Добри практики в образованието
по математика и ИТ
за развиване на

ключови компетентности

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Lifelong
Learning
Programme

Comenius Multilateral Project: Developing Key Competences by Mathematics Education Project
(Развиване на ключови компетентности чрез математическото образование)

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Lifelong Learning Programme

Developing Key Competences by Mathematics Education

СЪДЪРЖАНИЕ

Visualizing mathematical word problems

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Abstract. Through solving of word problems the students apply their mathematical knowledge to model and study real-life situations. Many traditional methods of problem solving implemented in K-12 school do not always lead to effective learning. The use of pictures and icons to accompany the texts of the word problems not only creates a friendly problem solving atmosphere, but also helps the students interpret the problems formulation and participate in inquiry-based learning. Applied in specialized dynamic mathematics software, the images turn the standard word problems into enthralling animated stories thus both attracting and keeping the students' interest to mathematics.

Key words: *word problems, humanitarian approach to math education, dynamic mathematics software, inquiry-based learning*

1. Introduction

Even the best-selling authors with no illustrations in their books value the proverb "A picture is worth a thousand words". The picture can be embedded in the book cover design to send a message from the publisher, sketched on a napkin in a fast food restaurant to seal the mood of the reader, or taken in the bookstore to show a line waiting for their signed copy. No matter what it looks like, it mirrors a piece of the contents and makes it intriguing or mysterious…

Solving word problems allows the students to experience the power of mathematics applications, but it also causes mathematics abundance and anxiety (Kelly & Tomhave, 1985; Tobias, 1994; Harper & Daane, 1998; Devine et al., 2012; Morsanyi, Busdraghi, & Primi, 2014).

A vivid approach to problem solving, which incorporates pictures, icons, and animation can make the mathematical concepts easier to teach, learn, explore, and perceive. Luckily, there are sufficient quantity of pictures from the internet accessible for educational purposes.

2. Bringing focus to word problems formulation

The findings of Chavez and Widmer (1982) show that quite a few elementary school teachers enjoy teaching mathematics. My teaching experience with prospective elementary school teachers explains such an attitude with their high school background, primarily in the humanities and arts (Gortcheva, Lalchev, & Tabov, 2006). It shows that a consistent approach to the university math curriculum can suppress undergraduate students' negative emotions towards mathematics and turn them into willingness to solve the great variety of word problems assigned.

The reflections of the prospective elementary school teachers reveal that they highly valued the humanitarian approach implemented in their math classes (Gortcheva, Lalchev, & Tabov, 2006; Tabov, Muirhead, & Vassileva, 1999). It required much consideration about the word problems formulation which included an attractive plot, well known literary or cartoon characters, elements of magic, humor, etc. The cute pictures accompaning the texts were also appreciated by the students. They were welcomed even when present in the tests because they mathched the problems' plots and characters and underlined important issues. For example, I used a picture of a fairy tale castle in a handout for prospective elementary school teachers to emphasize the topic of a specific assignment. The handout comprised word problems formulated as stories and fairy tales to represent mathematical ideas in a comprehensible, attractive, and memorable way. After having successfully completed their work, the prospective elementary school teachers noted that although they had not met such mathematical problems so far, they enjoyed reading about the unusual characters and plots, speculating, and inventing the solutions. They mentioned that the picture of the castle had a positive impact to successfully complete their work: it strengthened both the problems' magical atmosphere and their desire to crack them.

Often the pictures are an essential part of problems formulation and neglecting them may lead to the wrong result. The negative experience, however, can be highly educational and make the problem a students' favorite. Here is such an example:

The bookworm problem

Three volumes of fairy tales are placed on a bookshelf, as shown schematically in the figure to the right. Each volume has 200 pages. A tiny bookworm starts crawling from the first page of the first volume to the last page of the last volume, where it stops. It punches a hole into each page on its way except into the thick book covers which it bypasses. In how many pages is there a hole left after the bookworm completes its journey?

The immediate response which I received from the prospective elementary school teachers was that at the end of its journey the bookworm left a hole in all 600 pages of the three volumes. This answer, however, was incorrect. The late professor Arnold who is the author of the problem (Arnold, 2004) had similar observations. The picture of the bookshelf above is not just an illustration, but a part of the problem formulation. Therefore, it must be carefully "read" together with the text and used to "decipher" the bookworm's journey description. The attentive look at the bookshelf reveals that the 1st page of the 1^{st} volume and the 200^{th} page of the 3^{rd} volume are adjacent to the 2^{nd} volume.

To be on the safe side, the students checked these considerations on a real set of three textbooks they pulled out from someone's backpack. However, some ingenuity was needed to guess that when the bookworm punched the 1^{st} page of the 1^{st} volume, it also punched the 2^{nd} page of the volume. Similarly, when it punched the 200^{th} page of the 3^{rd} volume, it punched the 199^{th} page as well. This reasoning excited the students and with a lot of fun they concluded that the bookworm's journey was 2+200+2 pages long. Thus a seemingly "innocent" problem involved the whole class into inquiry-based learning and make them experience the joy of their own mathematical discoveries. It also showed that geometrical thinking is an important component of mathematical thinking and needs special care, especially when geometry classes in the prospective elementary school teachers' math curriculum are relatively few.

3. Bringing expressiveness to word problems' characters

The negative emotions towards mathematics which the prospective elementary school teachers carried from their high school period can be analyzed from various perspectives. They reveal that the knowledge gaps gradually become barriers to students' progress. As a result, boredom, a loss of interest, and avoidance of mathematics appear. With the potentials of modern learning environments and information technologies advancement in mind, such educational flaws are unacceptable. They are evidence that the 21^{st} century students need mathematical concepts to be represented according to their interests and views much like the computer games which are a part of their everyday life and allow them to play, socialize, and race in a dynamic and interactive way.

Modern researchers in math education successfully implement information technologies in their teaching practices (Lazarov & Vassileva, 2007; Vassileva, 2011; Denchev, Kovatcheva, & Sendova, 2012; Kenderov, Sendova, & Chehlarova, 2012; Gortcheva, 2013; Chehlarova et al., 2014; Zeljić & Dabić, 2014; Zsoldos-Marchiş, 2014*)*.

The Fisherman and the dog problem which follows shows my experience how the principles of humanitarian and interdisciplinary approach to math education allow to represent a serious mathematical idea through a story and an unexpected development of the plot through a dynamic mathematics software animation. To model the situation, the students use their knowledge in Bulgarian geography as well:

The fisherman and the dog problem. *Russy the Fisherman left the port of Russe on his boat, heading to the port of Tutrakan. At the same moment Tutty the Dog also left Russe, running on the beach of the Danube towards Tutrakan. Tutty's speed was equal to the speed of the boat in still water. When each of the two reached Tutrakan, he immediately headed back to Russe.*

Did Russy the Fisherman and Tutty the Dog arrive simultaneously back in Russe? If not, who arrived first?

To emotionally tie the students to the problem and help them keep a sense of reality throughout the inquiry-based learning, I constructed an animated applet[1] in *GeoGebra* (Figure 1).

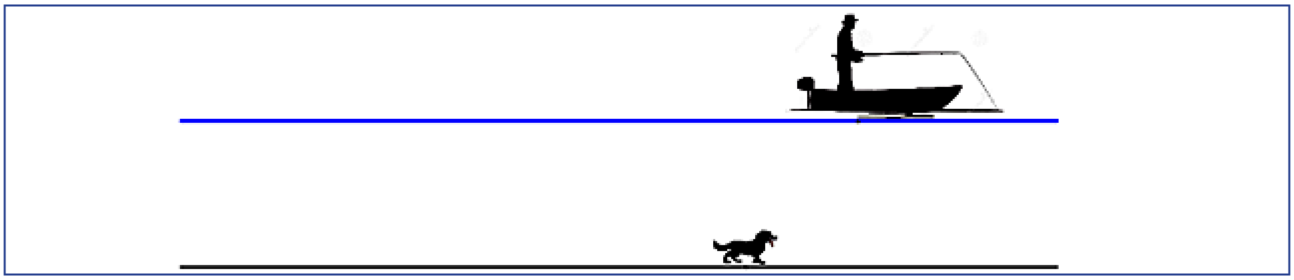

Figure 1. The icon selection brings expressiveness to the word problem characters

The students were happy to observe how Russy the Fisherman and Tutty the Dog started moving in opposite direction (*Figure 2*).

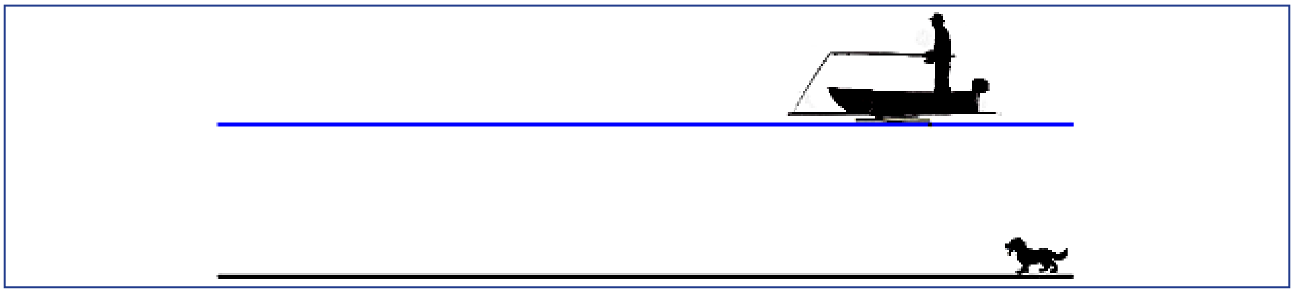

Figure 2. The "efforts" of the animated characters grab and keep the students' attention

Trying to guess who would arrive first, they explored the situation through the applet's sliders. This experience helped them find the solution and perform the algebraic operations to prove it.

[1] The pictures used in the applets here are taken from: www.clipartbest.com, http://www.shutterstock.com, http://itcteacheronthetrail.com, http://www.clker.com, http://www.dreamstime.com

4. Bringing attention to important societal issues such as road traffic safety

Animated applets of mathematical word problems can also help the students become literate participants in road traffic, both as pedestrians and drivers. The next problem models a common situation on a highway whose two lanes merge into one (Gortcheva, 2015):

The merging lanes problem. *Due to road construction work, vehicles keeping a distance of 25 meters and driving in the same direction with the same constant speed of 120 km/h in two separate lanes of the highway need to merge into one lane. After merging, they keep moving at an equal distance with equal constant speed, having adapted to the new traffic conditions. The traffic safety rules require the distance between the vehicles to be at least 9 meters. The length of the vehicles themselves is neglected.*

Rounded to tens, how many kilometers per hour is the speed of the vehicles allowed to be when entering the one-lane part of the highway?

The animated *GeoGebra* applet constructed by me simulates various road traffic situations. Thus not only does it help the teachers, students, and parents work on the solution, but hopefully trains them to act properly in complicated travel conditions.

To distinguish the vehicles travelling in the two lanes I used two different icons of cars in two different colors: red convertibles for the right lane and black sedans for the left. Implementing such an approach improved the visibility of the solution without a loss of generality. However, even a more sophisticated approach was needed:

The applet simulaions revealed that in order to keep the minimum distance of 9 meters, in some particular cases the vehicles were to wait before entering the one-lane part of the highway. To show the waiting cars I decided to keep their initial color, but reduce its intensity. Using this technique made it possible to visualize traffic conditions like bottlenecks (Figure 3) and multiple-vehicle collisions on the highway (Figure 4):

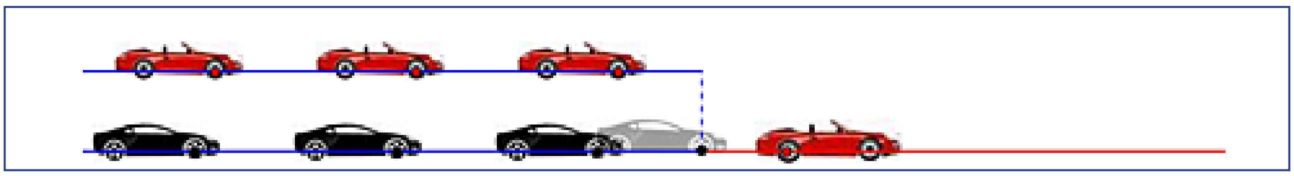

Figure 3. The animated applet demonstrates that when the speed of the cars in the one-lane part of the highway is not high enough, a bottleneck occurs

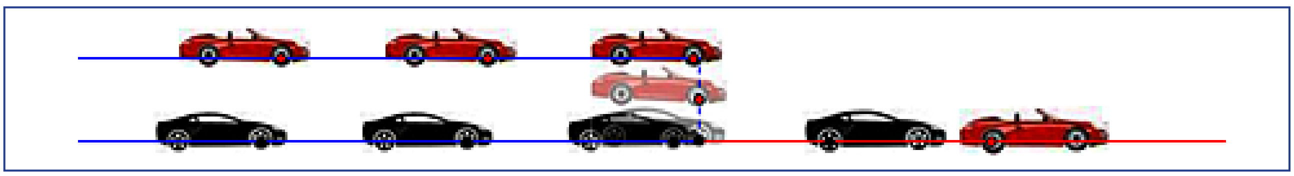

Figure 4. The animated applet shows that the inappropriate car speed can cause multiple-vehicle collisions on the highway

5. Concluding remarks

Visualization of word problems is not new in math education (Rubin, 1999; Atkinson, 2002; Yoon et al., 2006). However representations of mathematical ideas through dynamic geometry software bring in class a spirit of exploration, creativity, and fun.

Even if not a passion for mathematics, the students' passion for technology becomes a step towards understanding of mathematics and should be taken into account in lesson plans and the curriculum design.

Such an approach requires a lot of teachers' and educators' effort, but helps the students find their own paths in inquiry-based learning.

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