Dynamic tessellations based on regular hexagons and their transformations

Let us illustrate the idea of dynamic tessellations by transforming a hexagon tile in a tessellation tile of a new shape. We construct the regular hexagon **ABCDEF** as a partial case of the *polygon* tool, select a point **G** on its side**AB** and point **M** from its inside.We will transform the hexagon by *cutting out* the triangle **GBM** and *gluing* it to a neighboring side, e.g. **BC** (Fig.1). We are using BC with the idea of making the common vertex **B** as a center of rotation.

We then construct the images of **G** and **M** under rotation with a center **B** and angle of $-120°$**,** connect them and get a newly shaped tessellation tile.

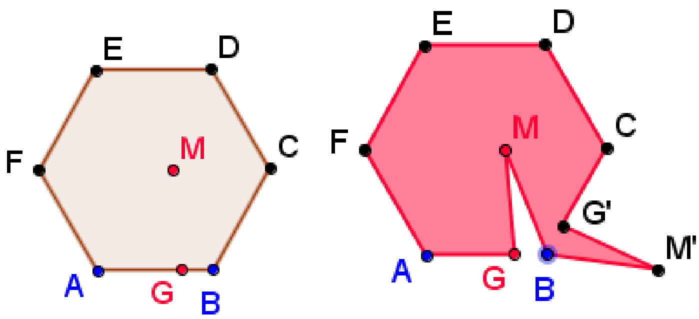


Fig.1 Transforming a regular hexagon in a newly shaped tessellation tile.

Now we could tessellate the plane with this tile by means of the same rotation, and then — by translation (Fig. 2):

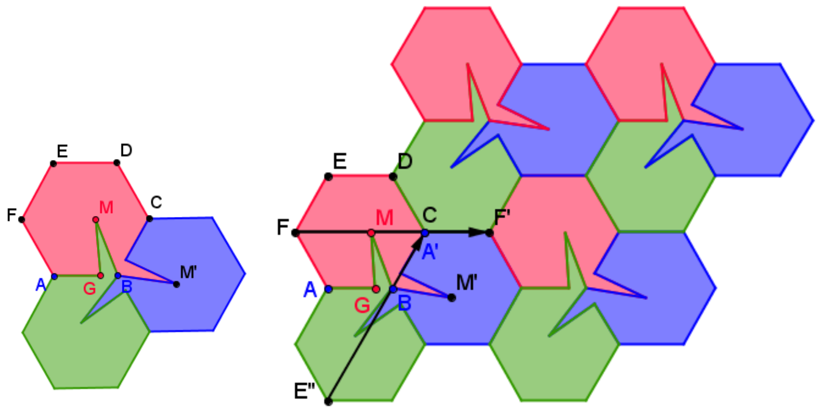


Fig. 2 Tessellating the plane with the new tile by means of rotation and translation

We have hidden some of the point and/or their names for convenience. We could use other ways for connecting the tiles but it is sufficient to move only **G** and **M** to modify the tessellation.

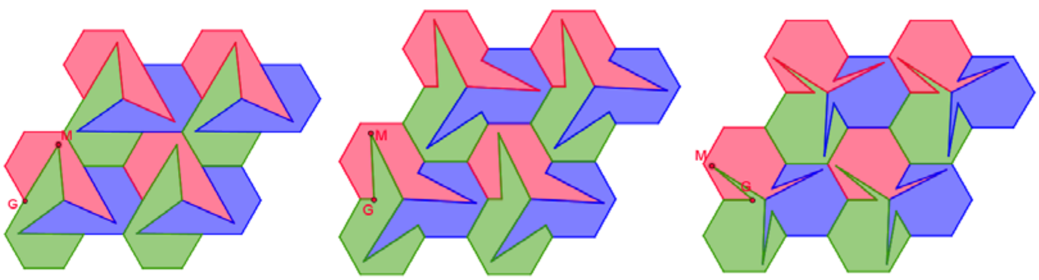


Fig. 3 Modifying the tessellations by moving two points only.

The tessellations in Fig. 3 are a result of moving the point **G** and **M** *by hand.* **W**e could *animate* the construction automatically by constructing sliders for the movement of **G** and **M**. The point **G** is the intersection point of the segment **AB** and the circle with a center **A** and radius **r** (a variable). To assure the existence of the point **G**, the upper limit of **r** is chosen to be close to (less than) the length of **AB.**
The point **M** will also lie on a preliminary constructed object, dependent on variables. In our case **M** is an intersection point of the circle with a center **H** (the center of the regular hexagon) and radius **k** (a variable), and the second ray of the angle with a vertex **H**, a measure α (a variable), and a fixed first ray (all the variables are represented by the sliders in Fig. 4).

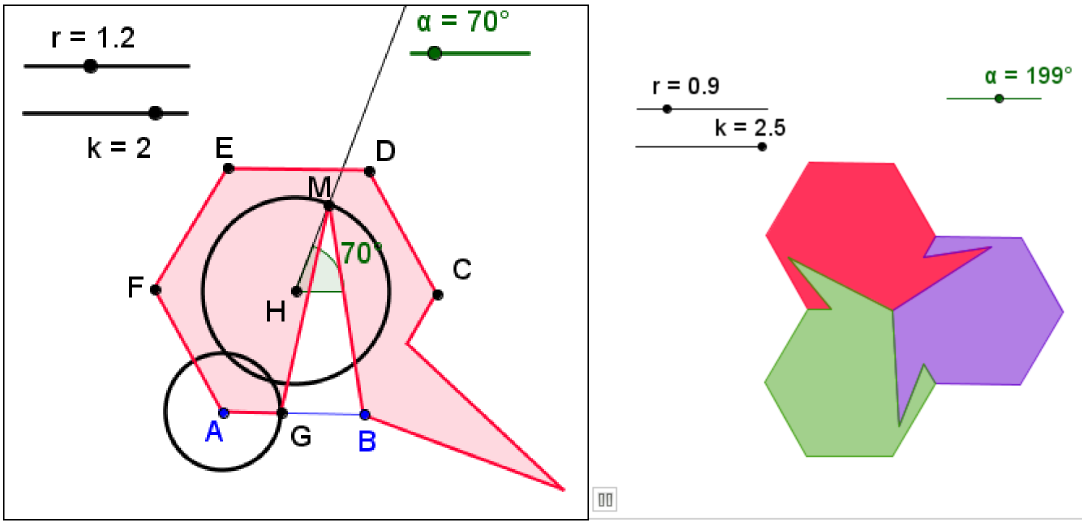


Fig.4 Organizing the animation of the tessellation.

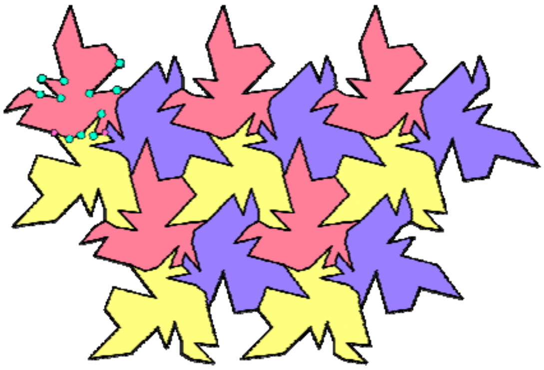
What is left is to hide the auxiliary elements and to run the sliders in animation mode.
These are just a few of the possible ways to create a newly shaped tile by transforming a regular hexagon, to tessellate the plane with it, and to animate the tessellations. But even they give an idea how the topic of dynamic tessellations could be used in support of the inquiry-based learning of mathematics and arts.

Let us remind that creating tile shapes was almost an obsession with the great Dutch artist Escher – *he would begin with a simple tile (often a polygon) that he* knew would tessellate the plane, then painstakingly coax the boundary into a recognizable shape. More formally, the tessellation of the plane in the style of Escher (known also as *Escherization*) could be formulated as follows:

The Escherization problem: *Given a shape S, find a new shape T such that:*

7 *T is as close as possible to S; and*

8 *Copies of T fit together to form a tiling of the plane.*



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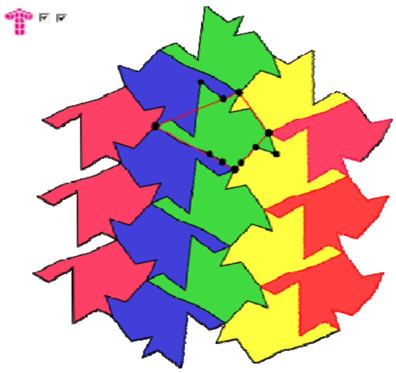

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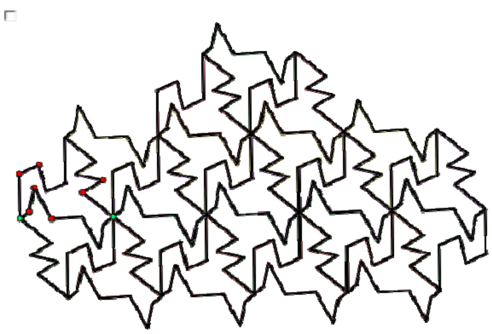

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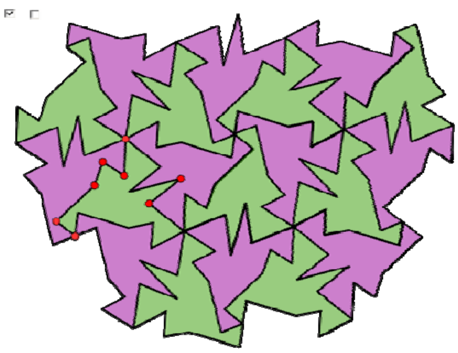

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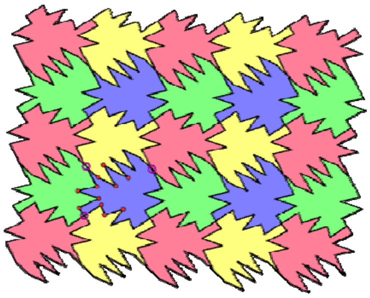

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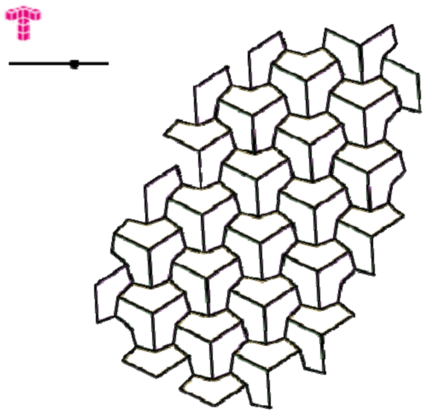

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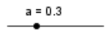
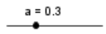
Let us now consider an algorithm suggested by Elisaveta Stefanova and her students on how to create Escher-style metamorphoses:

1 - Избираме еднакъв брой точки червени и зелени в нашия случай 3.

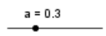
2 - Правим си плъзгач - число между 0 и 1 със стъпка примерно 0,1.
По късно стъпката може да се промени на по-малко число за по-плавно движение.

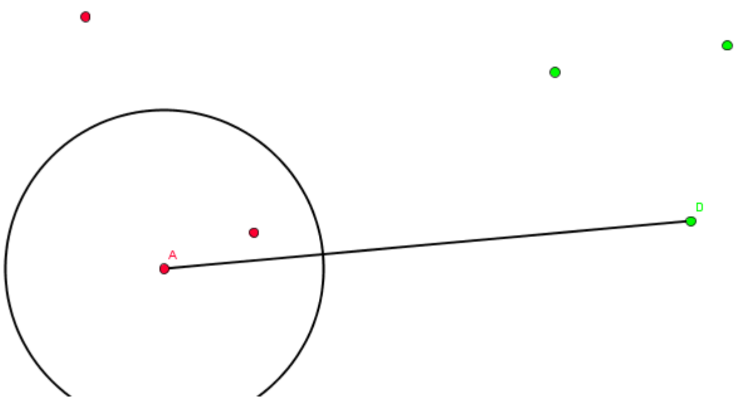


3 - Правим си отсечка между 2 съответни точки - червена и зелена.
В нашия случай точките са A и D.

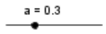


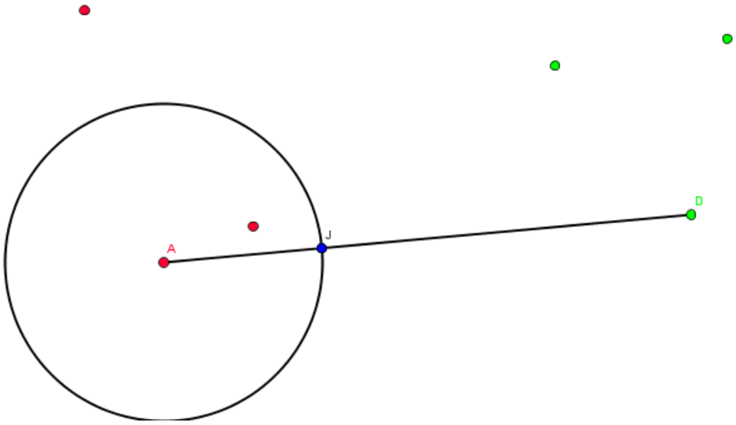
4 - Правим си окръжност с център A и радиус AD*a, където A е точката,
от която тръгваме, D е точката, в която ще се придвижим, а a е стойността
на плъзгача или практически когато a=0 окръжноста си остава в точката A,
когато a=1 окръжността минава през точката D.



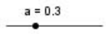


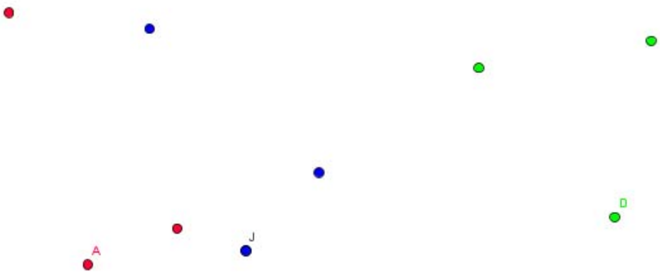
5 - Точката J - пресечна на отсечката и окръжността е точката, която ще се движи между A и D.



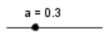


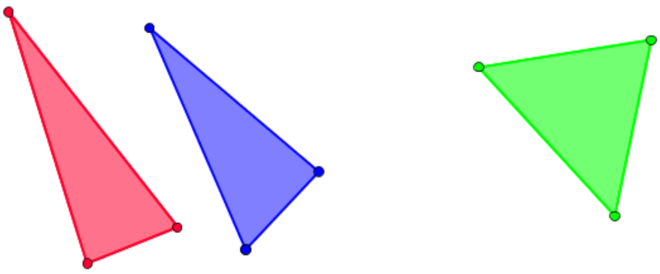
6 -По аналогичен начин намираме сините точки и на останалите двойки точки. Махаме окръжностите и отсечките.



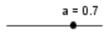


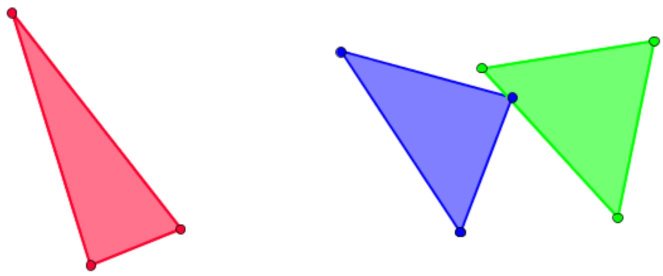
7 -Махаме имената на точките и рисуваме многоъгълниците - син, червен и зелен. Всъщност синият е важен - той ходи от червения до зеления и обратно. Всичко друго е от настройките на плъзгача, както и от първоначалните точки - червени и зелени.



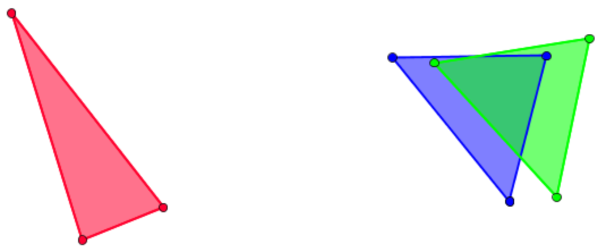


8 -Придвижваме плъзгача.

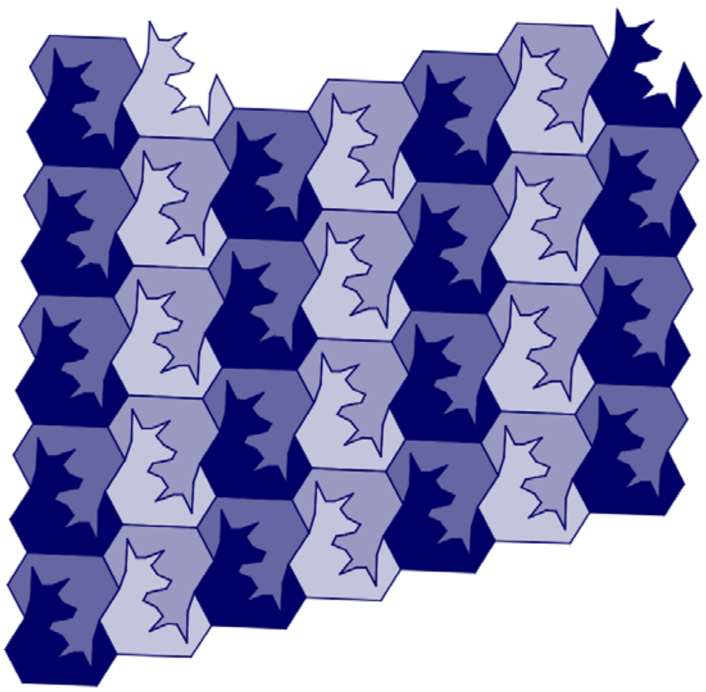


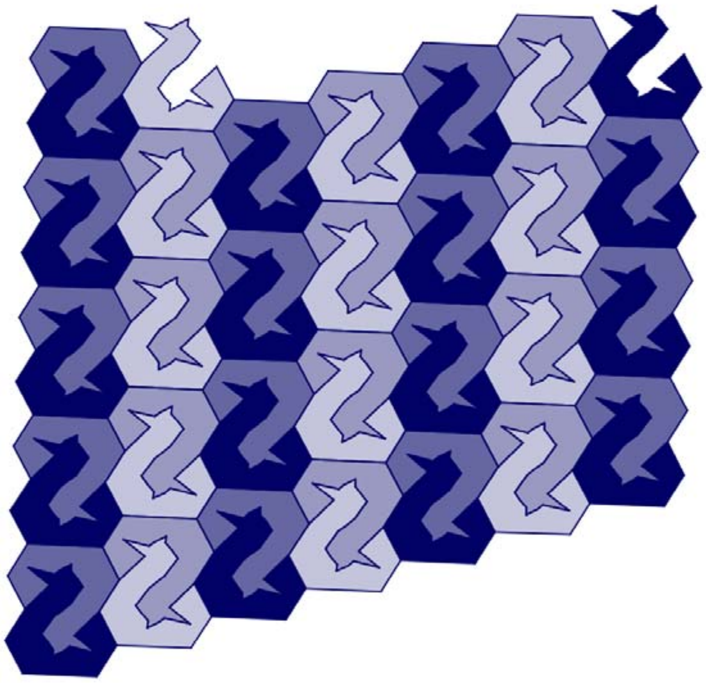


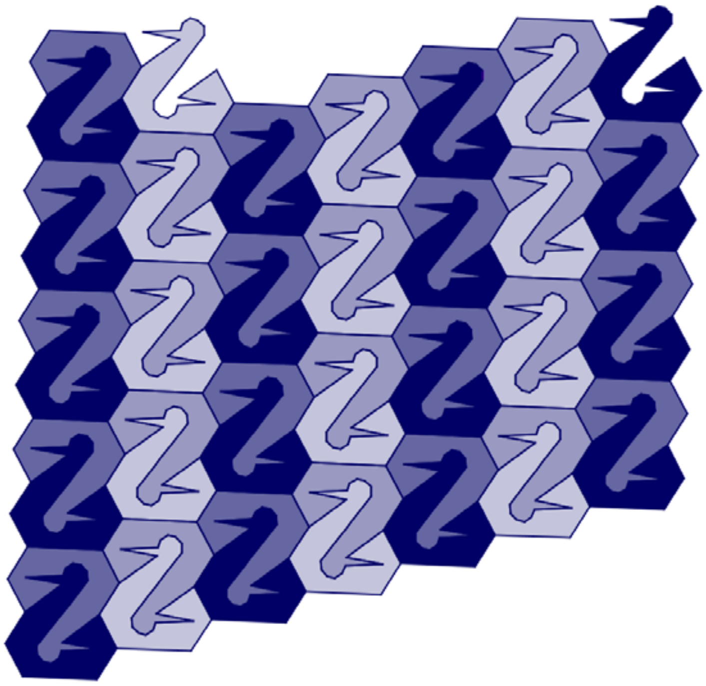
9 -Придвижваме плъзгача.



Now we can not only convert one triangle into another but to make various dynamic metamorphoses, like turning a dog into a swan.



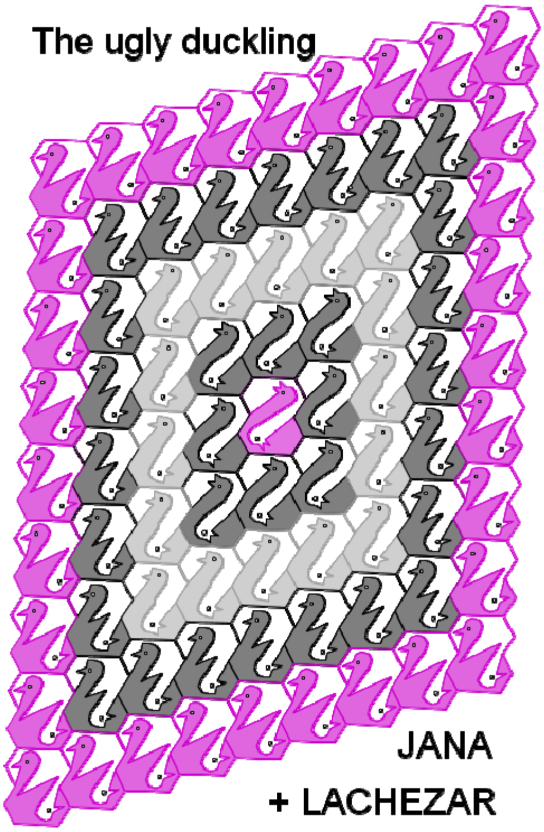




Create your own tessellation or metamorphosis motif and implement it in an artistic object of your liking - a book marker, a magnet, a poster, a greeting card (possibly a dynamic one).

Here are some interesting results of the students of Elisaveta Stefanova, teacher at 73rd General High School in Sofia:

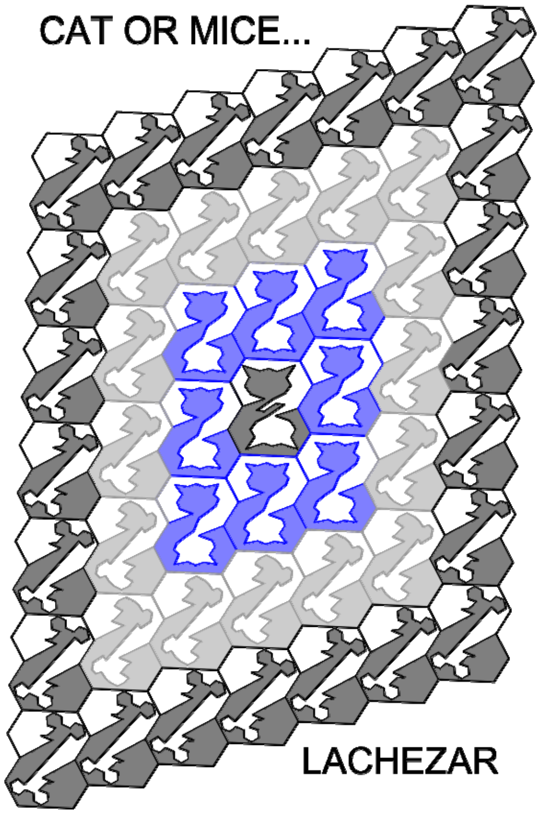
The ugly duckling



JANA

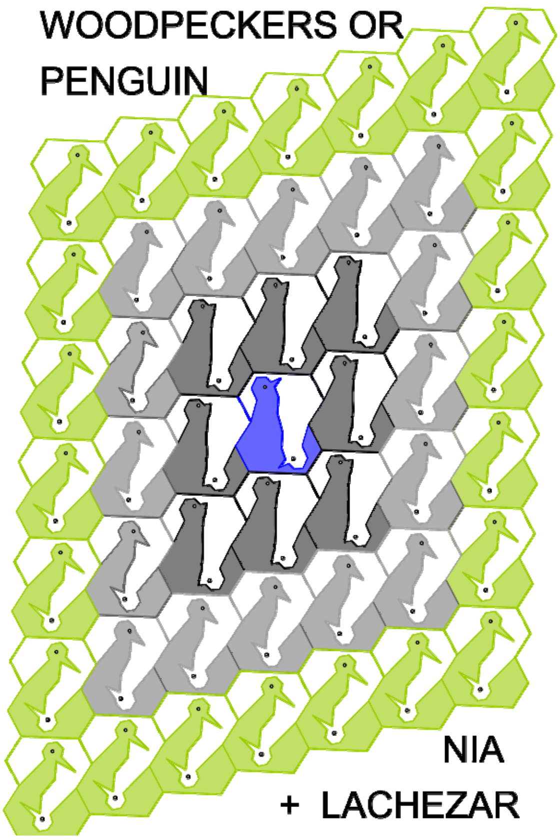
+ LACHEZAR

CAT OR MICE...



LACHEZAR

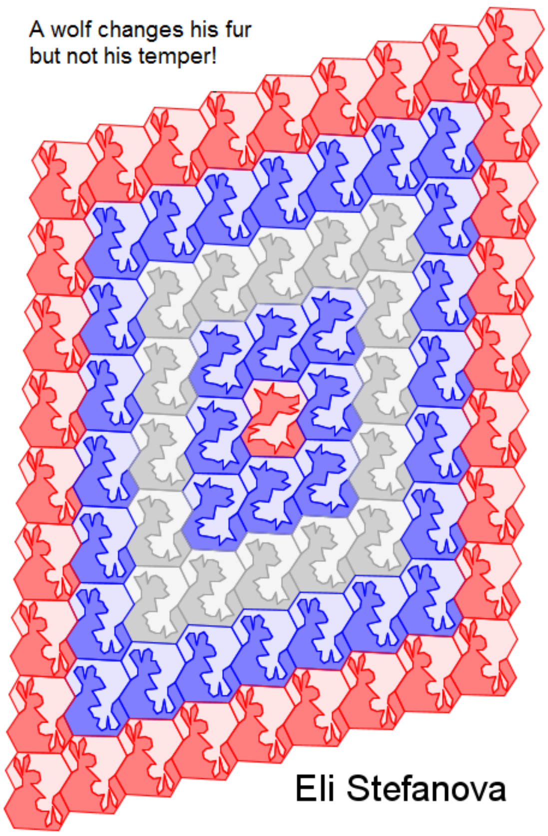
WOODPECKERS OR PENGUIN



NIA

+ LACHEZAR

A wolf changes his fur
but not his temper!



Eli Stefanova

Notes:

- The scenario is based on:
  - Chehlarova, T., Sendova, E., 2012. IBME in the secondary school: Overview and *examples in a Bulgarian context.* In: Baptist, P., D. Raab (eds.): *Implementing Inquiry in Mathematics Education*, Bayreuth pp. 114-124, ISBN 978-3-00-040752-9
  - Chehlarova, T., Sendova, E., Stefanova, E., 2012. *Dynamic tessellations insupport of the inquiry-based learning of mathematics and arts*, in *Theory, Practice and Impact - Proceedings of Constructionism 2012*, Athens, Kynigos, C., Clayson, J., Yiannoutsou, N. (Eds) August 21-25, pp.570-574.

- In order to work with the dynamic files you need **java**.
- You could download the above dynamic files as GG files at http://www.math.bas.bg/omi/cabinet/index.php?appletid=25
- To use them you should have **Geogebra** installed, e.g. from the address http://www.geogebra.org/cms/bg/