Serdica

Bulgariacae mathematicae publicationes

Сердика

Българско математическо списание

SOME PROPERTIES OF MULTISTATE *BW*-SYSTEMS

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In this paper we present a new approach to problems of reliability against hitherto binary systems. On the basis of multistate system (see [3]), we propose some general definitions of k-importance of a component (for the binary case, see [9]). Then, we outline the properties of k-importance and their proofs in a multistate system.

1. Introduction. Let us consider a multistate system consisting of n components which are not repaired. Let $C=\{1, 2, \dots, n\}$ denote the set of components of the system and for each $i \in C$, x_i denotes the performance of the i-th component. For $i \in C$ we distinguish M_i+1 performance levels ranging from perfect functioning — level M_i, to complete failure — level 0.

In addition we have given sets of components called min path sets $\{P_1, P_2, \dots, P_p\}$, where $\cup_{r=1}^{p} P_r = C$. The system state is defined to be the state of the "worst" component in the "best" min path, that is to say the structure function $\varphi(\boldsymbol{x}) = \max_{1 \leq r \leq p} \min_{i \in P_r} x_i$. We call this system a BW-system (see [3]).

Let $\{X_i(t), t \geq 0\}$ denote the stochastic process representing the state of the i-th component $(i \in C)$ at time t as t varies over the non-negative real numbers. The stochastic process $\{\varphi(\boldsymbol{X}(t)), t \geq 0\}$ represents the corresponding system state when t varies from 0 to ∞, where $\boldsymbol{X}(t) = (X_1(t), \dots, X_n(t))$ is known as a state vector. We assume that the processes $\{X_i(t), t \geq 0\}$, $i \in C$ are mutually independent. Furthemore, we assume $X_i(0) = M_i$, $i \in C$. This implies that when $\varphi(\boldsymbol{X}(0)) = M$, the system will be at the perfect operation state M for $t=0$.

2. Notations and definitions

(a) $(k_i, \boldsymbol{x})$ denotes $(x_1, \dots, x_{i-1}, k, x_{i+1}, \dots, x_n)$;

(b) the component $i (i \in C)$ of a system is irrelevant to $\{k_1, \dots, k_m\}$ if for all $s_i, u_i \in \{k_1, \dots, k_m\}$, $\varphi(s_i, \boldsymbol{x}) = \varphi(u_i, \boldsymbol{x})$ for each $(\cdot_i, \boldsymbol{x})$;

(c) a vector $\boldsymbol{x}$ is called an upper vector for a level k of a system if $\varphi(\boldsymbol{x}) \geq k$. It is called a critical upper vector for a level k, if in addition $\boldsymbol{y} \leq \boldsymbol{x}$ and $\boldsymbol{y} \neq \boldsymbol{x}$ implies $\varphi(\boldsymbol{y}) < k$;

(d) $U_k(\varphi)$ (or U_k) denotes the set of critical upper vectors for level k, $k=1, 2, \dots, M$;

(e) $\{P_1, \dots, P_p\}_k = \{P_j : \min_{i \in P_j} M_i \geq k\}$ is called the min path sets for a level k, $k=1, \dots, M$;

(f) $T_{ij} = \inf\{t : X_i(t) < j\}$, $(i \in C, j=1, 2, \dots, M_i)$ is called j-lifetime of a new i-th component, i. e. the time for which the i-th component first enters the states $\{0, 1, \dots, j-1\}$,

$T_j = \inf\{t : \varphi(\boldsymbol{X}(t)) < j\}$ $(j=1, 2, \dots, M)$ is called j-lifetime of the system;

(g) $$q_k(x_i) = \begin{cases} 1, & \text{if } x_i \geq k \\ 0, & \text{if } x_i < k \end{cases} \quad \text{and} \quad q_k(\boldsymbol{x}) = (q_k(x_1), \dots, q_k(x_n))$$

SERDICA Bulgaricae mathematicae publicationes. Vol. 13, 1987, p. 341—346.

$$v_k(x_i)=\begin{cases}k, \text{ if } x_i \geq k\\ 0, \text{ if } x_i<k.\end{cases} \quad \text{Let } x_{ik}=v_k(x_i),\ \boldsymbol{x}_k=(x_{1k},\ldots,x_{nk});$$

(h) $$q_k(U_k)=\{q_k(k_{P_j},\ 0_{C-P_j});\ P_j \in \{P_1,\ldots,P_p\}_k\};$$

(i) let the i-th component have an absolutely continuous j-life distribution $F_{ij}(t) = \mathbf{P}(T_{ij}<t)$ with density $f_{ij}(t)$, $j=1, 2, \ldots, M_i$;

(j) the j-reliability of the i-th component at time t is given by $r_{ij}(t)=1-F_{ij}(t)$;

(k) for $i \in C$ and $k=1, 2, \ldots, M$, $R_{ik}(t)=\begin{cases} r_{ik}(t), \text{ if } k \leq M_i\\ 0 \quad, \text{ if } k>M_i;\end{cases}$

(l) the j-reliability ($j=1, 2, \ldots, M$) of any system at time t is given by $H_j(t) = H_j(r_1(t), \ldots, r_n(t)) = \mathbf{E}q_j(\varphi(\mathbf{X}(t)))$, where $r_i(t)=(r_{i1}(t), \ldots, r_{iM_i}(t))$, $i \in C$;

(m) the $I^B_{ik}(t)=\mathbf{P}\{q_k(\varphi(x_{ik}=k,\ \boldsymbol{x}(t)))-q_k(\varphi(x_{ik}=0,\ \boldsymbol{X}(t))\}=1$, $k=1,\ldots,M_i$ is called a B-k-importance of the i-th component at time t. In the binary case this importance was defined in [4];

(n) the $I^{BP}_{ik}=\int\limits_0^\infty [H_k(x_{ik}=k,\ R_{1k}(t),\ldots,R_{nk}(t))-H_k(x_{ik}=0, R_{1k}(t),\ldots,R_{nk}(t))]\,f_{ik}(t)\,dt$ is the conditional probability that the i-th component causes the system to leave out the states $\{k, k+1, \ldots, M\}$ when the i-th component leaves out the states $\{k, k+1, \ldots, M\}$ ($k=1, 2, \ldots, \min(M_i, M)$). This measure is called BP-measure of k-importance for the i-th component (for the binary case see [1]);

(o) Y_{ik} = remaining system k-lifetime just before realizing the states $\{k, k+1, \ldots, M_i\}$ of the i-th component ($i \in C$), $k=1, 2, \ldots, \min(M_i, M)$, Z_{ik} = remaining system k-lifetime just after realizing the states $\{k, k+1, \ldots, M_i\}$ of the i-th component ($i \in C$), $k=1,2,\ldots,\min(M_i, M)$,
$Q_{ik}(u, t)=\mathbf{P}(Y_{ik}>u,\ Z_{ik}>t)$, $u \geq t \geq 0$,
$G_{ik}(t)=\mathbf{P}(Y_{ik}>t)$, $t \geq 0$, $S_{ik}(t)=\mathbf{P}(Z_{ik}>t)$, $t \geq 0$;

(p) $R^1_{ik,t}(u)=R_{ik}(t+u)/R_{ik}(t)$, $R^0_{ik,t}=0$
and for every binary vector $\boldsymbol{x}_k$ ($k=1, 2, \ldots, M$)

$$\boldsymbol{R}^{q_k(x_k)}_{k,t}(u)=(R^{q_k(x_1)}_{1k,t}(u),\ R^{q_k(x_2)}_{2k,t}(u), \ldots, R^{q_k(x_n)}_{nk,t}(u)).$$

Note that the vector $\boldsymbol{R}^{q_k(x_k)}_{k,t}(u)$ gives the conditional k-reliabilities of the components at time $t+u$, when the binary state vector $\boldsymbol{x}_k$ at time t is given.

3. Properties of a multistate BW-system. Now we formulate and prove several propositions.

(1) If $M=\max\limits_{1\leq j\leq p} \min\limits_{i \in P_j} M_i$ then $\{0, 1, \ldots, M\}$ will be the set of states of the system.

Proof. It is implied from the definition of BW-system.

(2) If $M_i>M$, then the i-th component is irrelevant to $\{M, M+1, \ldots, M_i\}$ and in this case each of the states $M+1, M+2, \ldots, M_i$ of the i-th component is not worth considering.

Proof. It is implied from (1) and (b) above.

(3) If $\min\limits_{i \in P_j} M_i=m>M$, then the min path P_j is irrelevant to $\{M, M+1, \ldots, m\}$.

Proof. It follows immediately from (2).

(4) For $k=1, 2, \ldots, M$, $U_k=\{(k_{P_j},\ 0_{C-P_j});\ P_j \in \{P_1, \ldots, P_p\}_k\}$.

Proof. It is implied from (c), (d) and (e).

(5) $q_M U_M(\varphi) \subset q_{M-1} U_{M-1}(\varphi) \subset \ldots \subset q_1 U_1(\varphi)=U_1(\varphi)$.

Proof. It is implied from (g), (h) and (4) above.

(6) $q_k(\varphi(\boldsymbol{x}))=\varphi(q_k(\boldsymbol{x}))$, $k=1, 2, \ldots, M$.

Proof. i) First we show that for every path, $\min\limits_{i\in P_j} q_k(x_i)=q_k(\min\limits_{i\in P_j} x_i)$.

Let $\min\limits_{i\in P_j} q_k(x_i)=1 \Leftrightarrow \min\limits_{i\in P_j} x_i\geqq k \Leftrightarrow q_k(\min\limits_{i\in P_j} x_i)=1$,

now let $\min\limits_{i\in P_j} q_k(x_i)=0 \Leftrightarrow \min\limits_{i\in P_j} x_i<k \Leftrightarrow q_k(\min\limits_{i\in P_j} x_i)=0$;

ii) We show that $\max\limits_{1\leqq j\leqq p} q_k(y_j)=q_k(\max\limits_{1\leqq j\leqq p} y_j)$.

Let $\max\limits_{1\leqq j\leqq p} q_k(y_j)=1 \Leftrightarrow \max\limits_{1\leqq j\leqq p} y_j\geqq k \Leftrightarrow q_k(\max\limits_{1\leqq j\leqq p} y_j)=1$.

Now let $\max\limits_{1\leqq j\leqq p} q_k(y_j)=0 \Leftrightarrow \max\limits_{1\leqq j\leqq p} y_j<k \Leftrightarrow q_k(\max\limits_{1\leqq j\leqq p} y_j)=0$.

From i) and ii) we see that

$q_k(\varphi(\boldsymbol{x}))=q_k(\max\limits_{1\leqq j\leqq p}\min\limits_{i\in P_j} x_i)=\max\limits_{1\leqq j\leqq p} q_k(\min\limits_{i\in P_j} x_i)=\max\limits_{1\leqq j\leqq p}\min\limits_{i\in P_j} q_k(x_i)=\varphi(q_k(\boldsymbol{x}))$.

(7) For $k=1, 2, \ldots, M$, $T_k=\max\limits_{P_j\in\{P_1,\ldots,P_p\}_k}\min\limits_{i\in P_j} T_{ik}$.

Proof. From (f) we have $T_k=\inf\{t:\varphi(\boldsymbol{x}(t))<k\}=\inf\{t:q_k(\varphi(\boldsymbol{x}(t)))=0\}$ $=\inf\{t;\varphi(q_k(\boldsymbol{x}(t)))=0\}=\inf\{t:\max\limits_{P_j\in\{P_1,\ldots,P_p\}_k}\min\limits_{i\in P_j} q_k(X_i(t))=0\}=\max\limits_{P_j\in\{P_1,\ldots,P_p\}_k}\min\limits_{i\in P_j}$ $\inf\{t:q_k(X_i(t))=0\}=\max\limits_{P_j\in\{P_1,\ldots,P_p\}_k}\min\limits_{i\in P_j}\inf\{t:X_i(t)<k\}=\max\limits_{P_j\in\{P_1,\ldots,P_p\}_k}\min\limits_{i\in P_j} T_{ik}$.

(8) For $k=1, 2, \ldots, M$, $H_k(\boldsymbol{r}_1(t), \ldots, \boldsymbol{r}_n(t))=H_k(R_{1k}(t), \ldots, R_{nk}(t))$.

Proof. It is implied from (k), (l) and (7).

(9) The expectation $\mathbf{E}\,T_k$ when exists, equals to:

$\mathbf{E}\,T_k=\int\limits_0^\infty H_k(R_{1k}(t), \ldots, R_{nk}(t))\,dt$, $k=1, 2, \ldots, M$.

Proof. It is analogous to the binary case.

(10) The function of the expected value of the system state is

$$\mathbf{E}\,\varphi(\boldsymbol{x}(t))=\sum_{j=1}^{M} H_j(R_{1j}(t), \ldots, R_{nj}(t)),\ t>0.$$

Proof. $\mathbf{E}\,\varphi(\boldsymbol{X}(t))=\sum\limits_{j=1}^{M} j\,.\,\mathbf{P}(\varphi(\boldsymbol{X}(t))=j)=\sum\limits_{j=1}^{M-1} j\,.\,[\mathbf{P}(\varphi(\boldsymbol{X}(t))\geqq j)-\mathbf{P}(\varphi(\boldsymbol{X}(t)\geqq j+1)]$ $+M\,.\,\mathbf{P}(\varphi(\boldsymbol{x}(t))=M)=\sum\limits_{j=1}^{M} H_j(R_{1j}(t), \ldots, R_{nj}(t))$.

(11) The probability density function of $T_k(k=1, 2, \ldots, M)$ is

$$f_k(t)=\sum_{i=1}^{n} f_{ik}(t)\,.\,\frac{\partial H_k(R_{1k}(t), \ldots, R_{nk}(t))}{\partial R_{ik}(t)},\ t>0.$$

Proof. $F_k(t)=1-H_k(R_{1k}(t), \ldots, R_{nk}(t))$ and from (i) we have

$$f_k(t)=-\sum_{i=1}^{n}\frac{\partial H_k(R_{1k}(t), \ldots, R_{nk}(t))}{\partial R_{ik}(t)}\ \frac{\partial R_{ik}(t}{\partial t}=\sum_{i=1}^{n} f_{ik}(t)\,\frac{\partial H_k(R_{1k}(t), \ldots, R_{nk}(t))}{\partial R_{ik}(t)};$$

and so $f_k(t)dt+o(dt)=\sum\limits_{i=1}^{n}[f_{ik}(t)+o(\partial t)]\,(\partial H_k(R_{1k}(t), \ldots, R_{nk}(t)))/\partial R_{ik}(t)$.

the probablity that the first enter of the system into the states $\{0, 1, \ldots, k-1\}$ in the interval $(t, t+dt)$ is equal to the sum (over i) of the product of the probability that the i-th component will enter into the states $\{0, 1, \ldots, k-1\}$ in the interval $(t, t+dt)$ and the probability that the last event will cause the system to enter these states (for the binary case, see [2], in particular, Proposition 3.2).

(12) The probability that the i-th component will cause the system to leave out the states $\{k, k+1, \ldots, M\}$ $(k=1, 2, \ldots, \min(M_i, M))$, under the condition that this system would leave out these states at time t, is given by $I_{ik}^{B}(t) \cdot f_{ik}(t) / \sum_{j=1}^{n} I_{jk}^{B}(t) \cdot f_{jk}(t)$.

Proof. From (m) above we see that $I_{ik}^{B}(t)$ is the probability that the system is functioning at time t at the states $\{k, k+1, \ldots, M\}$ if the i-th component is functioning at the states $\{k, k+1, \ldots, M_i\}$ but is not functioning at these states otherwise. Proposition (12) implies immediately.

(13) The probability that the i-th component causes that system to leave out the states $\{k, k+1, \ldots, M\}$ $(k=1, 2, \ldots, \min(M_i, M))$ in $[0, t]$ $(t>0)$, given this system leaves out these states in $[0, t]$, is

$$\int_0^t I_{ik}^{B}(u) f_{ik}(u) du \Big/ \sum_{j=1}^{n} \int_0^t I_{jk}^{B}(u) f_{jk}(u) du.$$

Proof. It is a consequence of proposition (12).

$$I_{ik}^{BP} = \int_0^\infty I_{ik}^{B}(t) f_{ik}(t)\, dt, \quad k=1, 2, \ldots, \min(M_i, M), \tag{14}$$

the Barlow-Proschan measure of the k-importance is a weighted average of the Birnbaum measure of the k-importance (the weight at time t being $f_{ik}(t)$).

Proof. Note that $H_k(q_k(x_i)=1, R_{1k}(t), \ldots, R_{nk}(t)) - H_k(q_k(x_i)=0, R_{1k}(t), \ldots, R_{nk}(t)) = \mathbf{P}[q_k\varphi(x_{ik}=k, \boldsymbol{X}(t)) - q_k\varphi(x_{ik}=0, \boldsymbol{X}(t))=1]$, $k=1, 2, \ldots, \min(M_i, M)$ and from definitions (m) and (n) we have proposition (14).

(15) For $i \in C$, $k=1, 2, \ldots, \min(M_i\, M)$,

$$I_{ik}^{B}(t) = \sum_{(\cdot_{ik},\, x_k)} [q_k\varphi(x_{ik}=k, \boldsymbol{x}_k) - q_k\varphi(x_{ik}=0, \boldsymbol{x}_k)] \times \prod_{j \neq i} [R_{jk}(t)^{q_k(x_{jk})} F_{jk}(t)^{1-q_k(x_{jk})}].$$

Proof. It follows from the independence of the processes $\{X_i(t), t>0\}$, $i \in C$.

(16) For $i \in C$, $k=1, 2, \ldots, \min(M_i, M)$, $0 \leq s \leq u$,

$$Q_{ik}(u, s) = \int_0^\infty \sum_{(\cdot_{ik},\, \boldsymbol{x}_k)} \prod_{j \neq i} [R_{jk}(t+s)^{q_k(x_{jk})} F_{jk}(t+s)^{1-q_k(x_{jk})}] f_{ik}(t) q_k\varphi(x_{ik}=0, \boldsymbol{x}_k)$$

$$\times \left[\left(1 - \frac{R_{ik}(t+u)}{R_{ik}(t)}\right) H_k(x_{ik}=0, \boldsymbol{R}_{k,\, t+s}^{q_k(\cdot_{ik},\, \boldsymbol{x}_k)}(u-s)) + \frac{R_{ik}(t+u)}{R_{ik}(t)} H_k(x_{ik}=k, \boldsymbol{R}_{k,\, t+s}^{q_k(\cdot_{ik},\, x_k)}(u-s))\right] dt.$$

Proof. We have $(0 \leq s \leq u)$

$$\mathbf{P}[Y_{ik}>u,\ Z_{ik}>s \mid (T_{ik}, (\cdot_{ik}, \boldsymbol{x}_k(T_{ik}+s))) = (t, (\cdot_{ik}, \boldsymbol{x}))]$$

$$= \mathbf{P}[q_k\varphi(\boldsymbol{X}_k(t+u)) q_k\varphi(x_{ik}=0, \boldsymbol{X}_k(t+s))=1 \mid q_k(X_{ik}(t))=1, (\cdot_{ik}, \boldsymbol{X}_k(t+s))=(\cdot_{ik}, \boldsymbol{x}_k)]$$

with pivotal decomposition

$$=q_k\varphi(x_{ik}=0,\ \boldsymbol{x}_k)\ [\frac{R_{ik}(t+u)}{R_{ik}(t)}H_k(x_{ik}=k,\ \boldsymbol{R}_{k,\,t+s}^{q_k(\cdot_{ik},\ \boldsymbol{x}_k)}(u-s))+(1-\frac{R_{ik}(t+u)}{R_{ik}(t)})$$

$$\times H_k(x_{ik}=0,\ \boldsymbol{R}_{k,\,t+s}^{q_k(\cdot_{ik},\ \boldsymbol{x}_k)}(u-s))],$$

by a conditional probability we have

$$\mathbf{P}(Y_{ik}>u,\ Z_{ik}>s\mid T_{ik}=t)=\sum_{(\cdot_{ik},\ \boldsymbol{x}_k)}\mathbf{P}[(\cdot_{ik},\ \boldsymbol{X}_k(T_{ik}+s))=(\cdot_{ik},\ \boldsymbol{x}_k)]$$

$$\times\mathbf{P}[Y_{ik}>u,\ Z_{ik}>s\mid(T_{ik},(\cdot_{ik},\ \boldsymbol{X}_k(T_{ik}+s)))=(t,(\cdot_{ik},\ \boldsymbol{x}_k))],$$

and $Q_{ik}(u,s)=\int_0^\infty\mathbf{P}(Y_{ik}>u,\ Z_{ik}>s\mid T_{ik}=t)\,f_{ik}(t)\,dt$, the expression for $Q_{ik}(u,s)$ is now immediate.

(17) $G_{ik}(u)=\int_0^\infty\sum_{(\cdot_{ik},\ \boldsymbol{x}_k)}\prod_{j\neq i}[(1-R_{jk}(t))^{1-q_k(x_{jk})}R_{jk}(t)^{q_k(x_{jk})}]\,H_k(\mathbf{R}_{k,\,t}^{q_k(x_{ik}=k;\ \boldsymbol{x}_k)}(u))\,f_{ik}(t)dt.$

P r o o f. For $u\geq 0$ we have

$$\mathbf{P}[Y_{ik}>u\mid(T_{ik},(\cdot_{ik},\ \boldsymbol{X}_k(T_{ik})))=(t,(\cdot_{ik},\ \boldsymbol{x}_k))]=\mathbf{P}[q_k\varphi(\boldsymbol{X}_k(t+u))=1\mid q_k(\boldsymbol{X}_{ik}(t),\ \boldsymbol{X}_k(t))$$

$$=(1,\ q_k(\boldsymbol{x}_k))]=H_k(\mathbf{R}_{k,\,t}^{q_k(x_{ik}=k,\ \boldsymbol{x}_k)}(u)),$$

the expression for $G_{ik}(u)$ is now immediate by a conditional probability argument.

(18) $S_{ik}(u)=\int_0^\infty\sum_{(\cdot_{ik},\ \boldsymbol{x}_k)}\prod_{j\neq i}[(1-R_{jk}(t))^{1-q_k(x_{jk})}R_{jk}(t)^{q_k(x_{jk})}]\,f_{ik}(t)\,H_k(\boldsymbol{R}_{k,\,t}^{q_k(x_{ik}=0,\ \boldsymbol{x}_k)}(u))dt,\ u\geq 0.$

P r o o f. The finding of the expression for $S_{ik}(u)$ is similar to (17).

(19) For $k=1, 2, \ldots, \min(M_i, M)$ we have $\mathbf{P}(Z_{ik}=0)=1-\mathbf{P}(Z_{ik}>0)=1-S_{ik}(0)$.

(20) The probability that the system left out the states $\{k, k+1, \ldots, M\}$ $(k=1, 2, \ldots \min M_i, M))$ before the i-th component leaves out the states $\{k, k+1, \ldots, M_i\}$ is, $\mathbf{P}(Y_{ik}=0,\ Z_{ik}=0)=1-G_{ik}(0)$.

P r o o f. $\mathbf{P}(Y_{ik}=0,\ Z_{ik}=0)=1-\mathbf{P}(Y_{ik}>0)-\mathbf{P}(Z_{ik}>0)+\mathbf{P}(Y_{ik}>0,\ Z_{ik}>0)=1-G_{ik}(0)-S_{ik}(0)+Q_{ik}(0,0)$, from (16), (17) (18) and since

$$H_k(\boldsymbol{R}_{k,\,t}^{q_k(x_{ik}=k,\ \boldsymbol{x}_k)}(0))=q_k\varphi(x_{ik}=k,\ \boldsymbol{x}_k),\ H_k(\boldsymbol{R}_{k,\,t}^{q_k(x_{ik}=0,\ \boldsymbol{x}_k)}(0))=q_k\varphi(x_{ik}=0,\ \boldsymbol{x}_k),$$

$$q_k\varphi(x_{ik}=1,\ \boldsymbol{x}_k)=q_k\varphi(x_{ik}=k,\ \boldsymbol{x}_k)+q_k\varphi(x_{ik}=0,\ \boldsymbol{x}_k)-q_k\varphi(x_{ik}=k,\ \boldsymbol{x}_k)\ q_k\varphi(x_{ik}=0,\ \boldsymbol{x}_k),$$

we have $1-G_{ik}(0)-S_{ik}(0)+Q_{ik}(0,0)=1-\int_0^\infty\sum_{(\cdot_{ik},\ \boldsymbol{x}_k)}\prod_{j\neq i}[(1-R_{jk}(t))^{1-q_k(x_{jk})}R_{jk}(t)^{q_k(x_{jk})}]$

$$\times q_k\varphi(x_{ik}=k,\ \boldsymbol{x}_k)\ f_{ik}(t)\,dt=1-G_{ik}(0).$$

(21) For $k=1, 2, \ldots, \min(M_i, M)$, we have $I_{ik}^{BP}=\mathbf{P}(Z_{ik}=0)-\mathbf{P}(Y_{ik}=0,\ Z_{ik}=0)$.

P r o o f. $I_{ik}^{BP}=\int_0^\infty\sum_{(\cdot_{ik},\ \boldsymbol{x}_k)}\prod_{j\neq i}[(1-R_{jk}(t))^{1-q_k(x_{jk})}R_{jk}(t)^{q_k(x_{jk})}]\,[q_k\varphi(x_{ik}=k,\ \boldsymbol{x}_k)-q_k$

$\times(x_{ik}=0,\ \boldsymbol{x}_k)]\,f_{ik}(t)\,dt=\int_0^\infty\sum_{(\cdot_{ik},\ \boldsymbol{x}_k)}\prod_{j\neq i}[(1-R_{jk}(t))^{1-q_k(x_{jk})}R_{jk}(t)^{q_k(x_{jk})}]\,[H_k(\boldsymbol{R}_{k,t}^{q_k(x_{ik}=k,\ x_k)}$

$$\times(0))-H_k(\boldsymbol{R}_{k,i}^{q_k(x_{ik}=0,\boldsymbol{x}_k)}(0))]\, f_{ik}(t)\, dt = G_{ik}(0)-S_{ik}(0)=\mathbf{P}(Z_{ik}=0)-\mathbf{P}(Y_{ik}=0,\ Z_{ik}=0).$$

We see that the Barlow-Proschan measure of the k-importance of the i-th component is just the difference $\mathbf{P}(Z_{ik}=0)-\mathbf{P}(Y_{ik}=0,\ Z_{ik}=0)$.

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Received 4. 10. 1985
Revised 4. 1. 1987