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A-COMPACT SPACES

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For each class **A** of topological spaces we have a closure operation [ ]: $P(X)\to P(X)$, called **A**-closure, where X is a topological space and $P(X)$ is the power set of X. In this paper we study the **A**-compact spaces, i. e. the topological spaces $(X, \tau)\in$ **A** such that the topology $\tau_{\mathbf{A}}$ generated by the **A**-closure is a compact topology.

We show that for many classess **A** the **A**-compact spaces play the same role in **A** that the compact Hausdorff spaces play in the class of Hausdorff spaces.

0. Introduction. For each class **A** of topological spaces we have a closure operation [ ]: $P(X)\to P(X)$, called **A**-closure, where X is a topological space and $P(X)$ is the power set of X [21]. Recently D. Dikranjan, E. Giuli ([2, 3]) characterized the closure operation [ ] for many classes of topological spaces **A**.

A topological space $(X, \tau)\in$ **A** is called **A**-compact with respect to **A** (in short **A**-compact) iff the topology $\tau_{\mathbf{A}}$ generated by the **A**-closure is a compact topology. The class of all **A**-compact spaces will be denoted by $K_{\mathbf{A}}$.

If **A** is the class of Hausdorff spaces $\mathbf{TOP}_2$ and $X\in\mathbf{TOP}_2$ then [ ] : $P(X)\to P(X)$ coincides with the ordinary closure operation in X ([2]), hence $K_{\mathbf{TOP}_2}$ is precisely the class of compact Hausdorff spaces.

In Section 1 we prove that many classical results about compact Hausdorff spaces are true for many classes **A** of topological spaces when we replace "compact Hausdorff" with "**A**-compact" and the ordinary closure with the **A**-closure, moreover we give an internal characterization of **A**-compact spaces using the Kuratowski theorem.

In Section 2 some general relations between **A**-compact and absolutely **A**-closed spaces are established.

Section 3 is devoted to the study of **Ury**-compact and absolutely **Ury**-closed spaces, where **Ury** is the class of Urysohn spaces. In this section we give an example of non **Ury**-minimal semiregular absolutely **Ury**-closed space.

In Section 4 we use the **A**-compactness to introduce the concept of $p(\mathbf{A})$ mapping.

Notation 0.1 The following categories are denoted as follows:

TOP the category of topological spaces and continuous functions;

$\mathbf{TOP}_i$ the category of topological spaces satisfying the T_i axiom $i=0, 1, 2$;

$\mathbf{FT}_2$ the category of functionally Hausdrorff spaces (points are separated by continuous real valued maps);

Ury the category of Urysohn spaces points are separated by disjoint closed neighborhoods);

$\mathbf{TOP}_3$ the category of regular Hausdorff spaces;

Tych the category of completely regular Hausdorff spaces;

0-dim the category of zero-dimensional spaces (i. e. Hausdorff spaces with a base of closen sets).

A full and isomorphism-closed subcategory **A** of **TOP** is said to be epireflective if for each topological space X there exist $rX\in$ **A** and an epimorphism $r_{\mathbf{A}}: X\to rX$

in **TOP** such that for each continuous function $f: X \to Y$, $Y \in A$, there exists a continuous function $f': rX \to Y$ such that $f' \circ r_A = f$. **A** is epireflective in **TOP** iff it is closed under the formation of products and subspaces [13].

All subcategories above are epireflective subcategories of **TOP**.

We define, now, a closure operator introduced by S. Salbany [21].

Definition 0.2 *Let* **A** *be an epireflective subcategory of* **TOP**, *let* X *be a topological space and* F *a subset of* X.

(a) *A point* X *of* X *is said to be* a point *of* **A**-*closure of* F *in* X *if for each* $f, g: X \to A$, $A \in \mathbf{A}$, *such that* $f|F = g|F$ (*where* $f|F$ *denotes the restruction of* f *to* F), $f(x) = g(x)$. *The set of all points of* **A**-*closure of* F *in* X *is said to be the* **A**-*closure of* F *in* X *and it is denoted by* $[F]^X_{\mathbf{A}}$.

(b) F is said to be **A**-closed in X if $[F]^X_{\mathbf{A}} = F$.

(c) *A function* $f: X \to Y$, $X, Y \in \mathbf{A}$, *is said to be* **A**-*continous if* $f([F]^X_{\mathbf{A}}) \subset [f(F)]^Y_{\mathbf{A}}$, $F \subset X$.

(d) *A function* $f: X \to Y$, $X, Y \in \mathbf{A}$, *is said to be* **A**-*closed if for every* **A**-*closed set* $F \subset X$ *the image* $f(F)$ *is* **A**-*closed in* Y.

(e) *the coarsest topology in* X *which contains all* **A**-*closed subsets as closed sets is said to be the* **A**-*closure topology of* X *and, if* τ *is the topology of* X, *it is denoted by* $\tau_{\mathbf{A}}$.

$F: \mathbf{TOP} \to \mathbf{TOP}$ will denote the functor which assigns to $(X, \tau) \in \mathbf{TOP}$ the space $(X, \tau_{\mathbf{A}})$. For each continuous map $f: (X, \tau) \to (Y, \sigma)$ in **TOP** the continuity of $f = F_A(f) : (X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ follows from 1.2(x) of [3].

The following results can be found in [2].

(1) $\tau_{\mathbf{A}} \leq \tau$ for all $(X, \tau) \in \mathbf{A}$ iff $\mathbf{A} \subset \mathbf{TOP_2}$.

(2) For $\mathbf{A} = \mathbf{TOP_2}$, $\mathbf{TOP_3}$, **Tych**, **0-dim** $\tau_{\mathbf{A}} = \tau$ for each $(X, \tau) \in \mathbf{A}$.

(3) The $\mathbf{TOP_0}$-closure is the front-closure defined in [19]: $Fr\,\mathrm{Cl}(A) = \{x \in X$: for each open nhood $(U$ of x, $\overline{\{x\}} \cap U \cap A \neq \emptyset\}$

(4) The $\mathbf{TOP_1}$-closure is the identity for all T_1-spaces.

(5) For $\mathbf{A} = \mathbf{Ury}$ let $X \in \mathbf{A}$ and $M \subset X$, we define $\mathrm{Cl}_\theta(M) = \{x \in X$: for each nhood V of x, $\overline{V} \cap M \neq \emptyset\}$, this is the θ-closure introduced by H. Velichko [23].

For $X \in \mathbf{Ury}$ and $M \subset X$ we have $\mathrm{Cl}_\theta(M) \subset [M]^X_{\mathbf{Ury}}$ and $M = \mathrm{Cl}_\theta(M)$ iff $M = [M]^X_{\mathbf{Ury}}$, thus the **Ury**-closure is the idempotent hull of Cl_θ.

Finally we recall that the topological terminology is that of [24].

1. A-Compact spaces.

Definition 1.1. *Let* **A** *be an epireflective subcategory of* **TOP**. $(X, \tau) \in \mathbf{A}$ *is said to be* **A**-*Compact iff* $(X, \tau_{\mathbf{A}})$ *is compact.*

We will denote by $K_{\mathbf{A}}$ the class of **A**-compact spaces, and by **A** Comp the class of compact spaces X such that $X \in \mathbf{A}$. The **A**-closure is said to be hereditary [7], if for $M \subset Y \subset X$ we have $[M]^Y_{\mathbf{A}} = [M]^X_{\mathbf{A}} \cap Y$ for all M, Y and X. In that case $i = F_{\mathbf{A}}(i) : (Y, \sigma_{\mathbf{A}}) \to (X, \tau_{\mathbf{A}})$ is an embedding, where τ is the topology of X, σ the relative topology of Y, and i is the embedding map of the subspace (Y, σ) in the space (X, τ).

Remark 1.2. For all categories **A** listed in 0. except $\mathbf{A} = \mathbf{Ury}$, $\mathbf{FT_2}$ and for every space $(X, \tau) \in \mathbf{A}$ the **A**-closure is hereditary in (X, τ).

Proposition 1.3. *Let* **A** *be such that for all* $(X, \tau) \in \mathbf{A}$ *the* **A**-*closure is hereditary in* (X, τ). *If* $(X, \tau) \in K_{\mathbf{A}}$ *and* $M \subset X$ is **A**-*closed then* M, *with the relative topology* τ_M, *is* **A**-*compact.*

Proof. M is closed in $(X, \tau_{\mathbf{A}}) \in$ Comp, therefore $(M, \tau_{\mathbf{A}})$ is compact, hence (M, τ_M) is **A**-compact.

Remark 1.4. The space $(X, \tau) \in \mathbf{Ury}$ given in ([6], ex. 4.2) belongs to $K_{\mathbf{Ury}}$ and it has an **Ury**-closed subset M such that (M, τ_M) is an infinite discrete space.

Remarks 1.5.

(1) $K_{\mathbf{TOP}_1}$ is the class of finite discrete spaces.

(2) $K = \mathbf{A}$Comp $= \mathbf{TOP}_2$ Comp of $\mathbf{A} = \mathbf{TOP}_2$, $\mathbf{TOP}_3$, **Tych**, **0-dim**

(3) Let **A** be the subcategory LM-T_2 of Lawson—Madison spaces (a space X is LM-T_2 iff every compact subspace of X is T_2, [16], [18]), since $\tau \leqq \tau_{\mathbf{A}}$ ([5], lemma 1.5) we have that $K_{\mathbf{A}} \subset \mathbf{A}$Comp, moreover if $(X, \tau) \in \mathbf{A}$Comp then $\tau = \tau_{\mathbf{A}}$ ([11], corollary 4.2 (b)) hence $K_{\mathbf{A}} = \mathbf{A}$Comp.

(4) Let X_j denote a T_1-space with cofinite topology and infinite cardinality j.

If $\mathbf{A} = \mathbf{Haus}$ $(\{X_j\}) = \{X \in \mathbf{TOP}$ such that every continuous map $f: X_j \rightarrow X$ is constant$\}$ [16], then $\tau_{\mathbf{A}}$ is discrete for every $(X, \tau) \in \mathbf{A}$ ([5], prop. 1.11), hence $K_{\mathbf{A}}$ is the class of finite discrete spaces and $K_{\mathbf{A}} \underset{\neq}{\subset} \mathbf{A}$Comp.

(5) The **Ury**-compact spaces are studied in the Section 3. A careful study of $\mathbf{TOP}_0$-compact spaces will be the object of a separate work.

Proposition 1.6. *For* $\mathbf{A} = \mathbf{Top}_0$, $\mathbf{Top}_1$, $\mathbf{Top}_2$, $\mathbf{Top}_3$, **Tych**, **0-dim** *let* $X \in K_{\mathbf{A}}$ *and* $M \subset X$, *then the following conditions are equivalent*:

(i) *M is* **A**-*compact*

(iii) *M is* **A**-*closed.*

Proof. (i) ⇒ (ii) For $\mathbf{A} = \mathbf{TOP}_1$ the **A**-closure is the identity for all T_1-spaces.

For $\mathbf{A} = \mathbf{TOP}_2$, $\mathbf{TOP}_3$, **Tych**, **0-dim** we recall that the **A**-closure coincides with the ordinary closure. For $\mathbf{A} = \mathbf{TOP}_0$ if $M \subset (X, \tau) \in \mathbf{TOP}_0$ is $\mathbf{TOP}_0$-compact then $(M, \tau_{\mathbf{A}})$ is compact in $(X, \tau_{\mathbf{A}})$, but $(X, \tau_{\mathbf{A}})$ is a T_2-space ([3], th. 2.2 (e)), hence M is closed in $(X, \tau_{\mathbf{A}})$, i. e. it is $\mathbf{TOP}_0$-closed in (X, τ).

The other implication follows from the proposition 1.3.

Observe that the implication (i) ⇒ (ii) holds for all categories **A** such that for each $(X, \tau) \in \mathbf{A}$ we have $\tau \leqq \tau_{\mathbf{A}}$ and every compact subspace of X is **A**-closed in X, for some examples see [10].

Proposition 1.7. *For* $\mathbf{A} = \mathbf{TOP}_0$, $\mathbf{TOP}_1$, $\mathbf{TOP}_2$, $\mathbf{TOP}_3$, **Tych**, **0-dim** *let* $(X, \tau) \in K_{\mathbf{A}}$ *and* $(Y, \sigma) \in \mathbf{A}$.

(1) *If* $f: (X, \tau) \rightarrow (Y, \sigma)$ *is a continuous and onto mapping then* $(Y, \sigma) \in K_{\mathbf{A}}$;

(2) *If* $f: (X, \tau) \rightarrow (Y, \sigma)$ *is continuous then it is* **A**-*closed.*

Proof. (1) Since $f = F_{\mathbf{A}}(f): (X, \tau_{\mathbf{A}}) \rightarrow (Y, \sigma_{\mathbf{A}})$ is continuous and onto, $(X, \tau_{\mathbf{A}})$ is compact, we have that $(Y, \sigma_{\mathbf{A}})$ is a compact space, hence $(Y, \sigma) \in K_{\mathbf{A}}$.

(2) Let G be an **A**-closed set in X, hence from prop. 1.3 follows that G is **A**-compact, so by (1) we have that $f(G)$ is **A**-compact, and from prop. 1.6 it follows that $f(G)$ is **A**-closed in Y, i. e. $f(G) = [f(G)]^Y_{\mathbf{A}}$.

Note that (1) holds for all subcategories **A** of **TOP**.

The functor $F_{\mathbf{A}}$ is said to be finitely multiplicative if it preserves finite products, i. e. $(\prod_i \tau_i)_{\mathbf{A}} = \prod_i (\tau_i)_{\mathbf{A}}$, $i = 1, \ldots, n$. We recall that if $F_{\mathbf{A}}$ is finitely multiplicative then for each $(X, \tau) \in \mathbf{A}$, $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$ (prop. 2.5, [3]).

Henceforth we consider only the epireflective subcategories **A** such that the **A**-closure is a Kuratowski operator.

The Kuratowski theorem. For a topological space X the following conditions are equivalent:

(i) The space X is compact.

(ii) For every topological space Y the projection $p: X \times Y \rightarrow Y$ is closed.

(iii) For every T_4-space Y the projection $p: X \times Y \rightarrow Y$ is closed.

Theorem 1.8. *Let* **A** *be an epireflective subcategory of* **TOP** *such that*

(1) $F_{\mathbf{A}}$ *is finitely multiplicative*;

(2) *For every* $X \in T_4$ *there exists* $Y \in$ **TOP** *such that* $F_{\mathbf{A}}(Y)=X$. *Then the following conditions are equivalent*:
(a) *the space* Z *is* **A**-*compact*
(b) *for every topological space* Y *the projection* $p: Z \times Y \to Y$ *is* **A**-*closed*.

Proof. (a) $\Rightarrow$ (b). Let $(Z, \tau) \in K_{\mathbf{A}}$, $(Y, \sigma) \in$ **TOP** and let $p:(Z, \tau)\times(Y, \sigma) \to (Y, \sigma)$ be the projection. Since $(Z, \tau_{\mathbf{A}}) \in$ Comp and $F_{\mathbf{A}}((Z, \tau)\times(Y, \delta))=(Z, \tau_{\mathbf{A}})\times(Y, \sigma_{\mathbf{A}})$ then the projection $p=F_{\mathbf{A}}(p):(Z, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is closed (by the Kuratowski theorem), hence $p:(Z, \tau)\times(Y, \sigma) \to (Y, \sigma)$ is **A**-closed.

(b) $\Rightarrow$ (a). If $(Z, \tau) \notin K_{\mathbf{A}}$ then $(Z, \tau_{\mathbf{A}}) \notin$ Comp hence (by the Kuratovski theorem) there exists $(X, \sigma_{\mathbf{A}}) \in T_4$ such that the projection $p=F_{\mathbf{A}}(p):(Z, \tau_{\mathbf{A}}) \times (X, \sigma_{\mathbf{A}}) \to (X, \sigma_{\mathbf{A}})$ is not closed, but there exists $(Y, \sigma) \in$ **TOP** such that $F_{\mathbf{A}}((Y, \sigma))=(X, \sigma_{\mathbf{A}})$ hence the projection $p:(Z, \tau)\times(Y, \sigma) \to (Y, \sigma)$ is not **A**-closed.

A categorical interpretation of Kuratowski's Theorem is given in [14] (see also [9]).

Next we give a list of well-known properties of compact spaces.

(1) X is compact iff every familly of closed subsets of X with the finite intersection property has a non-empty intersection.
(2) The continuous image of a compact space is compact.
(3) A subset of a compact Hausdorff space is compact iff it is closed.
(4) (A. Tychonoff) Let $\{X_i \mid i \in I\}$ be any family of spaces. Then $\Pi_I X_i$ is compact iff each X_i is compact.
(5) Let $Y \in \mathbf{TOP}_2$ Comp, $Z \in T_2$ and $f: Y \to Z$ continuous. Then f is a closed map.
(6) Let $X \in T_2$ and $Y \in \mathbf{TOP}_2$ Comp. Then $f: X \to Y$ is continuous iff its graph $G(f)$ is closed in $X \times Y$.

Remark 1.9. From property (1) above it follows easily that a space $X \in \mathbf{A}$ is **A**-compact iff every family of **A**-closed subsets of X with the finite intersection property has a non-empty intersection.

Theorem 1.10. *Let* **A** *be an epireflective subcategory of* **TOP** *such that* $F_{\mathbf{A}}$ *is finitely multiplicative. If* $(X, \tau) \in \mathbf{A}$ *and* $(Y, \sigma) \in K_{\mathbf{A}}$ *then the following conditions are equivalent*:
(a) $f:(X, \tau) \to (Y, \sigma)$ *is* **A**-*continuous*;
(b) *The graph* $G(f)$ *of* f *is* **A**-*closed*.

Proof. (a) $\Rightarrow$ (b). If $f:(X, \tau) \to (Y, \sigma)$ is **A**-continuous then $f=F_{\mathbf{A}}(f):(X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is continuous, since $(Y, \sigma_{\mathbf{A}}) \in \mathbf{TOP}_2$ we have that $G(f)$ is closed in $(X, \tau_{\mathbf{A}}) \times (Y, \sigma_{\mathbf{A}})=(X\times Y(\tau\times\sigma)_{\mathbf{A}})$, therefore $G(f)$ is **A**-closed in $(X, \tau)\times(Y, \sigma)$.

(b) $\Rightarrow$ (a), Let $G(f)$ be **A**-closed in $(X, \tau)\times(Y, \sigma)$, hence $G(f)$ is closed in $(X, \tau_{\mathbf{A}}) \times(Y, \sigma_{\mathbf{A}})$, but $(Y, \sigma_{\mathbf{A}}) \in$ Comp and it is a T_2-space, so we have that $f=F_{\mathbf{A}}(f):(X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is continuous, therefore $f:(X, \tau) \to (Y, \sigma)$ is **A**-continuous.

Proposition 1.11. *Let* **A** *be an epireflective subcategory of* **TOP** *such that* $F_{\mathbf{A}}$ *is finitely multiplicative, then* $\prod_{i=1}^{n} (X_i, \tau_i) \in K_{\mathbf{A}}$ *if and only if* $(X_i, \tau_i) \in K_{\mathbf{A}}$, $i=1, \ldots, n$.

Proof. Let $\prod_{i=1}^{n}(X_i, \tau_i)$ be **A**-Compact, since the projections $p_i: \prod_{i=1}^{n}(X_i, \tau_i) \to (X_i, \tau_i)$ are continuous and onto, it follows from prop. 1.7 that (X_i, τ_i) is **A**-Compact $i=1, \ldots, n$.

If (X_i, τ_i) is **A**-Compact, $i=1, \ldots, n$, then we have that $(X_i, (\tau_i)_{\mathbf{A}}) \in$ Comp for every i, hence $\prod_{i=1}^{n}(X_i, (\tau_i)_{\mathbf{A}}) \in$ Comp, but $\prod_{i=1}^{n}(X_i, (\tau_i)_{\mathbf{A}}) = (\prod_{i=1}^{n}(X_i, \tau_i))_{\mathbf{A}}$, so $\prod_{i=1}^{n}(X_i, \tau_i)$ is

A-Compact. Let $\{\tau_\alpha | \alpha \in I\}$ be a family of topologies on a fixed set X. The identity function from the set X to the space (X, τ_α) will be denoted i_α. Let $\tau = \sup\{\tau_\alpha \mid \alpha \in I\}$, i. e. the weak topology induced on X by the maps i_α. Let $\Delta = \{x \in \Pi(X, \tau_\alpha) \mid x_\alpha = x_\beta$ for all $\alpha, \beta\}$, (X, τ) is homeomorphic to Δ in the product space $\Pi(X, \tau_\alpha)$ ([24], 81.2.).

Proposition 1.12. *Let* **A** *be an epireflective subcategory of* **TOP**. *Let* $\{(X, \tau_\alpha) \mid \alpha \in I\}$ *be a family of topological spaces such that* $(X, \tau_\alpha) \in \mathbf{A}$ *for each* $\alpha \in I$, *then* $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} \leqq \tau_\mathbf{A}$.

Proof. (X, τ) is homeomorphic to Δ in the product space $\Pi(X, \tau_\alpha) \in \mathbf{A}$, hence $(X, \tau) \in \mathbf{A}$. Since every map $i_\alpha : (X, \tau) \to (X, \tau_\alpha)$ is continuous then every $i_\alpha = F_\mathbf{A}(i_\alpha)$: $(X, \tau_\mathbf{A}) \to (X, (\tau_\alpha)_\mathbf{A})$ is continuous too.

As $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\}$ is the weak topology induced on X by the maps $i_\alpha = F_\mathbf{A}(i_\alpha)$: $X \to (X, (\tau_\alpha)_\mathbf{A})$ we have that $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} \leqq \tau_\mathbf{A}$.

Corollary 1.13. *Let* $\{(X, \tau_\alpha) \mid \alpha \in I\}$ *be a family of topological spaces such that* $(X, \tau_\alpha) \in \mathbf{A}$ *for every* $\alpha \in I$. *If* $(X, \tau) \in K_\mathbf{A}$ *then* $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\}$ *is a compact topology.*

Corollary 1.14. *Let* $\{(X, \tau_\alpha) \mid \alpha \in I\}$ *be a family of topological spaces such that* $(X, \tau_\alpha) \in \mathbf{A}$ *and* $(X, (\tau_\alpha)_\mathbf{A}) \in \mathbf{TOP}_2$ *for every* $\alpha \in I$. *If* $(X, \tau) \in K_\mathbf{A}$ *then* $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} = \tau_\mathbf{A}$.

Proof. $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\}$ is a Hausdorff topology, it is weaker than $\tau_\mathbf{A}$ and $(X, \tau_\mathbf{A}) \in \mathbf{TOP}_2$ Comp, then $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} = \tau_\mathbf{A}$.

Proposition 1.15. *Let* $\mathbf{A} \subset \mathbf{TOP}_2$ *and* $\{(X, \tau_\alpha) \mid \alpha \in I\}$ *be a family of topoligical spaces such that* $(X, \tau_\alpha) \in \mathbf{A}$ *and* $(X, (\tau_\alpha)_\mathbf{A}) \in \mathbf{TOP}_2$ *for every* $\alpha \in I$. *If* (X, τ) *is a compact space then* $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} = (\tau_\alpha)_\mathbf{A} = \tau_\alpha = \tau = \tau_\mathbf{A}$ *for every* $\alpha \in I$.

Proof. From $\mathbf{A} \subset \mathbf{TOP}_2$ it follows that $\tau_\mathbf{A} \leqq \tau$, by virtue of prop. 1.12 we have that $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} \leqq \tau_\mathbf{A}$, from the definition of $\sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\}$ it follows that $(\tau_\alpha)_\mathbf{A} \leqq \sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\}$ for every $\alpha \in I$, then $(\tau_\alpha)_\mathbf{A} \leqq \sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} \leqq \tau_\mathbf{A} \leqq \tau$ for every $\alpha \in I$. Since $(X, (\tau_\alpha)_\mathbf{A}) \in \mathbf{TOP}_2$ and $(X, \tau) \in \mathbf{A}$-Comp $\subset \mathbf{TOP}_2$ Comp we have that $(\tau_\alpha)_\mathbf{A} = \sup\{(\tau_\alpha)_\mathbf{A} \mid \alpha \in I\} = \tau_\mathbf{A} = \tau$ for $\alpha \in I$. Moreover, by the definition of τ it follows that $\tau_\mathbf{A} \leqq \tau$ for every $\alpha \in I$, but $\tau = (\tau_\alpha)_\mathbf{A} \leqq \tau_\alpha$, hence $\tau_\alpha = \tau$ for every $\alpha \in I$.

2. A-Compact and absolutely A-closed space.

Definition 2.1 [4]. *Let* **A** *be an epireflective subcategory of* **TOP**.

$X \in \mathbf{A}$ *is said to be absolutely* **A**-*closed iff for each embedding* $f: X \to Y$, $Y \in \mathbf{A}$, $f(X)$ *is an* **A**-*closet subset of* Y.

The absolutely $\mathbf{TOP}_2$-closed spaces are the classical H-closed spaces [1]. For $\mathbf{A} = \mathbf{FT}_2$ (respectively $\mathbf{A} = \mathbf{TOP}_3$, **Tych, 0-dim**) we get the well-known functionally Hausdorff-closed spaces (resp. T_3-closed, Tychonoff-closed, 0-dimensional closed spaces). Some results about absolutely **Ury**-closed spaces (cf. [4], [6]) are given in the next section. Every T_1-space is an absolutely $\mathbf{TOP}_1$-closed space, and for $\mathbf{A} = \mathbf{TOP}_0$ we get the well-known sober spaces [15] (also called pc-spaces [19]).

Remark 2.2. By virtue of corollary 2.5 in [4] we have that $K_{\mathbf{FT}_2} =$ Absolutely $\mathbf{FT}_2$-closed.

Proposition 2.3. *Let* **A** *be such that for every* $(X, \tau) \in \mathbf{A}$ *either* $(X, \tau_\mathbf{A}) \in \mathbf{TOP}_2$ *or the* **A**-*closure is hereditary in* (X, τ). *Then every* **A**-*compact space is an absolutely* **A**-*closed space.*

Proof. Let $f: (X, \tau) \to (Y, \sigma) \in \mathbf{A}$ be an embedding and $(X, \tau) \in K_\mathbf{A}$, $(X, \tau_\mathbf{A})$ $\mathbf{TOP}_2$ Comp and $(Y, \sigma_\mathbf{A}) \in \mathbf{TOP}_2$, $f = F_\mathbf{A}(f): (X, \tau_\mathbf{A}) \to (Y, \sigma_\mathbf{A})$ is continuous, hence $f(X)$ is compact in $(Y, \sigma_\mathbf{A})$, therefore it is closed in $(Y, \sigma_\mathbf{A})$ and **A**-closed in (Y, σ) (recall that we consider only the **A**-closures that are Kuratowski operators), so (X, τ) is an absolutely **A**-closed space. The rest of the proof follows from prop. 1.7 (2).

Proposition 2.4. *If* $\mathbf{TOP}_2$ Comp $\subset \mathbf{A} \subset \mathbf{TOP}_2$ *then any locally compact and absolutely* **A**-*closed space is* **A**-*compact.*

Proof. Clearly we have $\mathbf{A}$ Comp$\subset K_{\mathbf{A}}$.
Let X be locally compact and absolutely **A**-closed, if X^* is the Alexandroff compactification of X and $h: X \to X^*$ is the embedding, since $X^* \in \mathbf{TOP}_2$ Comp $\subset \mathbf{A}$ we have that $h(X)$ is **A**-closed in X^* hence it is closed in X^*, therefore $h(X) \in \mathbf{A}$Comp$\subset K_{\mathbf{A}}$ and X is **A**-compact.

3. Ury-compact and absolutely Ury-closed space. From the theorem 2.4 in [6] it follows that every absolutely **Ury**-closed space is a **Ury**-compact space. It is an open question if there exists a **Ury**-compact space that it is not absolutely **Ury**-closed (see [6]). We will prove that every almost regular **Ury**-compact is absolutely **Ury**-closed. Let (X, τ) be a topological space, we will denote by τ_s, the coarsest topology on X which has as a base of the open sets all regularly open sets of (X, τ). (We recall that an open set U of a topological space (X, τ) is said to be regularly open iff $U = \mathrm{Int}(\bar{U})$). This topology is called the semiregularization of τ and (X, τ) is called semiregular space iff $\tau = \tau_s$. A Urysohn space (X, τ) is said to be almost regular [20] if $(X, \tau_s) \in \mathbf{TOP}_3$.

Lemma 3.1. ([6]) *For each* $(X, \tau) \in \mathbf{Ury}$, $(X, \tau_s) \in \mathbf{Ury}$ *and* $\tau_{\mathrm{Ury}} = (\tau_s)_{\mathrm{Ury}}$.
By lemma 3.1 it follows that

Proposition 3.2. *Let* $(X, \tau) \in \mathbf{Ury}$ *then* $(X, \tau) \in K_{\mathrm{Ury}}$ *iff* $(X, \tau_s) \in K_{\mathrm{Ury}}$.

Proposition 3.3. ([6]) *For a Uryson space* (X, τ) *the following conditions are equivalent*:

(a) (X, τ) *is almost regular*

(b) $\tau_s = \tau_{\mathrm{Ury}}$

We recall that a space (X, τ) *is said to be nearly compact iff* (X, τ_s) *is a compact Hausdorff space* [22].

Proposition 3.4. *Let* (X, τ) *be an almost regular space. Then the following conditions are equivalent*:

(*a*) (X, τ) *is absolutely* **Ury**-*closed*;

(b) (X, τ) *is* **Ury**-*compact*;

(c) (X, τ) *is nearly compact.*

Proof. (a)$\Leftrightarrow$(c) Corollary 4.4 in [4]. (a)$\Rightarrow$(b) is always true. (b)$\Rightarrow$(*c*) If (X, τ) is **Ury**-compact then (X, τ_{Ury}) is compact, hence by Lemma 3.1 and Proposition 3.3, we have that (X, τ_s) is a compact Hausdorff space, i. e. (X, τ) is nearly compact. A topological space $(X, \tau) \in \mathbf{A}$ is called **A**-minimal if $\tau' \leqq \tau$ and $(X, \tau') \in \mathbf{A}$ imply $\tau' = \tau$.

Proposition 3.5. *Let* $(X, \tau) \in \mathbf{Ury}$ *such that it is* **Ury**-*minimal and* $(X, \tau_{\mathrm{Ury}}) \in$ LM-T_2. *Then the following conditions are equivalent*:

(1) (X, τ) *is compact*;

(2) (X, τ) *is absolutely* **Ury**-*closed*;

(3) (X, τ) *is* **Ury**-*compact.*

Proof. (1)$\Rightarrow$(2) and (2)$\Rightarrow$(3) are well known. Now let (X, τ) be a **Ury**-compact space then $(X, \tau_{\mathrm{Ury}}) \in$ LM-T_2 Comp-$\mathbf{TOP}_2$ Comp, hence $(X, \tau_{\mathrm{Ury}}) \in \mathbf{Ury}$ and $\tau_{\mathrm{Ury}} \leqq \tau$, then $\tau_{\mathrm{Ury}} = \tau$ and (X, τ) is compact.

Theorem 3.6. *Let* $(X, \tau) \in \mathbf{FT}_2$. *If* (X, τ) *is* **Ury**-*compact then* $(X, \tau_{\mathrm{Ury}}) \in \mathbf{Tych}$.

Proof. For every $x, y \in X$ there exists a continuous function $f_k: (X, \tau) \to (I, \sigma)$, where $I = [0, 1]$ with the usual topology, such that $f(x) \neq f(y)$, then the collection $\{f_k: (X, \tau) \to (I, \sigma)\}_{k \in K}$ separates points in X, hence the mapping $f: (X, \tau) \to \Pi(I_k, \sigma_k)$ (where $(I_k, \sigma_k) = (I, \sigma)$ for each $k \in K$) assigning to the point $x \in X$ the point $\{f_k(x)\} \in \Pi(I_k, \sigma_k)$ is one-one and continuous ([8], 2. 3. 20). Since $\Pi(I_k, \sigma_k)$ is a T_3-space we have that $F_{\mathrm{Ury}}[\Pi(I_k, \sigma_k)] = \Pi(I_k, \sigma_k)$ ([2]), hence $f = F_{\mathrm{Ury}}(F): (X, \tau_{\mathrm{Ury}}) \to \Pi(I_k, \sigma_k)$ is a continuons and one-one mapping. Since $(X, \tau_{\mathbf{Ury}}) \in$ Comp and $\Pi(I_k, \sigma_k) \in \mathbf{TOP_2}$ we have that $f_{\mathbf{Ury}}(F)$ is closed, then $f = F_{\mathbf{Ury}}(F) : (X, \tau_{\mathbf{Ury}}) \to \Pi(I_k, \sigma_k)$ is an embedding, hence $(X, \tau_{\mathbf{Ury}}) \in$ **Tych.**

P r o p o s i t i o n 3.7. *Let* $(X, \tau) \in \mathbf{FT}_2$. *If* (X, τ) *is* **Ury**-*minimal and* **Ury**-*compact then it is compact.*

P r o o f. From the theorem 3.6 it follows that $(X, \tau_{\mathbf{Ury}}) \in$ **Tych** $\subset$ LM-T_2 hence by the prop. 3.5 we have that (X, τ) is compact.

It was proved by M. K a t e t o v [17] that a semiregular absolutely $\mathbf{TOP}_2$-closed space is $\mathbf{TOP}_2$-minimal. Now we give an example of a semiregular absolutely **Ury**-closed space that it is not **Ury**-minimal. (H. Herrlich produced an example of non **Ury**-minimal semiregular Urysohn space such that it is closed in each Urysohn space in which it is embedded [12]). The space (X, τ) described in [4] (example 5) is:

(a) functionally Hausdorff;

(b) absolutely **Ury**-closed;

but it is non nearly compact, hence from the prop. 3.4 it follows that (X, τ) is not almost regular, therefore $\tau_{\mathbf{Ury}} \underset{\neq}{<} \tau_s$ (by prop. 3.3). Since continuous images of absolutely **Ury**-closed spaces are absolutely **Ury**-closed spaces ([4], corollary 4.2), we have that (X, τ_s) is semiregular and absolutely **Ury**-closed. Now we prove that (X, τ_s) in not **Ury**-minimal. From Theorem 3.6 it follows that $(X, \tau_{\mathbf{Ury}}) \in$ **Tych**, hence $(X, \tau_{\mathbf{Ury}})$ is a Urysohn space and $\tau_{\mathbf{Ury}} \underset{\neq}{<} \tau_s$ then (X, τ_s) is not **Ury**-minimal.

4. p(A) mappings. In this section we consider only the epireflective subcategories **A** such that for all $(X, \tau) \in \mathbf{A}$ the **A**-closure is hereditary in (X, τ) and it is a Kuratowski operator.

D e f i n i t i o n 4.1. *Let* $X, Y \in \mathbf{A}$, *an* **A**-*continuous mapping* $f : X \to Y$ is $p(\mathbf{A})$ *if* f *is an* **A**-*closed mapping and all fibers* $f^{-1}(y)$ *are* **A**-*compact subsets of* X.

R e m a r k 4.2. For $\mathbf{A} = \mathbf{TOP_2}$, $\mathbf{TOP}_3$, **Tych**, **0-dim** every $p(\mathbf{A})$ mapping is perfect.

P r o p o s i t i o n 4.3. *Let* (X, τ), $(Y, \sigma) \in \mathbf{A}$ *with* $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$. $f : (X, \tau) \to (Y, \sigma)$ *is* $p(\mathbf{A})$ *iff* $f = F_{\mathbf{A}}(f) : (X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ *is perfect.*

P r o o f. If $f : (X, \tau) \to (Y, \sigma)$ is $p(\mathbf{A})$ then it is **A**-continuous, **A**-closed and for, each $y \in Y f^{-1}(y)$ is **A**-compact in (X, τ), then $f = F_{\mathbf{A}}(f) : (X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is continuous closed and all fibers $f^{-1}(y)$ are compact in $(X, \tau_{\mathbf{A}})$ (since the **A**-closure is hereditary, hence $f = F_{\mathbf{A}}(f)$ is perfect. If $f = F_{\mathbf{A}}(f)$ is perfect then it is continuous, closed and all fibers $f^{-1}(y)$ are compact in $(X, \tau_{\mathbf{A}})$, hence $f : (X, \tau) \to (Y, \sigma)$ is **A**-continuous, **A**-closed and all fibers $f^{-1}(y)$ are **A**-compact in (X, τ) (since the **A**-closure is hereditary), therefore f is $p(\mathbf{A})$.

P r o p o s i t i o n 4.4. *Let* (X, τ), $(Y, \sigma) \in \mathbf{A}$ *with* $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$. *An* **A**-*continuous and one-one mapping* $f : (X, \tau) \to (Y, \sigma)$ *is* $p(\mathbf{A})$ *iff it is* **A**-*closed.*

P r o o f. If $f : (X, \tau) \to (Y, \sigma)$ is **A**-closed then $f = F_{\mathbf{A}}(f) : (X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is continuous, one-one and closed, and $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$, hence $f = F_{\mathbf{A}}(f)$ is perfect, then by 4.3 follows that f is $p(\mathbf{A})$.

The converse is obvious.

C o r o l l a r y 4.5. *Let* $(X, \tau) \in \mathbf{A}$ *and let* (M, τ_M) *be a subspace of* X *such that* $(M, (\tau_M)_{\mathbf{A}}) \in \mathbf{TOP}_2$. *The embedding* $i_M : (M, \tau_M) \to (X, \tau)$ *is* $p(\mathbf{A})$ *iff* $M = [M]^X_{\mathbf{A}}$.

By 1.3 and 1.7 (2) follows that

P r o p o s i t i o n 4.6. *For* $\mathbf{A} = \mathbf{TOP}_0$, $\mathbf{TOP}_1$, $\mathbf{TOP}_2$, $\mathbf{TOP}_3$, **Tych**, **0-dim** *let* $X \in K_{\mathbf{A}}$ *and* $Y \in \mathbf{A}$, *then every A-continuous mapping* $f : X \to Y$ *is* $p(\mathbf{A})$.

Let **A** be such that: (1) $F_{\mathbf{A}}$ is finitely multiplicative, (2) for every $X \in T_4$ there exists $Y \in \mathbf{TOP}$ such that $F_{\mathbf{A}}(Y) = X$, then by 1.8 it follows that

Proposition 4.7. *Let* $X \in K_{\mathbf{A}}$ *and* $Y \in \mathbf{A}$, *then the projection* $p: X \times Y \to Y$ *is* $p(\mathbf{A})$.

Theorem 4.8. *Let* $\mathbf{A}$ *be such that* $F_{\mathbf{A}}$ *is finitely multiplicative. The cartesian product* $f = \prod_{s=1}^{k} f_s$, *where* $f_s: X_s \to Y_s$ *and* $X_s \neq \emptyset$ *for* $s = 1, \ldots, k$, *is* $p(\mathbf{A})$ *if and only if all mapping* f_s *are* $p(\mathbf{A})$.

Proof. Let $f = \prod_{s=1}^{k} f_s : \prod_{s=1}^{k} (X_s, \tau_s) \to \prod_{s=1}^{k} (Y_s, \sigma_s)$ be a $p(\mathbf{A})$ mapping, since $F_{\mathbf{A}}$ is finitely multiplicative we have that $\prod_{s=1}^{k} (X_s, (\tau_s)_{\mathbf{A}}) = (\prod_{s=1}^{k} (X_s, \tau_s))_{\mathbf{A}}$ is T_2 ([3], prop. 2.5), hence by 4.3 follows that $f = F_{\mathbf{A}}(f): \prod_{s=1}^{k} (X_s, (\tau_s)_{\mathbf{A}}) \to \prod_{s=1}^{k} (Y_s, (\sigma_s)_{\mathbf{A}})$ is perfect then every $f_s = F_{\mathbf{A}}(f_s): (X_s, (\tau_s)_{\mathbf{A}}) \to (Y_s, (\sigma_s)_{\mathbf{A}})$ is perfect ([8], th. 3. 7. 7), hence by 4.3 we have that $f_s: (X_s, \tau_s) \to (Y_s, \sigma_s)$ is $p(\mathbf{A})$, $s = 1, \ldots, k$.

If every $f_s: (X_s, \tau_s) \to (Y_s, \sigma_s)$ is $p(\mathbf{A})$ then $f_s = F_{\mathbf{A}}(f_s): (X_s, (\tau_s)_{\mathbf{A}} \to (Y_s, (\sigma_x)_{\mathbf{A}})$ is perfect for each s, hence $\prod_{s=1}^{k} f_s = F_{\mathbf{A}}(\prod_{s=1}^{k} f_s): \prod_{s=1}^{k} (X_s, (\tau_s)_{\mathbf{A}}) \to \prod_{s=1}^{k} (Y_s, (\sigma_s)_{\mathbf{A}})$ is perfect ([8], th. 3.7.7), therefore $\prod_{s=1}^{k} f_s: \prod_{s=1}^{k} (X_s, \tau_s) \to \prod_{s=1}^{k} (Y_s, \sigma_s)$ is $p(\mathbf{A})$.

Theorem 4.9. *Let* $\mathbf{A}$ *be such that* $F_{\mathbf{A}}$ *is finitely multiplicative and let* (X, τ), $(Z, \sigma) \in \mathbf{A}$. *For an* $\mathbf{A}$*-continuous mapping* $f: (X, \tau) \to (Z, \sigma)$ *the following conditions are equivalent:*

(1) *the mapping* f *is* $p(\mathbf{A})$;

(2) *for every* $(Y, \tau') \in \mathbf{A}$ *the cartesian product* $f \times id_Y$ *is* $p(\mathbf{A})$

(3) *for every* $(Y, \tau') \in \mathbf{A}$ *the cartesian product* $f \times id_Y$ *is* $\mathbf{A}$*-closed*

Proof. (1) $\Rightarrow$ (2) follows by th. 4.8 (2) $\Rightarrow$ (3) is obvious. (3) $\Rightarrow$ (1). Let $f \times id_Y: (X, \tau) \times (Y, \tau') \to (Z, \sigma) \times (Y, \tau')$ be $\mathbf{A}$-closed then $f \times id_Y = F_{\mathbf{A}}(f \times id_Y): (X, \tau_{\mathbf{A}}) \times (Y, \tau'_{\mathbf{A}})$, $\to (Z, \sigma_{\mathbf{A}}) \times (Y, \tau'_{\mathbf{A}})$ is closed, hence $F_{\mathbf{A}}(f): (X, \tau_{\mathbf{A}}) \to (Z, \sigma_{\mathbf{A}})$ is perfect ([8], th. 3.7.13) then by 4.3 is follows that $f: (X, \tau) \to (Y, \sigma)$ is $p(\mathbf{A})$.

Proposition 4.10. *Let* $\mathbf{A}$ *such that for every* $(X, \tau) \in \mathbf{A}$ *we have* $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$.

A $p(\mathbf{A})$ mapping $g: (X, \tau) \to (Y, \sigma)$ cannot be $\mathbf{A}$-continuously extended over any space $(Z, \tau') \in \mathbf{A}$ that contains X as a $\mathbf{A}$-dense proper subspace.

Proof. If $G: (Z, \tau') \to (Y, \sigma)$ is an $\mathbf{A}$-continuous extension of g over a space $(Z, \tau') \in \mathbf{A}$ that contains X as a $\mathbf{A}$-dense proper subspace then $G = F_{\mathbf{A}}(G): (Z, \tau'_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is a continuous extension of the perfect mapping $g = F_{\mathbf{A}}(g): (X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$, with $(Z, \tau'_{\mathbf{A}}) \in \mathbf{TOP}_2$ and X dense proper subspace of $(Z, \tau'_{\mathbf{A}})$, a contradiction (by Lemma 3.7.14, [8]).

Proposition 4.11. *Let* $\mathbf{A}$ *be such that for every* $(X, \tau) \in \mathbf{A}$ *we have* $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$, *and let* $f: (X, \tau) \to (F, \sigma)$ *be a* $p(\mathbf{A})$ *mapping.*

If $(Y, \sigma) \in K_{\mathbf{A}}$ *then* $(X, \tau) \in K_{\mathbf{A}}$.

Proof. If (Y, σ) is $\mathbf{A}$-Compact then $(Y, \sigma_{\mathbf{A}}) \in \mathbf{TOP}_2$ Comp, since $f = F_{\mathbf{A}}(f): (X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is perfect (by 4.3), we have that $(X, \tau_{\mathbf{A}})$ is a compact Hausdorff ([8], th. 3.7.24), hence (X, τ) is $\mathbf{A}$-Compact.

Theorem 4.12. *Let* $\mathbf{A}$ *be such that*:

(1) *For every* $(X, \tau) \in \mathbf{A}$ *we have* $(X, \tau_{\mathbf{A}}) \in \mathbf{TOP}_2$

(2) *For every* $(X, \tau) \in \mathbf{TOP}_2$ *there exists* $(Z, \rho) \in \mathbf{A}$ *such that* $(Z, \rho_{\mathbf{A}}) = (X, \tau)$.

If $(X, \tau), (Y, \sigma) \in K_{\mathbf{A}}$ *then the following conditions are equivalent:*
(a) *the* **A**-*continuous mapping* $g:(X, \tau) \to (Y, \sigma)$ *is* $p(\mathbf{A})$
(b) g *cannot be* **A**-*continuously extended over any space* $(Z, \rho) \in \mathbf{A}$ *that contains* X *as a* **A**-*dense proper subspace.*

Proof. (a) $\Rightarrow$ (b) follows by 4.10. If $g:(X, \tau) \to (Y, \sigma)$ is not $p(\mathbf{A})$ then $g = F_{\mathbf{A}}(g):(X, \tau_{\mathbf{A}}) \to (Y, \sigma_{\mathbf{A}})$ is not perfect, hence there exists a continuous extension $G:(Z, \rho) \to (Y, \sigma_{\mathbf{A}})$ of $g = F_{\mathbf{A}}(g)$ over a Hausdorff space (Z, ρ) that contains X as a dense proper subspace ([8], th. 37.16). But there exists $(X', \tau') \in \mathbf{A}$ such that $(X', \tau'_{\mathbf{A}}) = (Z, \rho)$ then the mapping $\bar{g}(X', \tau') \to (Y, \sigma)$ with $\bar{g} = F_{\mathbf{A}}(\bar{g}) = G$, is **A**-continuous extension of $g:(X, \tau) \to (Y, \sigma)$ over a space $(X', \tau') \in \mathbf{A}$ that contains X as a **A**-dense proper subspace.

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