Identifying and Cultivating Extraordinary Mathematical Talent

by Titu Andreescu, Joseph A. Gallian & Janet E. Mertz — November 26, 2008

We discuss here some specific programs through which extraordinary mathematical talent is typically identified and nurtured. Initial identification usually occurs through participation in school-sponsored talent searches, math competitions, and enrichment programs. This talent is subsequently nurtured through participation in additional higher-level math competitions, college-level courses, math camps, Math Circles, problem solving websites, and undergraduate research programs.

For some, extraordinary mathematical talent is apparent at a very early age. For others, it surfaces later in life. For example, when eighteenth century mathematician Carl Friedrich Gauss was asked at the age of five to sum the first 100 positive integers, he immediately came up with the right answer: 5050. When asked how he did this so quickly, his explanation was that the sum of 1 and 100 is 101, the sum of 2 and 99 is 101, and continuing in this way, the sum of 50 and 51 is 101, so fifty pairs of 101 gives a total of 5050.

An important question is how to identify children with exceptional mathematical talent that is not as apparent as in Gauss's case. One way is to have students participate in out-of-grade-level testing, for example, take the Scholastic Aptitude Test while in middle school (e.g., see http://cty.jhu.edu/set/index.html). Another way is to have students participate in regional math competitions. Middle and high school students who perform exceptionally well on regional math competitions are then encouraged by teachers and parents to participate in more advanced contests. The most important math contests for middle and high school students in the US are the American Mathematics Competitions sponsored by the Mathematical Association of America (MAA). Those who perform at the highest level in these competitions are invited to take additional, more difficult exams. More than 50 of the best performers on the USA Mathematical Olympiad are given the opportunity to attend the all expenses paid three week-long Mathematics Olympiad Summer Program (MOSP) at the University of Nebraska-Lincoln, where they are taught highly sophisticated problem solving techniques by world class problem solvers. After a series of exams given at MOSP, six of these students are selected to represent the United States in the International Mathematics Olympiad (IMO) against other six-person teams from around the world. (A documentary film about the 2006 IMO is available for purchase.)

Students who do not qualify for MOSP, but would like to have a better chance to qualify in future years, can participate in programs such as AwesomeMath, run by the first author. AwesomeMath offers a competitive and highly-intensive three-week summer camp that provides a supportive community of peers and an excellent environment to meet other students who share the same passion for the subject. The teaching, which emphasizes the importance of problem solving, is done by professors with national and international experience in math competitions, mentors, and assistants who are students at prestigious universities such as MIT, Harvard, and Princeton. AwesomeMath also includes a correspondence program that consists of six segments distributed throughout the school year. The segments, structured around important mathematical concepts, consist of lecture notes, problems with solutions, and assigned problems. The first author is also the editor of Mathematical Reflections, an online journal for advanced high school and college level students. Other summer math programs for middle and high school students that have a long successful track record, with more emphasis on enrichment than on problem solving, include the Ross Mathematics Program at Ohio State University, The Hampshire College Summer Studies in Mathematics at Hampshire College, Canada/USA Mathcamp, and the Program In Mathematics for Young Scientists at Boston University. A comprehensive listing of summer math programs is available.

Another way to identify students with exceptional ability in mathematics is to track students who rise to the top in enrichment programs. Many students learn advanced mathematics and hone their problem solving skills by taking college math classes at an early age. Later on, some do original, high-level mathematical research while they are undergraduate students in college. Math Circles have recently begun to make an impact in the United States. Typically, Math Circles are weekly 2-hour evening, Saturday, or Sunday sessions for middle and high school students given during the academic year by university faculty who teach problem solving skills and topics well beyond the 7^{th} through 12^{th}-grade curriculum. The annual Siemens/Westinghouse and Intel Science Talent Searches provide other mechanisms by which students are identified who possess exceptional talent in mathematics and science.

Online and correspondence programs provide additional ways by which students in grades 6 through 12 can develop their talent in mathematics. The Purple Comet! Math Meet, a free online international team contest for middle and high school students is co-sponsored by the University of Wisconsin-Whitewater and the University of Texas at Dallas. The meet is offered every April, and over 7,000 students from numerous

countries participated in 2008. The Art of Problem Solving, offering classes in problem solving that prepare students for mathematics contests, enables students from around the world to develop their problem solving skills and to join a virtual community of like-minded peers. The USA Mathematical Talent Search is a free competition in which middle and high school students are given a month to solve each of four sets of challenging problems per year, with mathematicians giving them feedback on the written justifications to their solutions.

For college students, the MAA sponsors an annual contest called The William Lowell Putnam Mathematical Competition. This is a 120-point, 12-problem exam taken in two three-hour sessions. Typically, only the top math students from each school enter the contest. The Putnam exam is so difficult that the median score among the nearly 4000 contestants is usually 0 or 1! The exam is intentionally made extremely difficult to distinguish students with world-class mathematical ability from ones with simply exceptional mathematical skills. Many people who excel in the Putnam have gone on to become leading mathematicians, including winners of the Fields Medal (the so-called Nobel Prize of Mathematics); several have won Nobel Prizes in Physics.

Over the past two decades, numerous top college math students have acquired their first research experience by taking part in summer Research Experiences for Undergraduates programs (REUs) sponsored by the National Science Foundation. In the REU directed by the second author, nine carefully selected students per year work on original, professional-level problems chosen by the Director. There are no instructional lectures or books. Two program alumni serve as research advisers to the undergraduate students. The students are given one or two background articles and work largely on their own in their on-campus apartments. Everyone involved gathers twice weekly to hear the students give presentations on their progress. Approximately 20 program alumni visit for a week or more throughout the summer to assist with the program by working and socializing with the nine "official" participants. This networking is extremely valuable as many of the program participants go on to become professional mathematicians in academia, government, or industry. By the end of the REU, most of the students have results that are subsequently published in mainstream research journals. There are currently more than 40 REUs in the United States.

In this article we have outlined a familiar path that many people with extraordinary math talent follow: begin with math contests in middle and high school; while in grades 6-12 take advanced math courses at a local college or via online programs (e.g., Stanford University's Education Program for Gifted Youth) or problem solving courses online; participate in academic year enhancement programs such as a Math Circle; attend high-level summer math camps; enroll in an elite college or university and go to an REU; obtain a Ph.D. in mathematics from a leading graduate school; become a professor or work for the federal government or a successful company. Following this path, students talented in mathematics who have supportive parents and teachers will become outstanding mathematicians.

While much is being done to cultivate mathematical talent in the United States, much more could be done. Starting in sixth grade at the latest, mathematics courses should be taught by teachers who majored in mathematics or mathematics education, or, at the very least, are certified to teach mathematics so that mathematically precocious students can continue to find challenging opportunities within their own schools. There should be more schools like the Bronx High School of Science, the Thomas Jefferson High School for Science and Technology, and the Illinois Math and Science Academy that specialize in science and mathematics and provide a socially and academically supportive environment with like-minded peers and highly qualified teachers. We should facilitate the ease with which mathematically talented students can study mathematics at local colleges and through online courses. And, to get more students onto the path described here, many more schools should participate in math competitions such as MATHCOUNTS®, the Purple Comet! Math Meet, and the ones sponsored by the Mathematical Association of America.

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