METHODICAL CONCEPTS,
INTRODUCED BY Prof. Doctor Habil. IVAN GANCHEV

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ABSTRACT

Some methodical concepts, introduced by Prof. Doctor Habil. I. Ganchev, have been presented. The ideas that have rendered their introduction indispensable have also been described for some of the concepts.

The development of the Methods of the Education in Mathematics as a science demands its set of concepts to be developed. The expansion of the set of concepts is connected with a change of the content of the terms used, as well as with the introduction of new ones. New concepts in methods spring up as a result of adaptation (and specification) of concepts from the spheres of psychology, pedagogy, philosophy, mathematics and others, or as a result of achieving new knowledge in the Methods of the education in mathematics.

Prof. Dr Habil. Ivan Ganchev proved to have introduced about 60 new methodical concepts. Some of them have acquired such immense popularity that they leave the impression of having existed for ages. Others, however, are less popular, and their usage adds accuracy to the language of the Methods of the Education in Mathematics. Some of the concepts will be accompanied by short comments relevant to the way the concepts were obtained and some basic ideas that have rendered their introduction indispensable. Others will be registered by their terms and source of origin.

1. Concepts and related comments.

While investigating and classifying cognitive activities of students, Prof. Ganchev uses and recommends an approach that he calls **hierarchical**, and which fulfils the following requirements: at the examination of each activity, it is easy to delimit those activities on which its adoption depends, as well as the activities, the adoption of which depends on it; at the classification of a system of activities, each activity is considered where there have already been adopted sufficient other activities which it depends on, and can be explained by means of already familiar concepts; at the classification of a system of assertions about the correlations between activities, each assertion is examined where both activities it relates have already been explained. [1]

The use of the materials of the propositional and predicative logic to investigate the process of education in mathematics and to develop a methods for its better management is called a **logical atomic-molecular approach**. The propositional computation does not render an account for the structure of the separate simple propositions – that is done in the predicative calculation. By analogy with the structure of substances, when propositional calculation is used in the methods of the education of mathematics, it is said that issues are viewed at a **molecular level**, and in the case when predicative calculation is used, they are viewed at an **atomic level**.

The level, at which the solutions to problems are examined as particular sequences of solutions to problems that are components, is called a **cellular level**. Thus the concepts **modelling of mathematical knowledge and activities at an atomic, molecular and cellular level** are formulated.

As a result of the ideas of hierarchy of mathematical concepts and assertions, and of order in the specified intellectual capabilities in the products of human labour, an index of mathematical objects is introduced, corresponding to the index of the specified intellectual capabilities. In a concept, in the definition of which a concept with the highest index k is used, there will be specified intellectual capabilities with an index k+1. Primary concepts and axioms have the index 1. In connection with the fact that the hierarchy of mathematical concepts and assertions is related to their understanding, the given index can be used as a measure of understanding as well. Thus, the following concepts have been introduced for the understanding of mathematical knowledge: **understanding with index 1 of a mathematical concept; understanding with index k+1 of a mathematical concept; understanding of a definition (description) of a mathematical concept; understanding of an axiom; understanding of a theorem; understanding of the proof of a theorem; understanding of a mathematical problem; understanding with an index 1 of the solution to a mathematical problem; understanding with an index k+1 of mathematical knowledge.**

The model of L. Vygotsky concerning the relation between education and development has been improved. At every stage of education, there is a specific **zone of actual development of a student** *ZAD* (representing the range of the mature psychological functions at a definite moment), and a student's zone of proximal development *ZPD*. The activities, which students can carry out by themselves, are called **activities, corresponding to the zone of actual development** *Azad*, while the activities, which a student can carry out using some help, are called **activities, corresponding to the zone of proximal development** *Azpd*. The higher psychological functions have been inducted by relevant knowledge, corresponding to these two zones.

The knowledge, corresponding to ZAD, has been designated by *KZAD,* while **the knowledge, corresponding to ZPD**, has been designated by *KZPD*. Under the terms of the syllabus, a range of knowledge *Zoб* has been defined in the

process of education, and through joint work with adults, it ensures maturing of higher psychological functions φ_{κ}, which are added to ZAD. This expansion of ZAD leads to an enhancement in the development of the individual [8].
The expansion and restoration of ZAD and ZPD in such a way that certain knowledge or skills would be rendered in $Z_{збр}$ and the formation of beliefs about their cognitive significance is called **preparation of the internal conditions** in students for the acquisition of this knowledge or skills.

The knowledge and skills, programmed for acquisition at a definite level, alongside with the setting up of components for the development and education of students at a certain stage, is called **trajectory of cognition** (the term was offered by Acad. Dr. Yuri Kolyagin)[1,p.27]. In the better case students "move" straight along the trajectory of cognition. The availability of a "deviation" from this trajectory indicates the presence of gaps in their knowledge and skills.

In [1,p.26] there has been made a classification of the manageable processes in the course of education in accordance with the time of examination of the deviation from the trajectory of cognition. When deviations are examined at the end of each cycle of the process, then the principle of the "black box" is manifested. Such a process is called a "**badly-managed process**". When the regulation of the process is carried out based on information about the whole process, then the principle of the "transparent box" is manifested. Such a process is called a "**well-managed process**".

There exist two types of theorems according to their relation to the concepts. A theorem, providing a necessary condition for the existence of a concept, is called a **theorem - attribute with respect to the concept.** A theorem, providing a sufficient condition for the existence of a concept, is called a **theorem - indication with respect to the concept.**

A sentence with the structure

$$(p_1(x) \wedge p_2(x) \wedge ... \wedge p_k(x)) \vee (p_{11}(x) \wedge p_{12}(x) \wedge ... \wedge p_{1m}(x)) \vee ... \\ \vee (p_{l1}(x) \wedge p_{l2}(x) \wedge ... \wedge p_{\ln}(x)) \to p(x)$$

that is a disjuncture from the conjuncture of properties in the definition of a given concept S, and conjunctions of properties in theorems - indications of that concept, is called a **didactic system of indications** of this concept [1,p.85].
Usually the theorems - attributes of a single concept are not studied all at once. This "dissipation" presents an obstacle for the provision of effective education. Collecting all currently known sufficient conditions for the concept S into one place is very convenient to support conjecturing. A starting point is provided for solving a problem. After studying a new sufficient condition for S, it is added to the currently considered ones. Such complex sentences are obtained for different concepts. They are very helpful and widely used.
In this way the necessity for a common denomination and an inscription is formed. Thus a new methodological concept is created – a didactic system of indications and the symbol of DSI [3]. A sentence with the structure

$p(x) \rightarrow (q_{11}(x) \wedge ... \wedge q_{lt}(x) \wedge ... \wedge q_{kl}(x) \wedge ... \wedge q_{kv}(x))$, that is a conjunction of the characteristics in the definition of the concept S, as well as from the characteristics of theorems - characteristics of that concept, is called a **didactic system of characteristics**.[1]

A mathematical problem is a sequence of thoughts, through which is established a subset R of a given set M of mathematical objects following the requirements listed below: a) to establish R constructively, or b) to ascertain that R is a subset of an already established subset M by means of a definition, or c) to show that the objects R can be obtained by means of certain rules, characterizing given drawing tools, or d) to show that R coincides with a certain set, which is assumed to be known. In connection with the concept – problem, prof. Ivan Ganchev specifies in [2] the content of the following concepts**: text of a problem, given entities of a problem, deduction of a problem, solution to a problem, solved problem**. With the purpose of finding the solution to a problem, it is important to master common schemes of reasoning **such as Euclid's analysis scheme (scheme of imperfect analysis) of a solution to a problem; Pappus analysis scheme (scheme of perfect analysis) of a solution to a problem; scheme of synthesis of a solution to a problem**. In order to form skills of providing direct proof, it is recommended to **change objectives** in the process of solving problems and proving theorems. When a student has difficulties in finding or reproducing a proof, he must be guided to start from the objective, which has to be achieved, and to search (from relevant indications systems) what other assertions are sufficient to prove.[7]

A problem Z_k with a solution A_k is called a **problem – component** of problem *Z* with a solution *A*, when the solution A_k of Z_k is contained in the solution *A* of *Z*. Since a problem can be solved using different methods, in the different solutions it may have different problems – components. Based on the model of solution to a problem on the cellular level, there has been made the following characteristic of the concepts "level of complexity of the solution to a problem" and "level of difficulty of the solution to a problem". If A_1, $\ldots, A_{n-1}$ are the solutions to the problems – components of the problem Z_n with a solution A_n, then: **the level of complexity of the solution** A_n is determined by the number *n*; **the level of difficulty of the solution** A_n depends on its level of difficulty and on how many and which of the solutions $A_1, \ldots, A_{n-1}$ have already been known to the problem – solver.

It is clear that the level of complexity of a given solution cannot be altered, while the level of difficulty can be changed by means of solving problems – components of the respective problem beforehand. The use of problems – components offers the possibility of changing the difficulty of the solution to a problem. Through them *ZAD* and *ZPD* are broadened. A group of problems, in which each problem (excluding the last one) is a problem – component of subsequent problems, is called a **didactic system of problems (a group of problems)**. [2,p.66-83]. **A dominant (strong) property of a theorem, axiom, or definition with respect to a given student** is displayed in the student's guessing right to use these bearers of mathematical information. **A recessive (weak) property** of a theorem, axiom, or definition with respect to a given student is displayed in the student's not guessing right to use these bearers of mathematical information.

In scientific papers, aided work has been accounted for (i.e. in the student's ZPD). The graded instructions are given emphasis to as especially useful (providing different variants for help as appropriate to the student's needs or the mistakes they have made). In the case of developing the ideas, connected with the use of computer technology for the solution to some methodical problems, related to the aided work, as well as in building a plan – scenario for the dialogue – teaching programs, there has been introduced the so-called **didactic block**.

By analogy to the property of equations, which led to the denomination of "algebra", the equivalence $p_1 \wedge p_2 \wedge ... \wedge p_i \wedge ... \wedge p_n \to q \Leftrightarrow p_1 \wedge p_2 \wedge ... \wedge \neg q \wedge ... p_n \to \neg p_i$ is called a **logical algebar**. Based on this equivalence, from each theorem there can be made problems, the usefulness of which is determined by two conditions: each of the problems made accordingly is easy to solve using the indirect method of proof; problems are provided for the immediate application of the proven theorems without using a lot of old knowledge, which is of primary importance for the initial consolidation of new theorems [1,p.96].

Inter-subjective correlations are a multitude of relations in the range P of subjects. The main types of relations p_1, p_2 and p_3 from this multitude are determined by the sentences: p_1: Knowledge and skills related to subject *x* are used to motivate the study of concepts in subject *y*; p_2: Knowledge and skills related to subject *x* are used for modelling in subject *y* while solving theoretical problems; p_3: Knowledge and skills related to subject *x* are used at the formulation and solution to problems in subject *y*, or at the conduction of other exercises in it [5,p.12].

Mathematical folklore (problems, which are folk art): those are problems, presented in a vivid, entertaining and easy-to-apprehend form, they are distributed verbally and their authors are anonymous [4].

2. Concepts of which the source of introduction is given, and of which Prof. Ganchev is the author or co-author.

An algorithm for the conjugation of verbs by means of using a personal computer [5]; a vector of the element composition of a molecule [5,p.29]; a vector of the mass composition of a molecule [5,p.33], a descriptive logical model of an activity in the study of mathematics [3,p.50]; ideals of human activities in mathematics – understanding, convincing, economy [1,p.175]; linear sequence of equivalents [2,p.26] and of implications [2,p.27]; branched sequence of equivalents

[2,p.26] and of implications [2,p.27]; logical structure of a solution to a mathematical problem [2,p.25]; national radio competition in mathematics [6,p.123]; sphere of search (orientation environment) in OM [7,p.28]; operating logical model of activity in OM [3,p.52]; maintaining of acquired knowledge [1,p.154]; principle of element - divisibility, repetition and dependency [1,p.180]; principle of the threefold objective [2,p.59]; system of primary school activities in mathematics lessons [1,p.165]; classification of concepts in OM with respect to their volumes [6,p.41].

As a conclusion, I would like to point out that my motivation to present these facts was not so much to estimate some of Prof. Dr Habil. Ivan Ganchev's contributions to the Methods of the education in mathematics. The information offered above is aimed at the next generation of specialists in educational methods, who will be working towards further development of the Methods of the education in mathematics as a system of knowledge.

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