2 Construct a model to help you explain your solution, using the Dalest Math Wheel application.









You can use the *Math button.*

DALEST

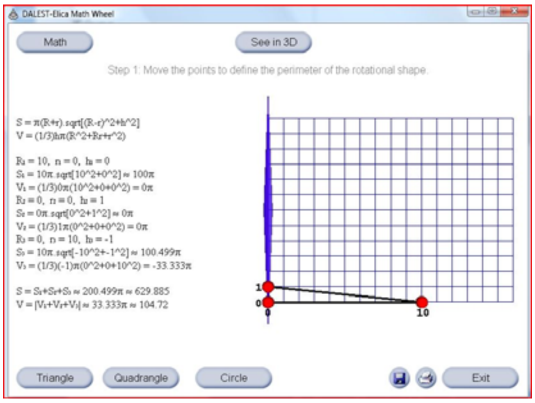
Extension

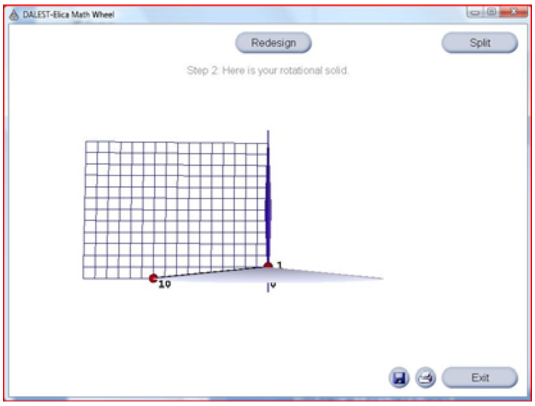
3

In the table below you have dimensions of a cone in each column (*r*—radius and *h*—height).
Using the Dalest Math Wheel application fill in the first four columns.
Make your conjecture for the numbers in the last two columns. It will be easier if you convert the decimal fraction into common fractions.
Using the known formulas examine the validity of your conjecture.
Give a general rule about the way in which the volume of the cone changes when its height changes.

Cone

r (radius)	10	10	10	10	10	10
h (height)	1	2	4	8	16	20
V (volume)						





DALEST

Extension

(4) In the table below you have dimensions of a cone (*r*—radius and *h*—height).

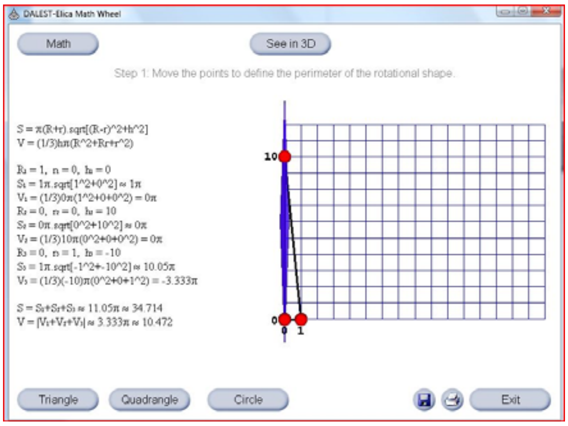
Using the Dalest Math Wheel application fill in the first four columns. Conjecture the numbers in the last two columns. It will be easier if you convert the decimal fractions into common fractions.

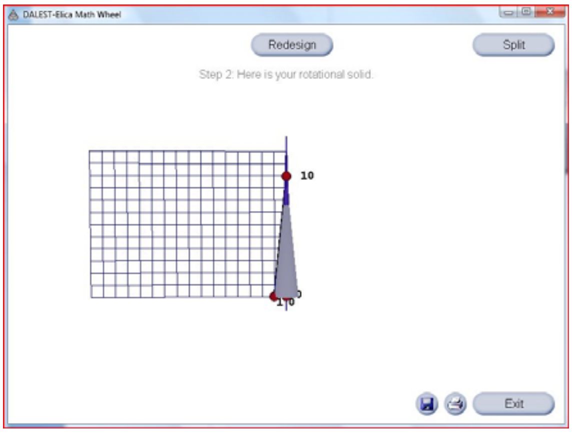
Using the known formulas examine the validity of your conjecture.

Give a general rule about the way in which the volume of the cone changes when the radius changes.

Cone

r (radius)	1	2	4	8	16	20
h (height)	10	10	10	10	10	10
V (volume)						





DALEST

Extension

5 Using the already found relations, compare the volumes of the cones with:

a) r = 5 см, h = 12см and r_1 = 12 см, h_1 = 5 см

b) r = 7 см, h = 24 см and r_1 = 24 см, h_1 = 7 см

c) r = 21 см, h = 20 см and r_1= 20 см, h_1 = 21 см

d) r = 45 см, h = 28 см and r_1 = 28 см, h_1= 45 см

- How will the volume of a cone change if:
  a) we decrease its height 7 times?
  b) we decrease its radius 7 times?
  c) we decrease its height 7 times and its radius 7 times?
  d) we decrease its height 7 times and increase its radius 7 times?

- In order to increase the cone volume 64 times, how many times we have to increase:
  a) its height?
  b) its radius?

Write a letter to a friend, explaining how you solved the problem. Can you generalize your findings?

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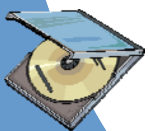
DALEST

Relations in a Cylinder

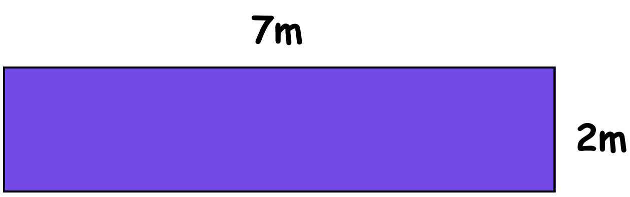
In this activity you will discover and use the relation between the dimensions of a cylinder, its surface area and its volume. For your explorations you will use the Dalest Math Wheel application.

Introduction

1 A firm wants to place an advertisement in the shape of a **7m x 2m** rectangle. The rectangle will turn around one of its sides to produce a cylinder.



The firm wants to know which of the two adjacent sides should be the axis of rotation, so that the least possible space is occupied by the advertisement.

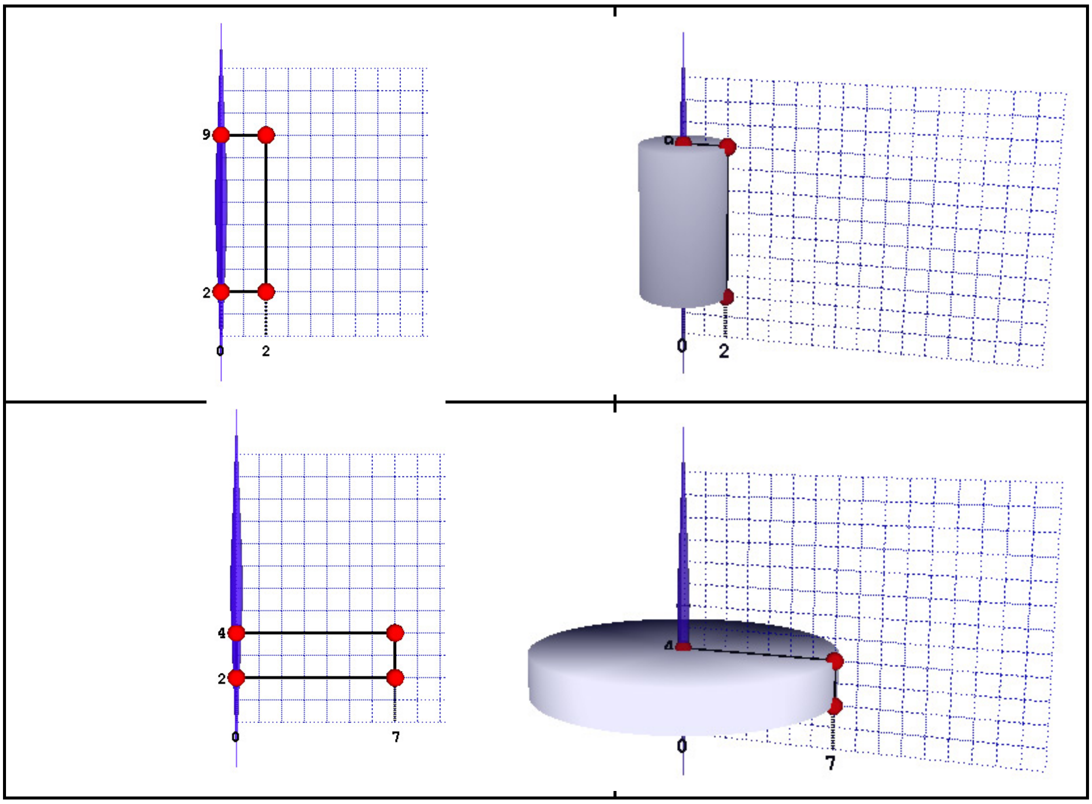


DALEST

Exploration

2 Construct a model of each cylinder in the *Math Wheel* application.

Use the *Math button* to check your answers.



DALEST

Extension

3 The table below contains the dimensions *r* *(radius)* and *h* *(height)* of different cylinders.

- Fill in the first 4 columns, using the *Math Wheel* software.
- Make a conjecture about the numbers in the last 2 columns.
- Use the formulae you know to check your conjecture.
- Provide a rule explaining how the cylinder's surface changes when its height changes.
- Provide a rule explaining how the cylinder's volume changes when its height changes.

R (radius)	10	10	10	10	10	10
h (height)	1	2	4	8	16	20
S (surface)						
V (volume)						

DALEST

Extension

4 The table below contains the dimensions *r* and *h* of different cylinders.

- Fill in the first 4 columns, using the *Math Wheel* application.
- Make a conjecture about the numbers in the last 2 columns.
- Use the formulae you know to check your conjecture.
- Provide a rule explaining how the cylinder's surface changes when its height changes.
- Provide a rule explaining how the cylinder's volume changes when its radius changes.

r (radius)	1	2	4	8	16	20
h (height)	10	10	10	10	10	10
V (Volume)						

5 Use the relations you found to compare the volumes of the cylinders with the following dimensions:

a) r = 7 cm, h = 12cm and r_1 = 12 cm, h_1 = 7 cm

b) r = 45cm, h = 24 cm and r_1 = 24 cm, h_1 = 45 cm

c) r = 25cm, h = 17 cm and r_1 = 17 cm, h_1 = 25 cm

d) r = 90 cm, h = 89 cm and r_1 = 89 cm, h_1 = 90 cm

How will the volume of a cylinder change if:

a) its height decreases 9 times ?
b) its radius decreases 9 times ?
c) its height decreases 9 times and its radius decreases 9 times?
d) its height decreases 9 times and its radius increases 9 times?

To increase the cylinder's volume 100 times, how many times do you have to increase:
a) its height?
b) its radius?

DALEST

6 Write a letter to a friend, explaining how you solved the problem.

Can you generalize your findings?

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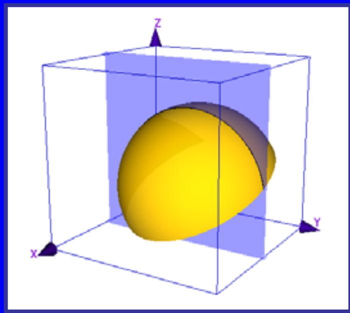
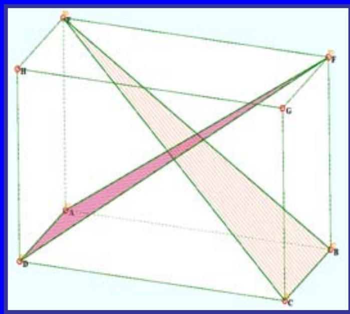
DALEST

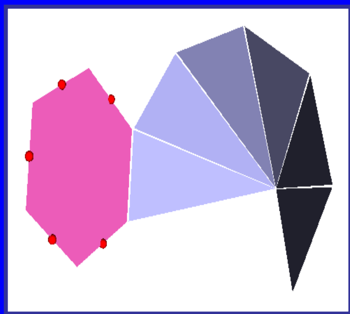
The book **"Teaching Stereometry with Dalest"** is a product of Dalest project. **D**eveloping an **A**ctive **L**earning **E**nvironment for **St**ereometry (DALEST) is a European research co funded project in the framework of SOCRATES 2005–2007, under MINERVA Action. In this book the software for teaching stereometry and the activities developed are presented.



The collaboration involved five universities – The University of Cyprus (coordinating institution), The University of Southampton, The University of Lisbon, The University of Sofia, The University of Athens, as well as the N.K.M. Netmasters, and the Cyprus Mathematics Teachers Association in an active joint work.



The main purpose of the Dalest project was to develop a dynamic set of microworlds and applications suitable for the teaching of stereometry and developing spatial thinking in elementary and middle schools. The software focuses on the development of learners' thinking abilities and on their abilities to model ideas, to analyze and solve problems in their everyday activities.



Education and Culture

Socrates

Minerva

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