Final Report (part 1)
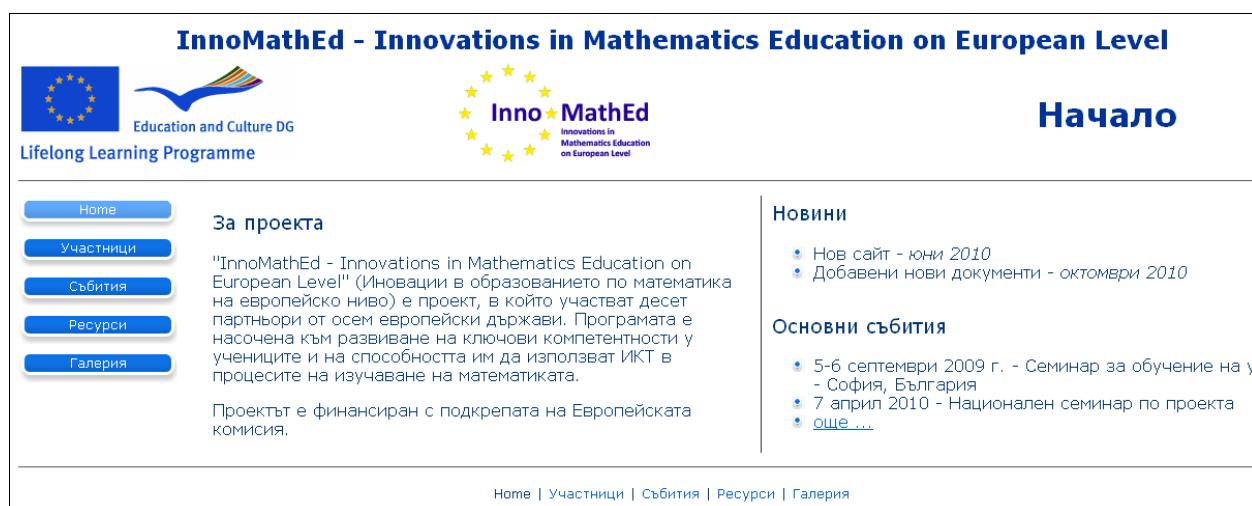## of the Bulgarian InnoMathEd team
(Bulgarian Academy of Sciences)

Deliverable 3.1: Learning environments

The learning environments developed within the *InnoMathEd* project include didactic scenarios and computer applications. The corresponding materials in Bulgarian together with the materials prepared for the workshops with teachers within the I*nnoMathEd* project are published on the site of the Mathematics and Informatics Education Department of the Institute of Mathematics and Informatics, BAS:

http://www.math.bas.bg/omi/InnoMathEd/index.htm

InnoMathEd – Innovations in Mathematics Education on European Level

Education and Culture DG
Lifelong Learning Programme

Inno MathEd
Innovations in Mathematics Education on European Level

Начало

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За проекта

"InnoMathEd - Innovations in Mathematics Education on European Level" (Иновации в образованието по математика на европейско ниво) е проект, в който участват десет партньори от осем европейски държави. Програмата е насочена към развиване на ключови компетентности у учениците и на способността им да използват ИКТ в процесите на изучаване на математиката.

Проектът е финансиран с подкрепата на Европейската комисия.

Новини

• Нов сайт - *юни 2010*
• Добавени нови документи - *октомври 2010*

Основни събития

• 5-6 септември 2009 г. - Семинар за обучение на уч - София, България
• 7 април 2010 - Национален семинар по проекта
• още ...

Home | Участници | Събития | Ресурси | Галерия

(NB The site has undergone several versions so far and is being dynamically updated taking into account the suggestions of the project-team members and the needs of the teachers having participated in the workshops).

3.1.1 Didactic scenarios

The didactic scenarios are available in English and/or in Bulgarian as follows:

In English:

- **Geometric transformation (**with *GeoGebra*) – developed by Dessislava Dimkova
  Reflection, Dilation, Inversion, Rotation, Fagnano's Problem
  http://www.math.uni-augsburg.de/de/prof/dida/innomath/internals/Learning_Environments/Bulgarian_Academy_of_Sciences/
- **Reflecting on the reflection** (with Elica *Geomland*) − developed by Evgenia Sendova and Pavel Boytchev
  http://www.math.bas.bg/omi/docs/Reflecting_on_the_reflection.pdf

- **Explorations with configurations of colored unit cubes (**with the Elica application *Cubix Editor***)** **-** developed by Toni Chehlarova and Evgenia Sendova

  http://www.math.uni-augsburg.de/de/prof/dida/innomath/internals/Learning_Environments/Bulgarian_Academy_of_Sciences/Explorations_with_Cubix_configurations.pdf

- **Nets** (with the Elica application *Origami Nets*) **-** developed by Toni Chehlarova and Evgenia Sendova

  http://www.math.uni-augsburg.de/de/prof/dida/innomath/internals/Learning_Environments/Bulgarian_Academy_of_Sciences/Nets.pdf

- **Rotational symmetry** (with *GeoGebra*) - developed by Evgenia Sendova and Toni Chehlarova

  http://www.math.uni-augsburg.de/de/prof/dida/innomath/internals/Learning_Environments/Bulgarian_Academy_of_Sciences/
  http://www.math.bas.bg/omi/docs/RotSymmetry/Scenario_ceilings.html

- **There are enough rotational solids to go around (**with the Elica applications *Potter's Wheel, Math Wheel, Bottle Design*) - developed by Toni Chehlarova and Evgenia Sendova

  http://www.math.uni-augsburg.de/de/prof/dida/innomath/internals/Learning_Environments/Bulgarian_Academy_of_Sciences/Rotational_Solids.pdf

In Bulgarian:

- **Geometric transformation** (with GeoGebra) – developed by Dessislava Dimkova

  - **Reflection**   http://www.math.bas.bg/omi/docs/ReflectionBG/GeoTransf.html
  - **Dilation**   http://www.math.bas.bg/omi/docs/DilationBG/GeoTransf.html
  - **Inversion**   http://www.math.bas.bg/omi/docs/InversionBG/GeoTransf.html
  - **Rotation**   http://www.math.bas.bg/omi/docs/RotationBG/Start_Rotation.html

- **Layers in configurations of colored unit cubes** (with the Elica application *Cubix Editor*) **-** developed by Toni Chehlarova and Evgenia Sendova

  http://www.math.bas.bg/~omi/docs/sloeve-cubixBG.pdf

- **Nets of Archimedean and Platonic solids** (with the Elica application *Origami Nets*) developed by Toni Chehlarova and Evgenia Sendova

  http://www.math.bas.bg/omi/docs/arh-tela-BG_JT.pdf

- **There are enough rotational solids to go around** (with the Elica applications *Math Wheel, Bottle Design*) - developed by Toni Chehlarova and Evgenia Sendova

  http://www.math.bas.bg/~omi/docs/rot-kompozBG.pdf

- **Elementary and basic constructions – 7. grade**

  http://www.math.bas.bg/omi/docs/EOPBG/

- **Teach=learn (or the role of the teachers)** (by E. Sendova – ppt in Bulgarian)
  http://www.math.bas.bg/~omi/docs/Inno_math_seminar1.pdf

- **Pick's theorem** (written by Sava Grozdev and Toni Chehlarova)
  http://www.math.bas.bg/omi/docs/PICK-BG.pdf

- **Cartesian coordinate system, 6. grade (**with *GeoGebra***) -** developed by Toni Chehlarova
  http://www.math.bas.bg/omi/docs/DEKART_6_BG/

- **Experiments with compositions of congruences (**with *GeoGebra*) - developed by Toni Chehlarova
  http://www.math.bas.bg/omi/docs/Proizvedenie_na_ednakvostiBG/

- **The congruences as compositions of relections (**with *GeoGebra*) - developed by Toni Chehlarova
  http://www.math.bas.bg/omi/docs/Ednakvosti_chres_simm2_BG/

- **Problems and games with matches** (with *GeoGebra*) **-** developed by Toni Chehlarova
  http://www.math.bas.bg/omi/docs/Kibr_klechki_BG/

- **Miniatures for the beginners in *GeoGebra*** developed by Evgenia Sendova and Toni Chehlarova
  http://www.math.bas.bg/omi/docs/Miniatures/Miniaturi_%20GeoGebra.pdf

- **How to make a dynamic ruler -** developed by Evgenia Sendova and Dessislava Dimkova
  http://www.math.bas.bg/omi/docs/Miniatures/Dividing_segment.pdf

The series of scenarios on the geometric transformations *Reflection in line*, *Dilation* and *Inversion* was developed based on the methodology presented in Bulgarian mathematics textbooks for the secondary school dating from 20 years ago. The aim has been to present the material in a "classical" way, but using the potential of the new information technologies in terms of visualization, interactivity and dynamic explorations:

A fragment by the *Inversion* scenario

The scenarios for working in 3D with the *Elica* (**E**ducational **L**ogo **I**nterface for **C**reative **A**ctivities) applications dealing with *rotational solids, nets of Archimedean and Platonic solids, layers in configurations of colored unit cubes* were developed with the idea that the exploratory style should be promoted as early as possible. Other scenarios meant for the junior-high school make use of *GeoGebra - Cartesian coordinate system, Problems and games with matches*, *Elementary and basic geometric constructions.*

3.1.2 Elica based virtual models

Virtual models **o**f mechanical devices for drawing mathematical curves are developed by Pavel Boytchev and uploaded in YouTube ElicaTeam channel at

http://www.youtube.com/user/ElicaTeam

and the project website:

http://www.math.uni-augsburg.de/de/prof/dida/innomath/internals/
Learning_Environments/Bulgarian_Academy_of_Sciences/

Virtual Mathematical Mechanisms (23 models in 2009; 39 models in 2010)

Snapshots of the virtual mathematical mechanisms

Non-mathematical models of conical sections (5 models in 2010)

These applications represent virtual experiments that students can replay at home with real objects. The figures below demonstrate various constructions of conic sections by using the light

cone of a torch and the shadow of a ball. The constructions are related to mathematical theorems thus made more approachable for the students.
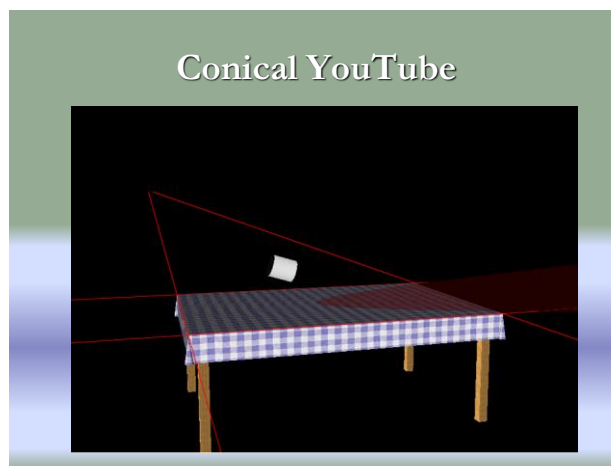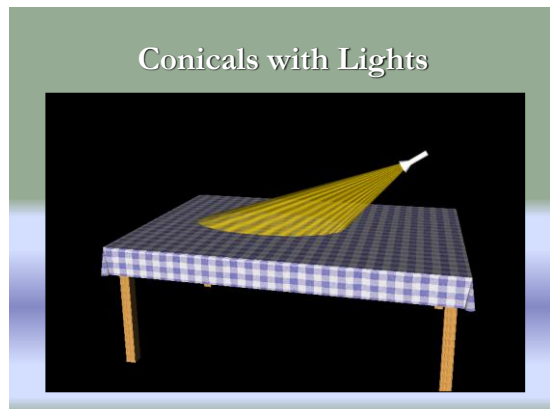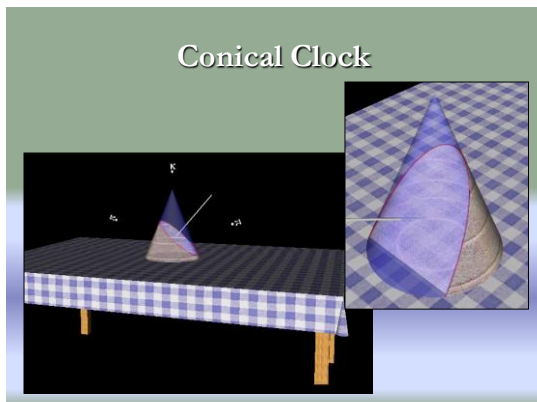
Snapshots of the *Conicals* computer applications

Models of mathematical theorems (2 models in 2010)

The Pythagoras theorem

An interactive dynamic bidirectional representation of the **Pythagoras theorem** is built using the *Elica* system. The proof is based on a sequence of 7 phases showing how the two smaller squares can be cut into pieces and how to regroup the pieces back to form the biggest square (see the figure above). The bidirectional feature means that one can start from the last phase and go backwards proving that the biggest square can be decomposed and recomposed into the two smaller ones. The interactive illustration is 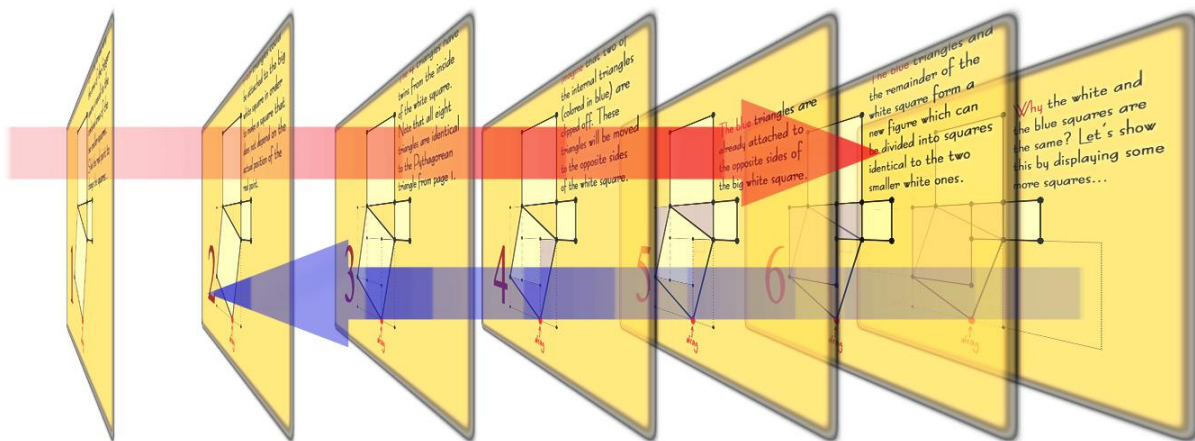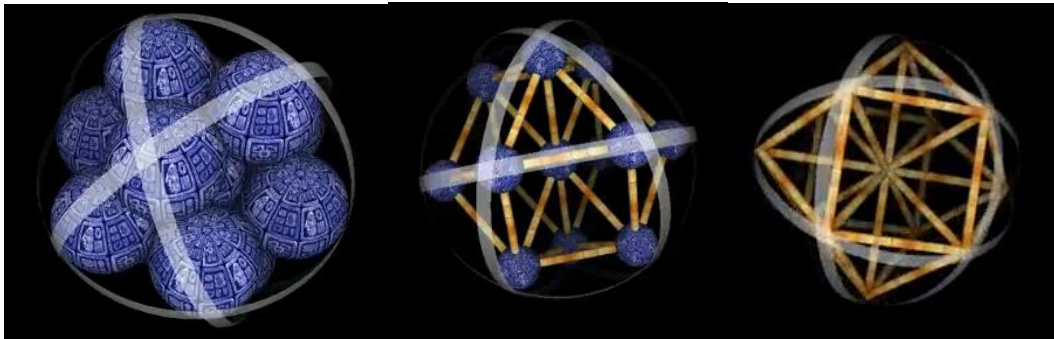constructed with one free parameter – an angle. The availability of this parameter defines a level of one-degree freedom. This freedom is utilised by the student who can modify the angle (and the whole illustration) at any phase.

The Kissing spheres theorem (http://www.youtube.com/watch?v=XlLMYlNz0n0)

Snapshots of *The Kissing spheres theorem*

How a sphere can *kiss* (i.e. osculate) twelve spheres at the same time? All 12 spheres are placed along 3 rings. Each ring contains 6 spheres and each sphere belongs to two rings. The sphere in the center (the 13th one) is the sphere that touches 12 spheres.

Kissing spheres theorem is an interactive *Elica* application enabling students to inspect a virtual 3D construction by rotating it in the space. They can change the radii of the spheres and study the inherent structure of the solution.

Other mathematical models (7 models in 2010)

- *Spherical Coordinate System* (http://www.youtube.com/watch?v=MiQWcD25K0w)

- *Polar Coordinate System* (http://www.youtube.com/watch?v=OmWEq930z34)

- *Cartesian Coordinate System* (http://www.youtube.com/watch?v=ylLR6gLRm-Y)

- *Topology "A hole through a hole in a hole"*
  http://www.youtube.com/watch?v=NuULlOCy4ps)

- *Sprinkler* (http://www.youtube.com/watch?v=CKSXVkjntXg)

- *Wet Wheel* (http://www.youtube.com/watch?v=Y2pujOMQJcg)

- *Normal distribution graph* (Galton board)
  (http://www.youtube.com/watch?v=PfCCf7b6Vyo)

The Equation Balance
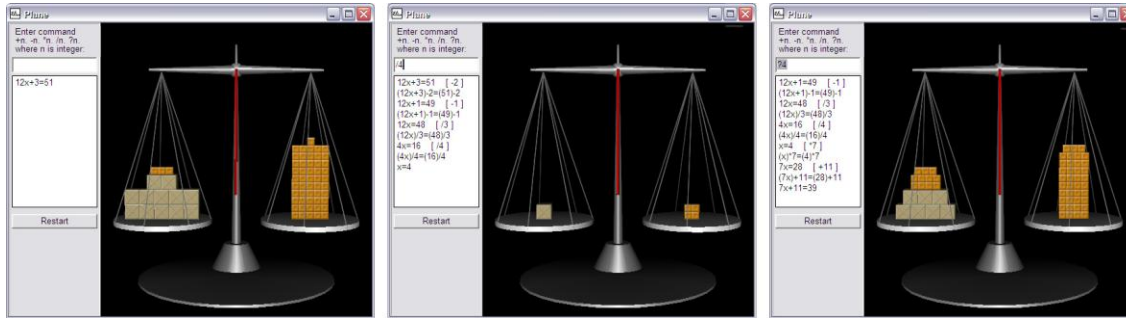
Solving linear equations by virtual balance

This *Elica* application features scales with two types of objects – golden bars (units) and boxes (variables) representing a linear equation. The figure above shows snapshots from the application. The first one is an initial problem – the left plate has 12 boxes and 3 bars, the right one has 51 bars. This setup is equivalent to the equation $12x+3=51$. By changing the number of objects and keeping the balance at any time, it is possible to reduce the number of boxes and bars to a minimum – 1 box and 4 bars. This configuration is shown in the middle image and it corresponds to the solution $x=4$. Finding this solution is not the end of using the application. The solution $x=4$ is an equation by itself, so it can be modified by the same rules. In this way, any future modification will actually generate a new equation. The right image in the figure above shows the new equation $7x+11=39$, which has the same solution $x=4$.

3.1.3 Educational movies

Ellipces

A mathematical movie about ellipses is produced in 2010. The movie represents seven methods of drawing ellipses.
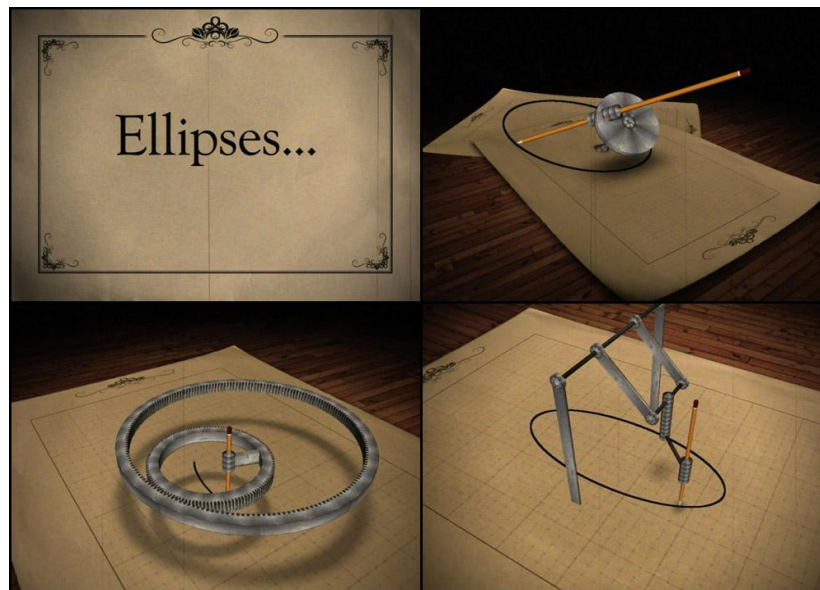
Snapshots of the movie *Ellipses*

Technical parameters: HD720p, 1000x720, 4 min, 24 fps, 5760 frames, 0.3 GB

The movie is available online at

http://www.youtube.com/watch?v=1v5Aqo6PaFw, and

http://www.youtube.com/watch?v=Q-2_WhwDhjw

It was presented at:

- Courses *Computer Environments and Languages for Education* and *Computer Graphics*.

- Conferences: Constructionism 2010 (France), HCICTE 2010 (Greece); EL 2010 (Lathvia), S3T 2010 (Bulgaria), ARIO 2010 (Bulgaria), Tech 2010 (Bulgaria)

- National IT Olympiad 2010 (Bulgaria)

A Journey in the Mandelbrot set

This film is based on the exhibition "Seduction". It blends computer-generated images from the Mandelbrot set with digital photographs from the real life.

Techical specification:HD720p, 1280x720, 5 min, 25 fps, 7500 frames, 1.5 GB

- Online access:

- 640x360 http://www.youtube.com/watch?v=93akxnQ1xxw

- 800x450 http://www.youtube.com/watch?v=J_0mXNy_ec8

- 1280x720 http://www.youtube.com/watch?v=JGxbhdr3w2I

- **Awards**: First place in Spring 2010 Fractal Art Competition, section Fractal Movies, organized by FractalForums.com.

Presented:

in Paris (Constructionism 2010), London, Berlin (Science Days Berlin-southwest), national conferences in Bulgaria. Snapshots are included in the open-air exhibition on *Computer Art* in Sofia.

An open-air exhibition on computer art in Sofia with snapshots from the film
A Journey in the Mandelbrot set

3.1.4 Video-films from the teachers presentations

A film about our trainee in-service teachers presenting their project was made at the workshop in Dimitovgrad (see **Deliberable 5.1.5**) The data were used to analyze typical problems the teachers come across while working with dynamic software (*GeoGewbra* and *Elica* in this case) and to discuss them with the next groups of teachers to undergo our workshops.

Deliverable 4.1: Short description of university courses involving *InnoMathEd* ideas

4.1.1 Course title: *Computer Environments and Languages for Education*

Lecturer: Pavel Boytchev

Type of the course: elective (for the 3d year students specializing in mathematics and informaitcs) and compulsory (for the rest of the specialties from 2-4th year)

Level of course: MSc for pre-service teachers in mathematics and informatics

Objective of the course: Introduction to the *Elica* computer system and its facilities for creating animated models and simulations in support of mathematics and informatics teachers. A special attention is paid to the Object-oriented programming and to the development of interactive applications.

Assessment methods: defense of a course project

Prerequisites: Minimal experience with programming, affinity to 3D graphics

Course contents:

- **Part 1** (basic themes): The history of *Elica*, introduction to the environment, basic types of data. Processing of numbers, words, and lists. Variables. User-defined command and functions. Operators.

- **Part 2** (advanced themes): Object-oriented programming. Graphical objects. Size, orientation, material, lighting texture, properties, application and parameters, local and global characteristics.

- **Part 3** (practical themes): Virtual environments. Course projects. Various topics related to the course projects.

The projects assigned to the students include interactive models of theorems or mathematical concepts in the spirit of the *InnoMathEd* project.

The Software applications from 3.1.2 have been demonstrated to the above course as follows:

       a. Year 2009-2010, elective, 27 students

       b. Year 2009-2010, compulsory, 27 students, future teachers

       c. Year 2010-2011, elective, 16 students

       d. Year 2010-2011, compulsory, 35 students, future teachers

> e. Year 2010-2011, extramural, 6 students, future teachers

In addition, these applications have been used in the Master courses *Computer Graphics in Education*:

> f. Year 2009-2010, compulsory, 10 master students

The educational projects of students in 2009-2010 are already received and evaluated; and 28 of them are related to the *InnoMathEd* project. Others are related to computer science and other non-mathematical sciences.

1. Thales theorem
2. Foerbach theorem
3. Thebault theorem
4. Inversion
5. Sine theorem
6. Pythagoras theorem
7. Middle segment in a triangle
8. Interactive trigonometric circle
9. Center of weights
10. Inscribed and circumflexed circle
11. Inscribed polygon
12. Cone in a sphere
13. The theorem of the three cosines
14. Desargue theorem
15. Fibonacci numbers
16. Sphere and triangular pyramid
17. Volume of triangular pyramid
18. Volume of truncated cone
19. Polke-Schwartz theorem
20. Routh theorem
21. Volume and area of pyramid
22. Inscribed sphere in a pyramid
23. Right triangle in a circle
24. Atlas of mathematical theorems
25. Quadric equations with complex coefficients
26. Menelaus theorem
27. Volumes of cones and cylinders
28. Orthogonal projection of a cube onto rotating plane

The projects of students 2010-2011 will be evaluated in February and March 2011.

4.1.2 Course title: *Dynamic software in the mathematics education*

Lecturer: Toni Chehlarova

Type of the course: elective (for graduate (BS) students at the Faculty of mathematics and informatics, Plovdiv university "Paisii Hilendarski" :
- 30 students – correspondence studies (December, 2009)
- 15 students – regular studies (March-May, 2010)

The description and the content of the courses is available (in Bulgarian) at:

http://www.fmi-plovdiv.org/index.jsp?id=1023&ln=1

4.1.3 Course title: *Didactical means*

Lecturer: Toni Chehlarova

Type of the course: compulsory (for postgraduate (MS) students at the Faculty of mathematics and informatics, Plovdiv university   "Paisii Hilendarski" - 15 students, October 2010

The description and the content of the courses is available (in Bulgarian) at:

http://www.fmi-plovdiv.org/index.jsp?id=364&ln=1

http://www.fmi-plovdiv.org/news.jsp?ln=1&newsId=260&newsPageNumber=3&id=2&archive=1

There are two projects developed by students in the frames of *InnoMathEd :*

> 29. Area of quadrilateral (for 5. grade)
>
> 30. Locus of points (for 8. grade)

Final Report (part 2.1)
of the Bulgarian InnoMathEd team
(Bulgarian Academy of Sciences)

Deliverable 5.1: Short description of in-service teacher education activities

5.1.1 *InnoMathEd* Workshop for in-service teachers − Sofia, 4-5 September, 2009

After intensive correspondence with several dozens potential participants, 22 persons from 12 towns from various Bulgarian regions (Montana, Vidin, Shumen, Veliko Turnovo, Varna, Plovdiv, Troyan, Rousse, Lovech, Vratza, Gorna Oryahovitza and Sofia) were invited officially to take part in the first *InnoMathEd* workshop which was held at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS).

The materials for the workshop included a brochure of six exemplary didactic scenarios associated with specific computer environments – *GeoGebra, Elica Applications* (*Origami Nets, Cubix Editor, Math Wheel, Bottle Design, Potter Wheel*) and *Elica Geomland*. Three of the scenarios were in Bulgarian, the rest – in English.

The workshop was opened on **4th of September** by **Petar Kenderov** who presented the *InnoMathEd* Project to the participants and the guests (colleagues and a journalist from CIO-ICT media).

Animated moments from the workshop with teachers in Sofia

Then **Evgenia Sendova** talked about the **role of the teachers as partners of the students** in a research process rather than a reality-check of researchers. She emphasized on the importance of the computer environments of laboratory type for the math education.

After the lunch break **Dessislava Dimkova** presented her scenarios on *Geometric transformations* developed in *GeoGebra* based on ideas of the late **Ljubomir Davidov.** The participants were very active in trying out the presented problems and shared their personal experience with this environment.

The day ended with the presentation of **Toni Chehlarova** who presented the *Elica* applications for developing the spatial imagination of students based on the experience within the DALEST European project together with newly developed scenarios: *Nets of Archimedean and Platonic solids*, *Color arrangements in a cube*, *Going around rotational solids, Layers in configurations of colored unit cubes.*

The applications are easy to start working with and the teachers were able to concentrate on the very problems rather than worrying about technical details.

Albena Vassileva presented to the participants the site of the IMI's *Department of Mathematics and Informatics Education* and concentrated on publishing some of the InnoMathEd project materials in Bulgarian. The idea of having a mirror-site of the project with the evaluation forms in Bulgarian was found to be very appropriate.

The second day started with presenting the constructionist ideas in the context of *Comenius Logo.* **Evgenia Sendova** demonstrated the main principles behind the Logo educational philosophy and the participants experienced the "no ceiling, no threshold" one by themselves. Starting with the simplest polygons, they were able to implement the symmetry in terms of the Turtle geometry,

and finally to see how natural it is to introduce the notion of *fractals* and integrate it with the traditional curriculum of the school algebra.

The notion of *reflection* was introduced in the *Elica Geomland* by means of a procedure with the objects *point* and *line* as parameters and then generalized by modifying the original procedure.

Finally the floor was given to the teachers. Three of them shared their experience with enriching their math classes by means of *GeoGebra* (**Daniela Petrova**), *GeoNext* (**Angel Gushev**) and the TI graphic calculators (**Boriana Kujumdzhieva**) with censors.

During the last break the teachers were given a short inquiry:

1) *With a picture (figure) and one word try to describe your mood*.
(This question was based on previous very interesting experience with teachers and young students alike.)
2) *What was the most useful thing for you?*
3) *What were the main problems and difficulties you experienced?*
4) *What do you suggest for a better dissemination of the project materials in Bulgarian context?*
(This is related to the fact that a great part of our teachers are not fluent in using English.)
5) *What options do you see for applying in your work some of the learning environments considered at the workshop?*
6) *Do you have any ideas for a scenario of your own or a modification of the scenarios we considered at the workshop? In what computer environment?*
7) *Any suggestions for the next workshops?*

Many of the pictures comprised smiley faces and were combined with phrases such as *splendid, thank you, I am ready to try, sunny and joyful, intrigued, optimistic, good, festive, beauty – I am full of it, positive…*

It was with a great satisfaction that we would read the feedback of the teachers – not only in the inquiry forms, but also – in e-mails sent immediately after the workshops.

- *We are very grateful to the InnoMathEd project and its team of lecturers for loading us with energy and enthusiasm at the beginning of the school year and for offering us new challenges for creative work.*
- *We greet the idea for breaking the classical frames of the lesson and involving the students in active learning which would make them feel like explorers and discoverers and will change crucially their attitude to the math classes. Our main concern is the limited number of classes according to the syllabus of the professional schools.*
- *It is my first time to participate in such type of a course in which the lecturers "teach the way they preach"… My main problem is that you told us not to take notes but to work with you in parallel mode, so I would appreciate sending some materials refreshing our memory…*
- *Find attached my first attempt to show mathematics in action. I made it by the trial and error method and I liked it! It is not directly related to the traditional curriculum but I had fun!*
- *The two days of the workshop were full of novelties for me and this was the best part. Everything was at an accessible level and I was happy to create something on my own (however immodest this may sound) and this was something I wanted to have a long time ago!*
- *For me this course was like going back to Paradise.*

The most frequent recommendations dealt with:
- publishing all the materials also in Bulgarian
- the necessity of additional stimuli for the innovative teachers

- organizing travelling seminars in various regions
- periodical meetings for disseminating the good practices
- organizing small working groups which would develop specific topics
- envisaging more time for discussions
- organizing a competition among teachers/students for developing didactic scenarios on a specific topic; evaluation and dissemination of the best team products
- formulating open problems to be investigated by student teams in appropriate computer environments
- developing scenarios fitting the traditional curriculum
- structuring the scenarios on the project website according to the age of the students
- creating video-films of working with specific learning environments
- popularizing the Project by means of the national media with demonstrations
- teacher training on a large-scale in various towns with various teachers
- maintaining an active network among the workshop participants and the Project team
- publishing short tutorials in Bulgarian for working with *Geogebra*, GEONExT, etc.

The environments mentioned by the teachers as something they were ready to implement in their activities were: *GeoGebra, Elica, Geonext, Java* script.

The workshop was attended by the journalist Anelia Stoyanova from the CIO journal who interviewed **Petar Kenderov** about the most recent tendencies of using IT in education. The interview included information about the *InnoMathed* project and appeared in the September issue of the journal dedicated to the IT & Education.

5.1.2. Meeting of experts in mathematics – Banki, 24 September, 2009

Evgenia Sendova delivered a lecture on the project at a meeting of experts in mathematics, in Banki. As an immediate post effect we received invitations from 4 regions (Montana, Stara Zagora, Vratsa and Dimitrovgrad) to organize **specialized workshops on *InnoMathEd*** taking into account teachers working with students of different age groups.

In response of the invitation of the experts **Yordanka Elenkova (Montana)** and **Rumiana Nestorova (Vratsa)** the project team members **Dessislava Dimkova, Evgenia Sendova** and **Georgi Dimkov** went to these towns and delivered lectures on the *InnoMathEd* ideas implemented in the learning environments.

5.1.3 Workshop in Montana – 14-15 November, 2009

The participants of the workshop were **19 teachers** in mathematics, informatics and IT from the junior-high and the secondary school.

The audience in Montana getting ready to start their own dynamic explorations

5.1. 4 Workshop in Vratsa – 12-13 December, 2009

The participants of were also teachers (20) in mathematics, informatics and IT from the junior-high and the secondary school.

Active reflections in the workshop in Vratsa

The organization of both workshops was excellent. The participants showed a great enthusiasm although they had to sacrifice their weekend. Some of the teachers had to travel for more than 100 km till the place of the event.

5.1. 5 National Seminar on creativity and innovations in the mathematics education – 21-22 December, 2009

The first results of the teachers having participated in the workshops, were reported at a **National seminar on creativity and innovations in the mathematics education**. It took place at the Institute of Mathematics and Informatics, BAS. The teacher **Daniela Petrova (**from the German Language School, Sofia**)** presented her scenarios on graphs of quadratic functions **(**http://www.math.bas.bg/omi/InnoMathEd/archive.htm**)** which she had already tested with her students in the presence of interested colleagues. Other teachers having attended the seminar to share their experience with students from the junior high school and the secondary school were **Antoaneta Stoimenova** (from *St. Kliment Ohridski* school, in a small village near Veliko Tarnovo), the expert **Yordanka Elenkova** (Montana**)** (see the pictures below), and **Angel Gushev** (from the Mathematics High School from Veliko Tarnovo)

Teachers sharing their valuable experience

The impressions of the participants presented at the discussions show that the teachers are motivated to implement the *InnoMathEd* learning environments demonstrated at the National seminar. Furthermore, they are ready to enrich the existing data-base of didactic scenarios. The most often recommendations were related to the necessity of distributing the project materials in Bulgarian, and our team dedicated serious efforts in this direction. The teachers who had already started implementing the *InnoMathEd* ideas in their classes encouraged their colleagues to use innovative means and approaches in their work so as to show to their students the beauty of mathematics.

5.1. 6 Workshop "Teachers and students in the role of explorers" − 6-10. April, 2010

http://www.math.bas.bg/smb/2010_PK/tom/pdf/099-100.pdf

This workshop was organized in the frames of the 39th spring conference of the Union of the Bulgarian Mathematicians. The main ideas of the *InnoMathEd* project were presented by Petar Kenderov (http://www.math.bas.bg/smb/2010_PK/tom/pdf/063-072.pdf)

Professors, teachers, students – all had the chance of actively participating in the workshop by solving specific problems.

The goal was to involve the whole audience including the novices in using dynamic software..

Stimulating acts…

The audience of all ages had fun and acted in teams

D. Dimkova, B. Lazarov, A. Vassileva, T. Chehlarova, T. Terzieva, P. Boytchev presented various interesting ideas for using dynamic constructions.

Pavel Boytchev presenting his virtual mathematical mechanisms

Is it easy to represent a square by means of your hands only?

.

Sharing the first experience in the class settings and ideas for the future
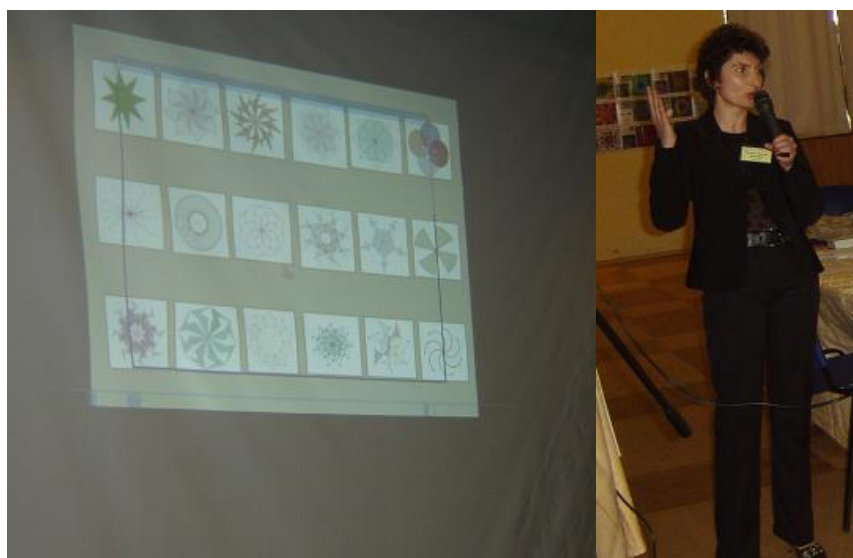
The teacher D. Kuncheva is proud of her students from a small village near Vratsa

The projects developed by the **high-school students Anton Belev, Kaloyan Buhovski** (with a mentor **Nikolay Dimitrov**, undergraduate from the Faculty of Mathematics and Informatics),

Rumen Dimov, and **Yanitsa Pehova** showed that the young researchers do not need additional motivations for using dynamic software – they are convinced in its applicability for solving complex problems and generating new hypotheses.
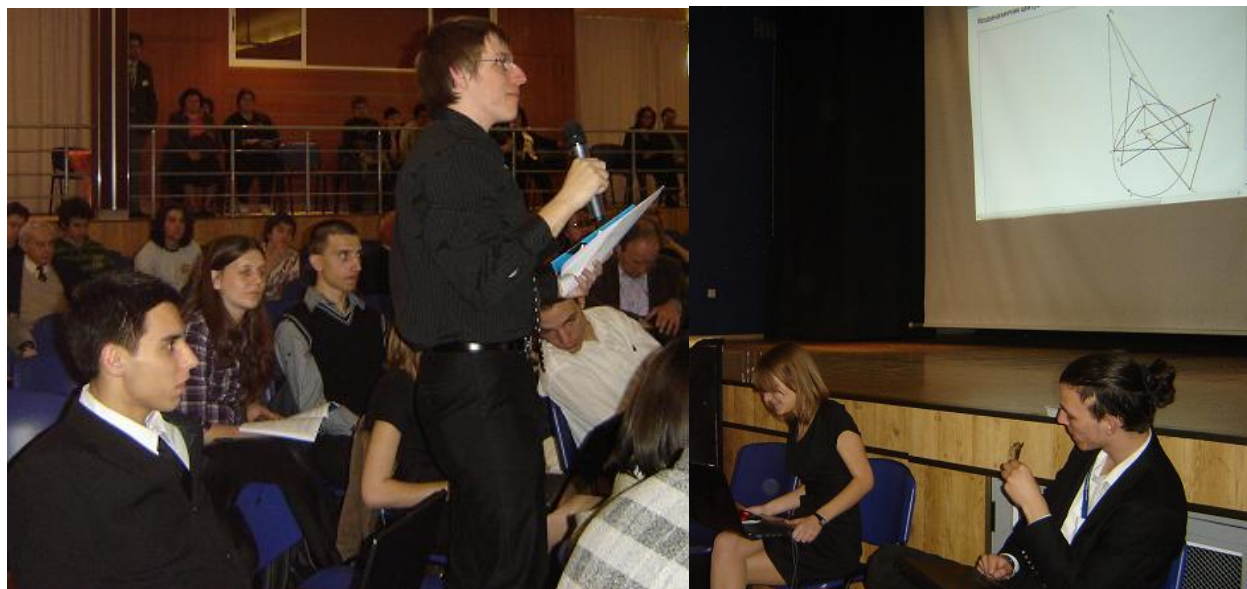
The students in action

Final Report (part 2.2)
of the Bulgarian InnoMathEd team
(Bulgarian Academy of Sciences)

5.1.7 Workshop in Dimitrovgrad − 19-23 April, 2010

From 19 to 23 April a workshop was organized with 17 teachers in mathematics and IT in Dimitrobgrad. The lecturers were **D. Dimkova, T. Chehlarova, B. Lazarov, A. Vassilieva, K. Chalukova, G. Kozhuharova.** The ideas of the *InnoMathED* project were presented together with the dynamic software GEONExT, *Geogebra*, *Elica* and didactial scenarios developed by the Bulgarian team. Various strategies for using the software in the compulsory and the elective education were demonstrated, as well as ideas for extra-class activities. The course ended with the teachers presenting their projects and scenarios for future activities.

The team work was very fruitful

The ideas behind *InnoMathEd* were presented to the local community via the mass media, the cable TV, visiting the mayor of Dimitrovgrad, the educational policymakers, as well as the regional expert of Haskovo and the expert in mathematics and IT education in Haskovo municipality. Here are some original ideas from the teachers' projects:
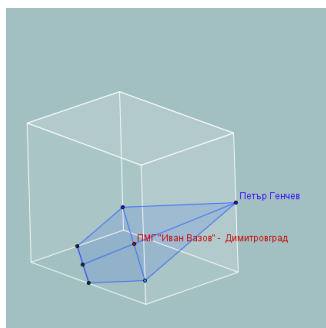
Interesection of a cube with a plane, Peter Genchev
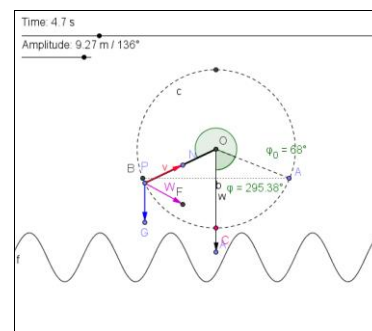
Mathematical pendulum, Georgi Gochev

When asked if they were ready for implementing the *InnoMathEd* learning environments in their practice the teachers not only gave positive answers, some of them already had started doing so…

5.1.8 Workshop in Pravets − 1- 2 May, 2010

The teachers participating in the course were 19. The first day the lecturer was **D. Dimkova**, who presented an introductory course on *GeoGebra* based on the geometric transformation scenarios. **B. Lazarov** lead the course during the second day and presented the notion of *parabola* from different perspectives. In spite of some technical difficulties the course was successful and the participants worked with enthusiasm.

5.1.9 Workshop in Shumen − 4-8 July, 2010

Moments from the meeting in the principal's office and the worksop

The workshop was organized in collaboration with Prof. **Rusanka Petrova** - Dean of the Faculty of Mathematics and Informatics at the University of Shumen "Episcope Konstantin Preslavski" , (also a municipality counselor), **Madlen Hristova** - the regional expert in mathematics education, and the principal of the Mathematics and Science High School "Nancho Popovich". The participating teachers were 32 and the workshop took place in the Mathematics and Science High Chool. **P. Kenderov** presented the ideas and goals of the project, then **T. Chehlarova, B. Lazarov** and **A. Vassileva** acquainted the teachers with the dynamic mathematics environments developed by the Project team. An interesting contribution to the project ideas was the lecture of **Radostina Encheva** from the Shumen University on dynamic constructions in the context of *Centers in a triangle*.

5.1.10 Workshop in Blagoevgrad − 14-15 July, 2010

This workshop was organized by **Elena Karashtranova** in the South-West University of Blagoevgrad "Neofit Rilski" and lead by **Maria Brauchle**. The participating teachers were 22 coming from the town of Blagoevgrad and its region. The dynamic software used was GEONExT. The interest of the teachers to learn how to use this environment was great although they were aware of the problems ahead of them in terms of limited computer resources in the schools and the amount of time needed for developing their own scenarios.

5.1.11 Workshop in Blagoevgrad − 8-9 September, 2010

This workshop was a follow up of the one held in Blagoevgrad in July (5.1.10). The schedule included some more advanced topics, e.g. creating a background of the plotting plane by means of a picture; integrating dynamic constructions with a slide show presentation, and combining GEONExT, HTML and Java Script. In addition, the teachers **Nadka Sheinkova** and **Zdravka Hajiivanova** presented their scenarios on functions. The scenario on trigonometric functions developed by Ms. **Hajiivanova** in the frames of the workshop was found to be directly applicable in the class setting by the rest of the participants.

5.1.12 Workshop in Uzana − 23-24 August, 2010

Uzana (a Mountain place around Gabrovo) provided very good conditions for a creative and *dynamic* atmosphere during a workshop with 17 teachers and 20 students. This time we wanted to get directly the impressions of students when working with dynamic mathematics environments. The mixture of students and teachers gave rise to hot discussions on the potential of the dynamic software for generating interesting mathematical results including mathematics "never done before" (students' phrasing). The challenges the teachers often face when working with mathematically gifted students were one of the topics the present teachers shared with professional mathematicians form the Institute of Mathematics and Informatics, BAS. After intensive discussions (formal and informal alike) the dynamic software was found to be an appropriate solution when working with students at different level of understanding and motivation (from the lowest to highest), as well as a real support for the creative endeavors of the teachers themselves.

Dynamic presentations

The discussions enriched the "catalog of ideas"
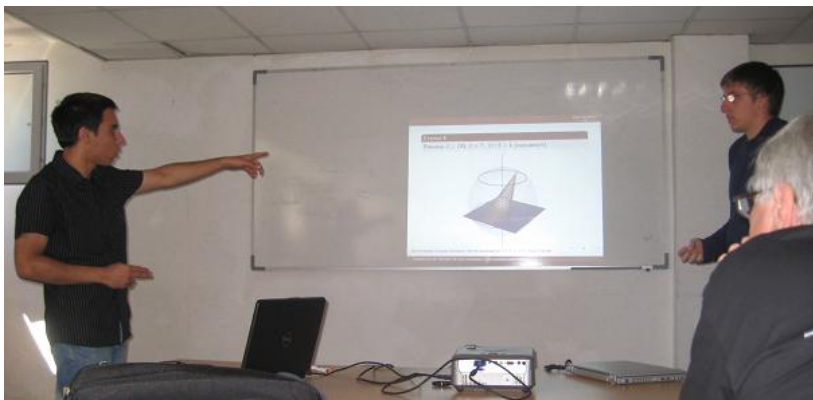
Moments from the teachers' and students' presentations of projects

Filling the evaluation forms was taken very seriously by teachers and students alike

5.1.13 Workshop in Shumen − 2-4 October, 2010

This workshop was a follow up of the one in July (see 5.1.9) – the idea was for us, the Project team, to check the progress of the teachers with using dynamic environments in a class setting and to present the most recent learning environments developed by the team.

Our impressions after the workshop were that the prevailing part of the teachers are still seeing the dynamic software as a means for visualization of mathematical facts (envisaged within the curriculum) rather than for organizing experiments and explorations, for discovering patterns, for making conjectures. Thus, it is still a challenge for us as promoters of the inquiry-based learning to encourage them to apply the full potential of the dynamic mathematics software in support of that style in their class setting.

Discussing the success of the teachers and the students in an IT environment

The following **5 workshops which have taken place in Sofia** are briefly described below – the first two of them are dealing with teacher training, the other three – with the teachers' presentations of their own project ideas and their first impressions from implementing the *InnoMathEd* approaches in a class setting. The local organizers were **Zdravka Jeleva** (administrative support) and **Daneil Kamburov** (technical support).

5.1.14 Workshop in Sofia − 7-8 September, 2010

Two workshops (each with 20 participants) were organized in Sofia with the organizational collaboration of the experts in mathematics **Tania Stoeva** and **Penka Ivanova**. The participating teachers were split according to the age of their students – junior-high school and secondary school. The lecturers (**T. Chehlarova**, **D. Dimkova** and **E. Sendova**) were rewarded with the enthusiasm of all the participants (some of them experienced mathematics teachers but lacking experience not only with dynamic software, but with ICT, in general).
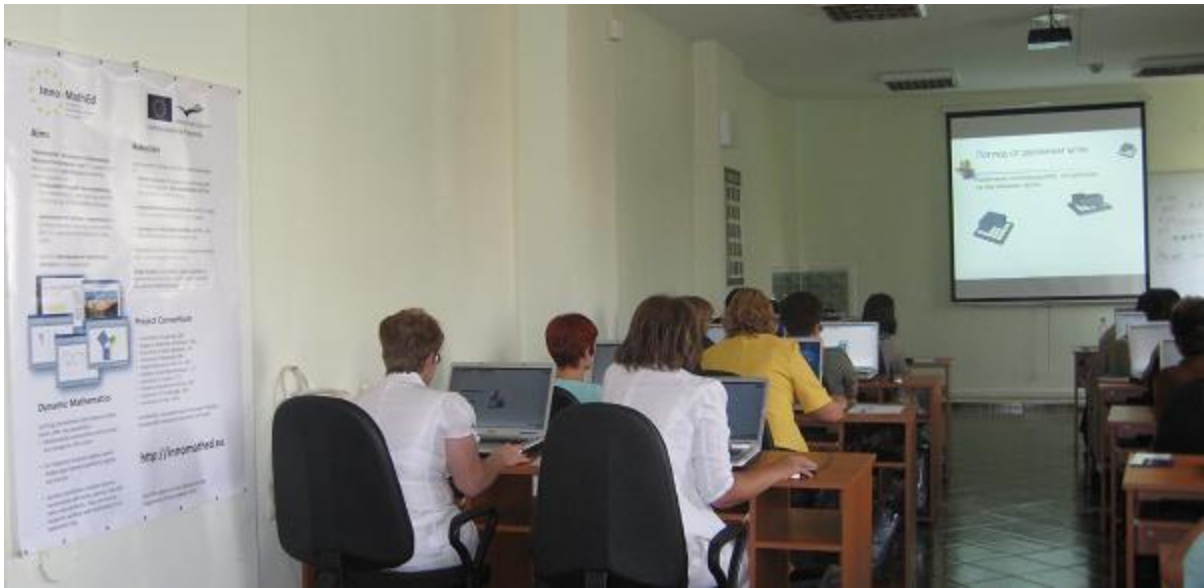
"Well known" topics presented dynamically found a warm reaction

5.1.15 Workshop in Sofia − 9-10 September, 2010

The teachers are enthusiastic in the role of students

5.1.16 Teachers' presentation of their own scenarios – Sofia, 23 October, 2010

The presentations of teachers demonstrated that they had already gained self-confidence to use the dynamic mathematics software with their students. Discussing a scenario proposed by a colleague working on the same topic proved to be very fruitful – the teachers realized that the richness of solutions and approaches is something they should also encourage in their classes.
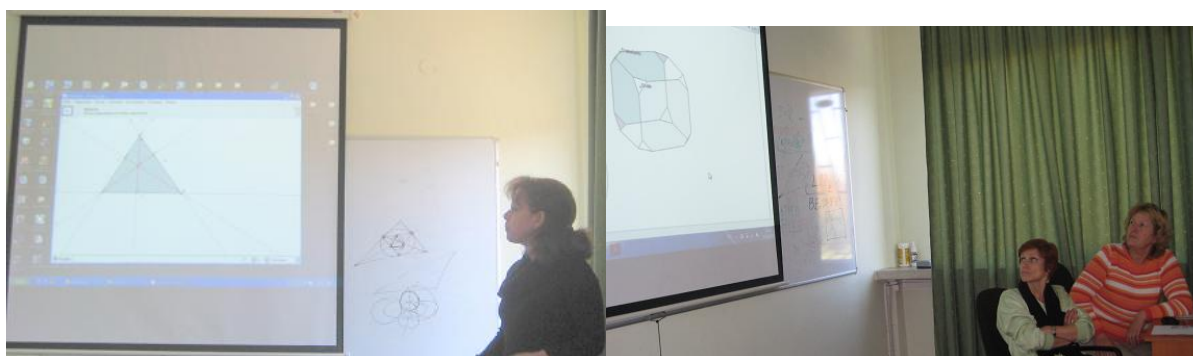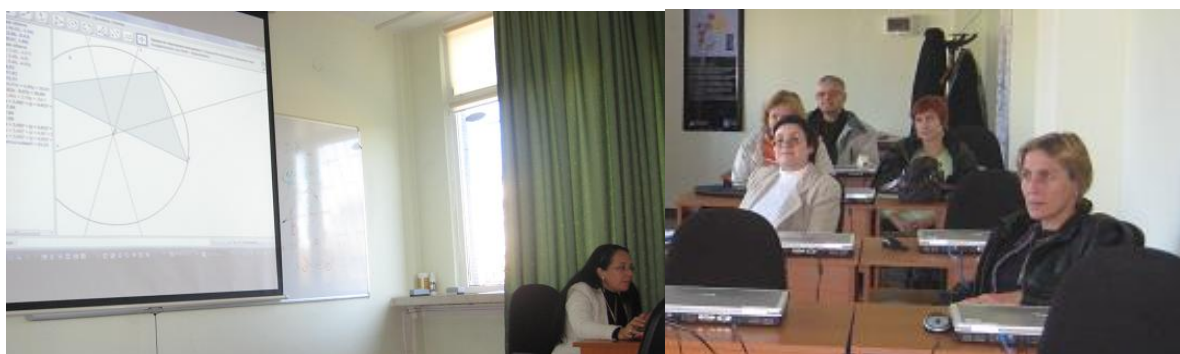
Various ideas for implementing dynamic mathematics in class had to be defended in front of appreciative and demanding audience

The received sertificates were well deserved!

It is after defending their projects that the teachers would receive a certificate for finishing the *InnoMathEd* course.

5.1.17 Teachers' presentation of their own scenarios – Sofia, 6 November, 2010
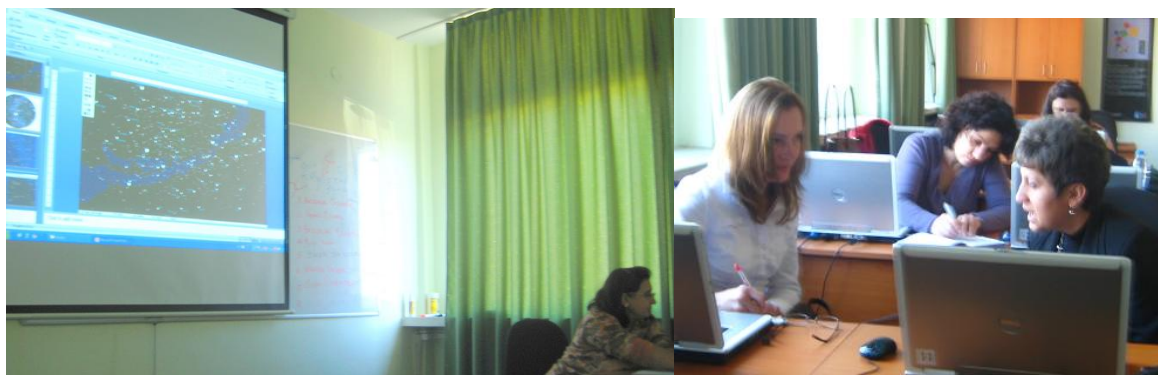
An interesting perspective from an astronomer's point ov view

5.1.18 Teachers' presentation of their own scenarios – Sofia, 8 November, 2010

My project might be modest but I am ready to try it with my students!

Final Report (part 2.3)
of the Bulgarian InnoMathEd team
(Bulgarian Academy of Sciences)

5.1.19 Workshop in Plovdiv − 25-26 September

This workshop was organized with the collaboration of Prof. **Asen Rahnev** - the Dean of the Faculty of Mathematics and Informatics (FMI), Plovdiv University "Paisii Hilendarski". The announcement on the FMI website about the course attracted several colleagues working with pre-service students there.

http://www.fmi-plovdiv.org/news.jsp?ln=1&newsId=260&newsPageNumber=3&id=2&archive=1

http://www.fmi-plovdiv.org/GetResource?id=325
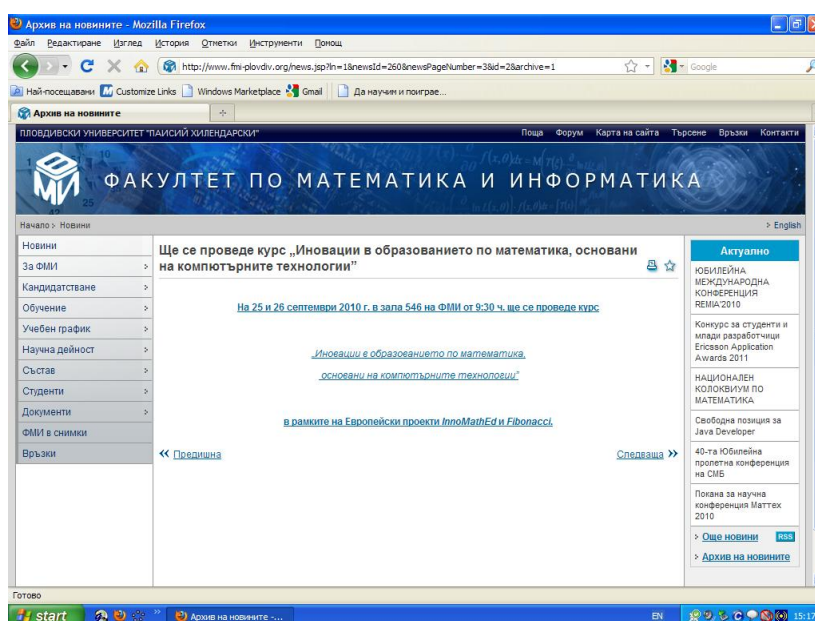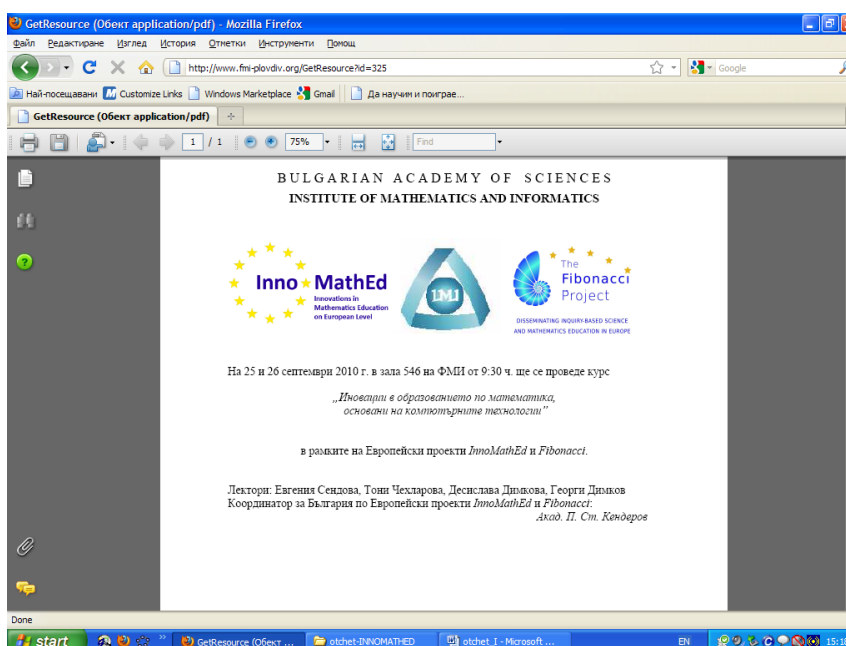
The participants were 22 teachers and university lecturers. Especially meaningful for the project team was the opinion of the teachers having participated in a previous educational experiment with implementing software for enhancing the 3D imagination of 5-6 graders. Thus, their work was one of partners of the Project team, not just a "reality-check" for researchers…
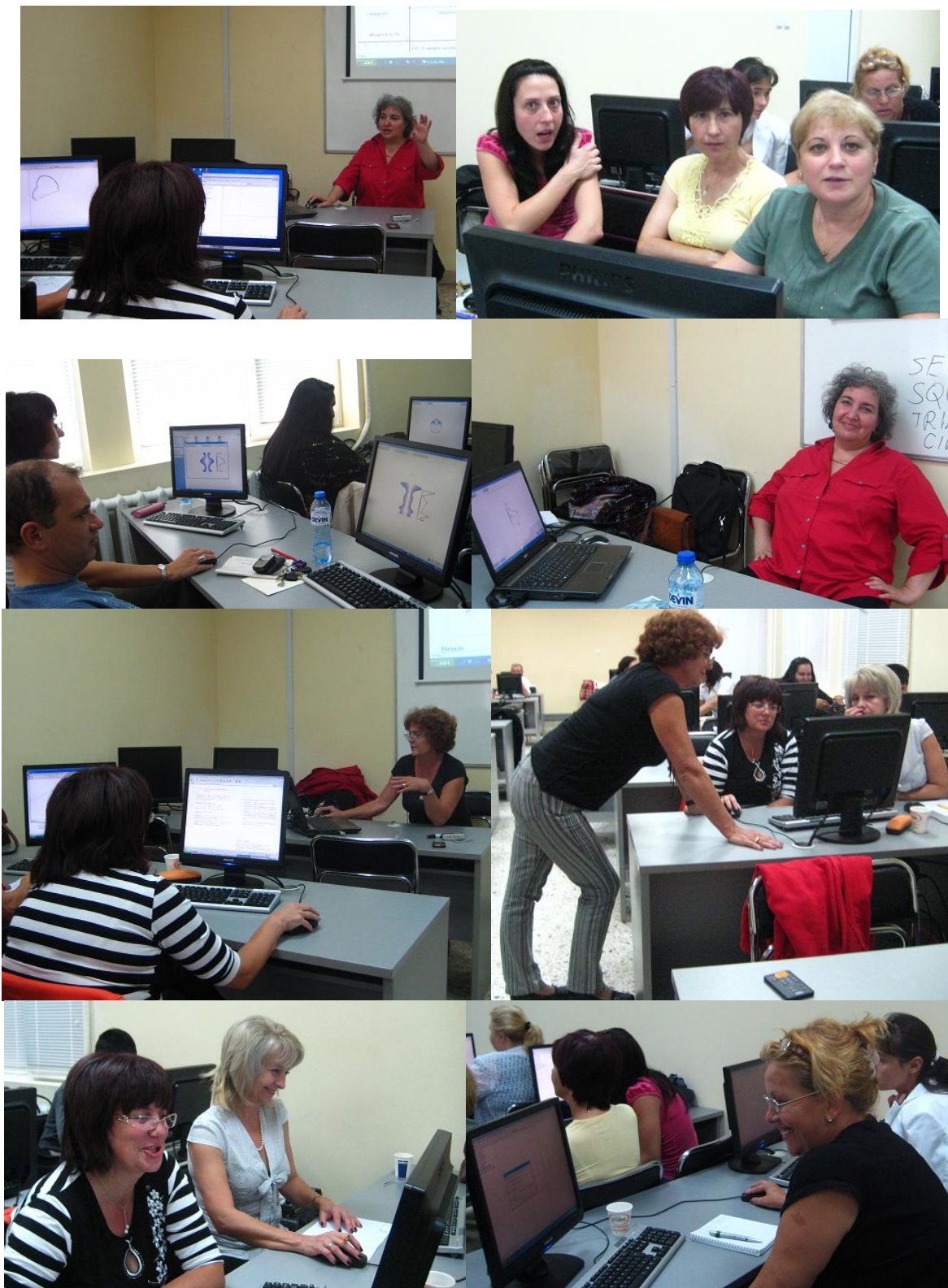
To be in *the skin* of students is always motivaitng

5.1.20 Workshop in Stara Zagora − 26-28 October, 2010

This workshop was organized in collaboration with **Evtim Kunchev**, a chair of the Stara Zagora's section of the Union of Bulgarian Mathematicians, **Nikolina Kulicheva** – principal of the Science and Mathematics High School *Geo Milev,* and **Alexander Burov** – a senior regional expert in mathematics. The participating teachers were distributed in groups depending on their professional duties during the day. An interesting detail was the fact that **Pavlin Peev, the teacher of Pavel Boytchev** (the Elica's developer), was this time in the role of a student in the course.

The experience from previous courses let us encourage the teachers with different levels of technical skills to explore the dynamic software environments on their own. Thus, even teachers with relatively modest technical skills could gain self-confidence and show promising results.

Opening the seminar

Using only their hands as "dynamic tools" for making squares

Creating virtual models of a school trophy
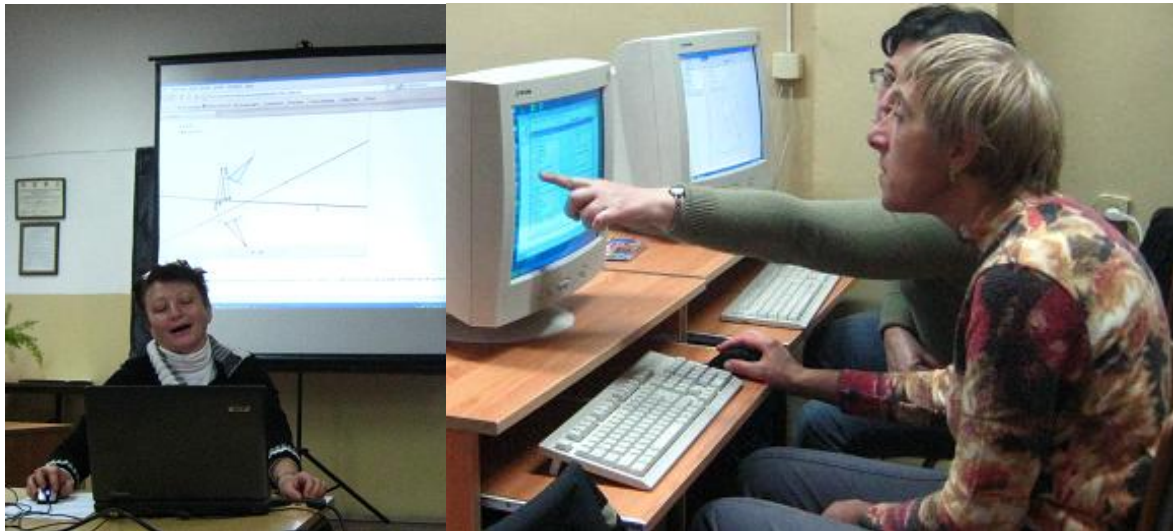
Proud of the newly acquired skills

At the end of the course we asked the teachers to share with a sentence their impressions. Here follow some answers:

- *We leave inspired and enthusiastic*
- *Now I can say: I CAN do it!*
- *I am loaded with ideas!*
- *It was interesting and new to me – I learned so many things…*

Deliverable 5.2, 5.3:
Learning environments and evaluation data from teachers

The first feedback in terms of development of educational materials by teachers was by **Daniela Petrova, a math teacher** in mathematics and informatics at the German Language High School No 91 in Sofia. She sent us **five activities in *GeoGebra* for exploring quadratic function and quadratic inequalities** and after getting our feedback she experimented them with her 10th grade students. These modules are uploaded on the Project website and will be further experimented with other Bulgarian teachers. Here is a fragment of her letter to the organizers of the workshop:

> *It was already during the workshop that I figured out how to enrich my math lessons on quadratic function and quadratic inequalities. And even then I managed to do something in this direction. I am very grateful to the project for making it possible for teachers like me to enrich the traditional teaching style.*

Later on she described her impressions from the *InnoMathEd* course and her class activities in an article which was published in the *Mathematics and Informatics* journal (see **Deliverable 8.6**)

Angel Gushev, a **math teacher** from Turnovo, is developing educational modules on geometric transformation with both GEONExT and *GeoGebra* together with students of his. He has sent them in Bulgarian and English to be published on the project website. He also published an article about his class experience with GEONExT and *Geogebra* (see **Deliverable 8.6**)

After the courses we gave the evaluation forms translated in Bulgarian which were later on translated in English and entered electronically by **Victoria Naumova**. The total number of evaluation forms filled in by **Bulgarian teachers is 186**, by **high school students – 21**, and by **undergraduates - 3**.

Final Report (part 3)
of the Bulgarian InnoMathEd team
(Bulgarian Academy of Sciences)

Deliverable 8.6: Web-based networks with other projects

8.6.1 Important related links

Here are some **links** to current and recent projects related to the activities of *InnoMathEd:*

- *Math to Earth Project* - http://www.km.fpv.ukf.sk/math2earth/

- *Unite project***:** http://www.unite-ist.org/

- *ShareTec* - *Sharing Digital Resources in the Teaching Education Community* Project http://www.share-tec.eu/

- http://math.fau.edu/yiu/geometry.html (a link suggested by Dr, Gunetcho Skordev, Bremen University after the workshop in Ohrid)

- *Innovative Didactics via Web Based Learning* (IDWBL) http://wad.fmi.uni-sofia.bg/wad/

Links to *InnoMathEd* are envisaged to be inserted in the websites of *ShareTec* and *Innovative Didactics via Web Based Learning*

8.6.2 Papers referring to the *InnoMathEd*

- **Boytchev, P**., Kamenova, S., **Sendova, E**., Stefanova, E., Kovatcheva, E., Nikolova, N., "IT for Innovative Educational Environments: Exploring, Authoring and Programming", In Proceedings of International Conference for Interactive Computer Aided learning – *The Challenges of Life Long Learning* ICL 2009, Ed. Michael E. Auer, Kassel University Press, Villach, Austria, 2009, pp. 434-444, ISBN 978-3-89958-481-3

- **Boytchev, P.,** *Science and Art*. In Proceedings of the National conference "Education in the information society", Plovdiv, Bulgaria, 2010, pp. 46-53, ISSN 1314-0752

- **Boytchev, P.,** *The Mandelbrot Set Fractal as a Benchmark for Software Performance and Human Creativity"*, In Proceedings of Constructionism 2010 and the 12th European Logo Conference – Constructionist approaches to creative learning, thinking and education: Lessons for the 21st century, Eds.: J. Clayson

- **Boytchev, P., Sendova, E**., Kovatcheva, E., *Geometry of Motion*, In Proceedings of the International Conference on e-Learning and The Knowledge Society (e-Learning 2010), Riga, Latvia, 26-27 August, 2010, ISBN 978-9984-30-181-5, pp.203-208

- **Boytchev, P**., *Bridging the Gap between Abstract Math and Reality*. In Proceedings (CD) and Book of Abstracts (hardcopy) of the 7th Pan-Hellenic Conference with International Participation "Information and Communication Technologies in

Education" (HCICTE 2010), Ed. Athanassios Jimoyiannis, Korinthos, Greece, 2010, Abstract: p. 23, Paper: pp. 103-110, ISBN 978-960-88359-5-5, ISSN 1792-5010

- **Boytchev, P.,** *Technology Enhanced Learning (in Italian:"Apprendimento doppiamente potenziato dalla tecnologia")*, Journal of e-Learning and Knowledge Society (Je-LKS), Focus on: Artificial Intelligence and e-Learning, Vol. 6, n. 2, May 2010, pp. 13-26, ISSN: 1826-6223, eISSN: 1971-8829, Italian e-Learning Association, Italy

- **Boytchev, P.,** *Re-experiencing Engineering Inventions within a Modern Virtual Environment"*. The 2nd International Conference Software, Services & Semantic Technologies (S3T 2010), Varna, Bulgaria, September 11-12, 2010

- **Boytchev, P.,** *Science and art: a story of mutual inspiration*, In Mathematics and Mathematical Education 2010, Proceedings of 39th spring conference of the Union of Bulgarian Mathematicians, Albena, Bulgaria, 2010, pp. 39-48, ISSN 1313-3330

- **P. Boychev,** *Reincarnations of Mandelbrot set*, Mathematics and informatics, vol. 4, p. 21

- **Chehlarova, T., E. Sendova**, Unity and Variety in the Context of Symmetry – a Dynamic Approach. In: Mathematics Education with Technology -Experiences in Europe, Tamara Bianco. Volker Ulm (Ed.) University of Augsburg, Augsburg, 2010. ISBN 978-3-00-032628-8

- **Chehlarova, T., E. Sendova**, Stimulating different intelligences in a congruence context. In: Constructionist approaches to creative learning, thinking and education: Lessons for the 21st century. Proceedings for Constructionism 2010. The 12th EuroLogo conference. 16-20 August, Paris, France. 2010. ISBN 978-80-89186-65-5 (Proc) ISBN 978-80-89186-66-2 (CD)

- **Chehlarova, T.,** Explorations in mathematics education by dynamic software simulations, in Education in the information age, Plovdiv, 27-28 May, 2010, pp . 205-212 , ISSN 1314-0752

- **Chehlarova, T., D. Dimkova, E. Sendova,** *Are five days enough? What about five hours?* Mathematics and Informatics, vol. 3, p. 3 (in Bulgarian)

- **Chehlarova, T., E. Sendova,** Variety in the similarities, Mathematics and Informatics, vol.4, p. 3. (in Bulgarian)

- **Chehlarova, T.**, Experiments with compositions of congruences in 8th grade, in the proceedings of *Reflection and synergy in mathematics education*, Bachinovo, 2010

- **Chehlarova, T., D. Dimkova, E. Sendova,** Air trackers with *GeoGebra (Mathematical fairytale about the falling ladder),* Mathematics and Informatics, vol. 6, 2010, p. 3. (in Bulgarian)

- **Dimkova, D., E. Sendova,** About the newest (but not forgetting the not so new) in mathematics education, Mathematics and Informatics, vol.1, p. 3. (in Bulgarian)

- **Dimkova, D.,** *20 Years Later – Inquiry Based Learning Again* , in: Mathematics Education with Technology -Experiences in Europe Tamara Bianco. Volker Ulm (Ed.) University of Augsburg, Augsburg, 2010. ISBN 978-3-00-032628-8

- **Gushev, A.,** *Dynamic mathematics for everybody*. Mathematics and Informatics, vol.4, p. 15 (in Bulgarian)

- **Kenderov**, P. Innovations in mathematics education: European projects *InnoMathEd* and *Fibonacci.* Proc. of the 39th Spring Conference of the Union of Bulgarian Mathematicians, Albena, Bulgaria, 2010. pp 63-72

- **Kenderov, P**., Higher Ability Students and Inquiry Based Learning in Bulgaria – the Role of European Projects *InnoMathEd* and *Fibonacci,* Proceedings of the 6-th Conference of the World Federation of National Mathematics Competitions (WFNMC), July 25 - 30, 2010. Riga

- **Kuncheva, D**., *Let's not be shy to experiment*, Mathematics and Informatics, vol.6, 2010, p. 3. (in Bulgarian)

- **Pehova, Y.,** *The history of a project (... or how GeoGebra helps in difficult situations),* Mathematics and Informatics, vol. 5, pp. 2-13

- **Petrova, D.,** *Dynamics on the screen and among the students*, Mathematics and Informatics, vol.3, p. 11-14 (in Bulgarian)

- **Sendova, E., Boytchev, P**., Stefanova,E., Nikolova, N., Kovatcheva, E., *Creating a Natural Environment for Synergy of Disciplines,* in the Proceedings of 4th European Conference on Technology Enhanced Learning, Nice, France, September 29–October 2, 2009 - Learning in the Synergy of Multiple Disciplines, Eds.: U. Cress, V. Dimitrova, and M. Specht, EC-TEL 2009, LNCS 5794, pp. 549–555, 2009. © Springer-Verlag Berlin Heidelberg 2009

- **Sendova, E.,** Nikolova, N, **Boytchev,P**., Stefanova,E., Kovatcheva, E., *Harnessing ICT for building a creativity-based society*, IT STAR Conference on *ICT Skills, Education and Certification: the Multi-stakeholder Partnership*, 27 – 28 November, 2009, Villa Aurelia –Via Leone XIII, 459 - 00165 Rome, Italy

- **Sendova, E., T. Chehlarova**, Explorations with rotational solids. 6th Mediterranean Conference on Mathematics Education 22 – 26 April 2009, Plovdiv, pp. 259–268, ISBN 978-9963-9277-9-1

- **Sendova, E., T. Chehlarova**, Explorations around the Rotational Solids. In: Mathematics Education with Technology − Experiences in Europe Tamara Bianco. Volker Ulm (Ed.) University of Augsburg, Augsburg, 2010. ISBN 978-3-00-032628-8

- **Sendova, E., T. Chehlarova,** *Breathing life back into inquiry-based learning of mathematics*, Mathematics and Informatics, vol. 5, p. 3-10 (in Bulgarian)

- Stefanova, E., Nikolova, N., Kovatcheva, E. **Sendova, E.** *et al.*, *From a "Flap of a Butterfly Wing" to the "Wind of Change",* in: Constructionist approaches to creative learning, thinking and education: Lessons for the 21st century. Proceedings for Constructionism 2010. The 12th EuroLogo conference. 16-20 August, Paris, France. 2010. ISBN 978-80-89186-65-5 (Proc) ISBN 978-80-89186-66-2 (CD)

Deliverable 8.7 Conference Contributions

8.7.1 *InnoMathEd* Workshop organizised in the frames of the MASSEE congress, Ohrid, 16-20 September, 2009

The workshop was included in the Congress program and a team of 5 members was in charge of preparing materials for the participants and presenting the *InnoMathEd* project:

The materials for the workshop included a brochure of didactic scenarios associated with specific computer environments – *GeoGebra, Elica Applications* (*Origami Nets, Cubix Editor, Math Wheel, Bottle Design, Potter Wheel*) and GEONExT. All the scenarios were in English (20 copies, 50 pages each, printed in colors).

It was decided to dedicate the workshop on the memory of the late Prof. Ljubomir Davidov, a well-known Bulgarian mathematics educator. The learning environments dealing with geometric transformations in *GeoGebra* were developed based on his draft materials.

The atmosphere of the workshop was very warm although the room was not conventional for that sort of events

The workshop was opened by **Petar Kenderov** who presented the *InnoMathEd* Project to the participants – mathematicians and math educators from Bulgaria, Republic of Macedonia, Serbia, Croatia, Romania, and Germany.

Then **Evgenia Sendova** introduced the teaching strategies envisaged in the project emphasizing on the *learning by doing* principle.

Dessislava Dimkova presented her scenarios on *Geometric transformation* developed in *GeoGebra* focusing on symmetry and inversion.

Angel Gushev demonstrated possible approaches to solving a math contest problem by means of geometric transformations in GEONExT.

Toni Chehlarova ended the presentations with showing didactical scenarios and possible approaches to problem solving and problem formulating in *Elica* applications for developing the spatial imagination of students. The topics were: *Nets of Archimedean and Platonic solids*, *Color arrangements in a cube*, and a newly developed scenario: *Going around rotational solids*.

Unfortunately the conditions were not very favorable for hands-on activities since there was only one computer for demonstrations. Still the audience was very active in making interesting proposals and providing food for reflection.

The last day of the Congress the Bulgarian project team attended the workshop lead by **Matthias Brandl**. He encouraged the audience to use their laptops and thus achieved a good interactivity. The materials spread by Matthias Brandl were very helpful. At the end of the workshop the Bulgarian teacher **Angel Gushev** presented learning environments **developed by students under his guidance** by means of GEONExT. The two workshops gave a very good idea of the project as a whole and launched very fruitful ideas for further explorations.

In the workshop organized by Matthias Brandl (left) there was a chance for Angel Gushev (right) to present learning environments based on GEONExT

8.7.2 The Interactive Computer Aided Learning Conference ICL 2009 - September 23-25, 2009, Villach, Austria

Pavel Boytchev presented the paper *IT for Innovative Educational Environments: Exploring, Authoring and Programming* (in co-authorship) at the Conference ICL2009 September 23 - 25, 2009 Villach, Austria, in which the authors talk about some novel *Elica* applications in the context of the *InnoMathEd* project. The paper is published on the project website and in the proceedings of the conference (see 8.6.2):

Pavel Boytchev demonstrating an innovative view on recursion

8.7.3 National conference *Information technologies for a new education*, 30 October, 2009, Sofia

Petar Kenderov and **Evgenia Sendova** presented the ideas and the current contributions of the *InnoMathEd* project as well as the progress of implementing them in a Bulgarian context. Their presentation was entitled: *About some innovations in the mathematics education – the InnoMathEd project in a Bulgarian context*. The authors offered leaflets about the project to the participants and encouraged them to get in touch for further information and collaboration.

8.7.4 Conference on *ICT Skills, Education and Certification: the Multi-stakeholder Partnership*, 27 – 28 November, 2009, Villa Aurelia – Via Leone XIII, 459 – 00165, Rome

http://www.starbus.org/ws4/ws4.htm

Evgenia Sendova submitted (in co-authorship) the paper *Harnessing ICT for building a creativity-based society* and presented it at the Conference on *ICT Skills, Education and Certification: the Multi-stakeholder Partnership*, 27 – 28 November, 2009, Villa Aurelia –Via Leone XIII, 459 - 00165 Rome, Italy. The paper reflects the authors' experience in the context of several European projects related to IT in education including *InnoMathEd*.

8.7.5 Sixth Congress of the World Federation of National Mathematics Competitions - Riga, Latvia, July 25 -30, 2010

http://nms.lu.lv/WFNMC

Petar Kenderov delivered a talk entitled: *Higher Ability Students and Inquiry Based Learning in Bulgaria – the Role of European Projects* **InnoMathEd** *and* **Fibonacci.**

8.7.6 The annual National Seminar on e-Learning *New Programs and Projects, Best Practices, Innovations and Trends*– 9 June 2010, Sofia.

 (see http://www.bvu-bg.eu/index.php?Clip=sem9)

 P. Kenderov and **E. Sendova** delivered a talk on the projects *InnoMathEd* and *Fibonacci* under the title *European Projects for Implementation of Innovations in Mathematics Education*

- **D. Dimkova** presented a talk on *Geometrical Transformations - Innovative Presentation of Classics.*

8.7.7 XXXIX Spring Conference of the Union of Bulgarian Mathematicians, Albena, Bulgaria, 2010.

Talks were delivered by:

- **P. Kenderov**: *Innovations in mathematics education: European projects InnoMathEd и Fibonacci*
- members of the Bulgarian *InnoMathEd* team in the frames of the workshop dedicated on the project.

Albena Vassileva presenting another dynamic representation of a parabola

(The details of the workshop are provided in **Deliverable 5.1.6)**

8.7.8 National Conference *Education in the information age*, Plovdiv, 27-28 May, 2010

E. Sendova delivered the talk (on invitation) *Challenging the limitations instead of limiting the challenges in the IT education*, which was published in the conference proceedings.

8.7.9. Seminar of CIESE, Stevens Institute of Technology (NJ, USA), 5 August, 2010

Evgenia Sendova delivered an **invited talk** *On some innovations of mathematics education – a European perspective* to the CIESE team on recent European Projects related to the IT in mathematics education (including *InnoMathEd*):

(CIESE (http://www.ciese.org/engineering/stevens.html) stands for *Center for Innovation in Engineering and Science Education.*)

8.7.10 Constructionism 2010. *Constructionist approaches to creative learning, thinking and education: Lessons for the 21st century* Paris, France. 17-24 August, 2010

http://my.aup.edu/conference/2010/constructionism

T. Chehlarova and **E. Sendova** delivered a **plenary talk** on *Stimulating different intelligences in a congruence context.*
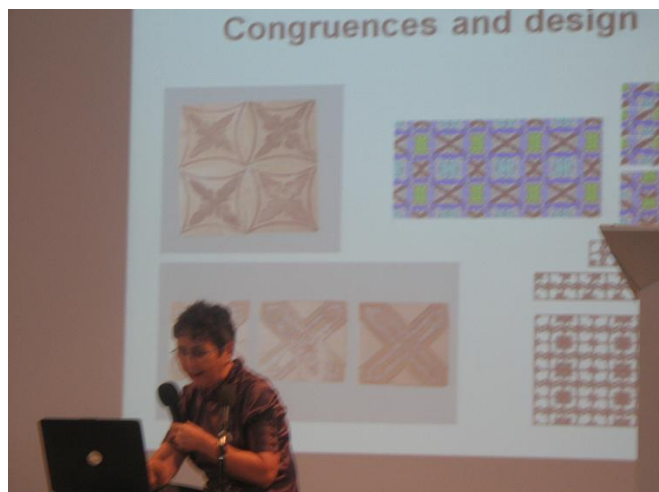
An *InnoMathEd* scenario in the core of a plenary talk – Constructionism 2010

Constuctionists from all over the world − Paris, France. 17-24 August 2010

***Dynamic mathematics* on the scene – the performers are mathematics educators −
Constructionism 2010**

E. Sendova (with co-authors) presented a talk: *From a "Flap of a Butterfly Wing" to the "Wind of Change".*

A team of 4 lecturers and 4 students presented their experience within a course on ICT in education embracing the team-work ideas of *InnoMathEd* − Constructionism 2010

8.7.11 VIII Mathematics Conference , Nitra, 14 September, 2010

http://www.nmk2010.fpv.ukf.sk/

Evgenia Sendova – delivered a talk**:** *The beauty in mathematics and the mathematics in the beautiful,* on projects developed by in-service and pre-service teachers under the title's theme.

8.7.12 A scientific conference *Traditions and Innovations in Education* – 24-25 September, Dimitrovgrad

Katia Chalukova, the chair of the Dimitrovgrad section of the Union of the Bulgarian Mathematicians, opened the Conference and delivered an overview of the teachers' achievements in the frames of the *InnoMathEd* course in the town (see 5.1.7).

A conference on traditions and innovations is a good platform for presenting the achievements of teachers from Dimitrovgrad who participated in *InnoMathEd* courses

The audience embraced young and older teachers – equally excited about innovations in the mathematics education

Evgenia Sendova delivered a talk on invitation: *Innovations in the mathematics education – the European projects InnoMathEd and Fibonacci.* Then teachers from the junior-high and the secondary school reported on their most recent experience in implementing *InnoMathEd* learning environments.

Teachers from Dimitrovgrad presenting their *InnoMathEd* inspired projects

The *InnoMathEd* scenario on modeling rotational solids by means of the Elica application *Potter's wheel* was an example of a natural integration of real with virtual objects since the participants had the chance of observing a workshop on pottery for young students organized in the local museum. This lead to a discussion of the various possibilities of personalizing the scenarios offered by the Project team.

Not only virtual, but tangible rotational solids were available to the conference participants

8.7.13 Regional Autumn Conference on IT in Education, Burgas, Sunny Beach, 5-7 November, 2010.

T. Chehlarova and **E. Sendova** presented *InnoMathed* inspired ideas on *Stimulating the creativity of the students*. These ideas were put in the context of teaching IT as a separate object for the junior-high school the emphasis being on *IT as a means for expressing oneself.*

Some impressions of the Bulgarian Project team

In a *nut shell*, the impressions of the Bulgarian members of the Project team are that the prevailing number of teachers getting familiar with the *InnoMathEd* are motivated to try out the ideas presented at the workshops and to be involved in implementing, evaluating and possibly modifying the proposed scenarios, as well as in developing some new ones.

The discussions (in formal and informal setting) showed that the question: *What software should be used in the math classes and to what extent?* doesn't have a unique answer. It depends on multiple factors including the level of the students and their mathematics and informatics culture. In any case it is the nature of the mathematical problems that matters, and the environment has just an auxiliary role.

The specific options of the computer environment should be used only after the learners have acquired the standard methods for geometric constructions. Thus the learners would realize that the basic geometric constructions or graphs are not a matter of simply pressing a button or selecting a menu option.

In the case of the Bulgarian teachers, the native language is an essential issue and we have published most of our Project materials in Bulgarian.

The activities within the InnoMathEd project have been reported to the traditional seminars of two departments of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences - *Operations Research* and *Education in Mathematics and Informatics (EMI)*. A guest lecturer of the *Didactical Modeling* seminar at EMI, **Boyko Banchev**, made a *A Brief Tour to Dynamic Geometry Software* (http://www.math.bas.bg/omi/DidMod/Articles/BB-

dgs.pdf) and provoked a vivid discussion among the participants which is expected to continue with sharing personal experience.

The Department of *Education in Mathematics and Informatics* at IMI has been appointed to work part of the time on the project (starting on October 15, 2009) by a special order of the IMI Director, Acad. Stefan Dodunekov. Evgenia Sendova (in charge of the Department) has been distributing the tasks among its members, which include

- studying the existing learning environments, posted by the project partners;
- developing new ones;
- implementing some selected scenarios in the teacher training;
- giving (virtual or face to face) feedback to the teachers having attended the Project workshops in relations to their ideas, problems and requests.

The first presentations of teachers' projects for dynamic mathematics scenarios have demonstrated that teachers still see it as a great challenge to develop original resources (since it is *difficult and time consuming*). They express great interest in having access to ready-made dynamic constructions, or even to complete lessons that could be used directly or after small modifications. At the same time, the teachers express their appreciation of scenarios relating mathematics in a natural way to other fields – art, dance, nature, architecture, etc.

This explains the direction of our efforts in two main streams:
- embedding lessons from classical math textbooks in HTML structures
- enriching the existing curriculum with scenarios for extracurricular activities

In addition, we encouraged the teachers to use the homework for organizing specific stages of the inquiry-based process. To work actively and on their own is crucial for the learners to achieve the goals behind the inquiry-based learning strategies. The mere demonstrations would never accomplish these goals. The time dedicated to exploring, observing, conjecturing, generalizing and studying special cases will not only lead to a better understanding of the subject matter, but will enhance the thinking and the imagination of the learners, the acquirement of research skills and contribute to a relevant attitude to science in general.

In conclusion, the development of resources making use of dynamic constructions is just an element of the **dynamic mathematics education**. The discoveries, the representations and the implementation of mathematical objects and ideas could be related to the enhancement of the creative potential of learners by providing appropriate conditions and our on-going efforts are in this direction.

The achievements of the Bulgarian Project team are due to the professionalism and the enthusiastic attitude of all its members – scientists, administrative and technical specialists alike.

The financial team of our Project team

These achievements wouldn't be possible without the on-going support by the Project Coordinator - the University of Augsburg's team, who provided an excellent professionalism and leadership.

From the kick-off meeting till the end of the Project the coordinators acted as genuine leaders

Even though it is hard for a single organization or a research team to do very much to affect the overall system directly, addressing the problems of the current educational system with *InnoMathEd*-like strategies will hopefully influence the situation in the whole country.

Further activities

The Bulgarian *.InnoMathEd* team will continue its efforts even after the completion of the Project by:

- organizing an annual seminar on inquiry-based mathematics learning within the framework of the *Spring Conference of the Union of the Bulgarian Mathematicians*;

- conducting competitions for teachers developing educational environments for inquiry-based mathematics education;

- developing new educational environments

- publishing (on a regular basis) related materials on the pages of the most popular mathematics magazines and on the web-site of IMI-BAS (via a bulletin)

- moderating a forum on implementing the inquiry-based learning in the mathematics and informatics education

- publishing a book (in Bulgarian) with examples of best practices in using dynamic constructions for inquiry-based learning of mathematics

- working intensively in preparing the next generation of lecturers promoting and disseminating the *InnoMathEd* ideas country-wide.