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FUNCTIONALLY COUNTABLE SPACES AND BAIRE FUNCTIONS

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Abstract. The concept of the distinguished sets is applied to the investigation of the functionally countable spaces. It is proved that every Baire function on a functionally countable space has a countable image. This is a positive answer to a question of R. Levy and W. D. Rice.

0. Introduction. The present work deals with the properties of distinguished subsets in Tychonov spaces and spaces of functions.

Section 1 presents the basic definitions and notions used in this paper. In Section 2 we discuss the concept of a distinguished subset. Every Baire set is a distinguished set. In Section 3 a factorization theorem for measurable mappings into a separable metric space is proved. For Baire measurable mappings more general results were obtained in [2]. Section 4 contains a positive answer to the question posed by R. Levy and M. D. Rice [11]. The extensions of measurable mappings are studied in Sections 5 and 6. Section 7 presents some results on extensions of zero-sets.

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1. Preliminary results, definitions and notations. We consider only Tychonov spaces. We shall use the notations and terminology from [4]. In particular, βX is the Stone-Čech compactification of the space X, $cl_X H$ or $cl\, H$ denotes the closure of a set H in X, $|X|$ is the cardinality of X, $\mathbb{N} = \{1, 2, 3, \ldots\}$, the symbol $\mathbb{R}$ will denote the topological field of real numbers.

Fix a space X. By $C(X)$ we denote the space of all continuous real-valued functions on X. Let $B_0(X) = C(X)$ and inductively define the α Baire class $B_\alpha(X)$ for each ordinal $\alpha \leq \Omega$ (Ω denotes the first uncountable ordinal) to be the space of pointwise limits of sequences of functions in $\cup\{B_\beta(X) : \beta < \alpha\}$. For every function $f : X \to \mathbb{R}$ we denote $Z(f) = f^{-1}(0)$ and $CZ(f) = X \setminus Z(f)$. We put $Z_\alpha(X) = \{Z(f) : f \in B_\alpha(X)\} = \{f^{-1}F : F$ is a closed subset of $\mathbb{R}\}$, $CZ_\alpha(X) = \{CZ(f) : f \in B_\alpha(X)\}$, $Z_\alpha(X) \cap CZ_\alpha(X) = A_\alpha(X)$. The class $Z_\alpha(X)$ (class $CZ_\alpha(X)$) is a multiplicative (additive) class α of Baire sets of the space X. The sets $A_\alpha(X)$ are called the sets of ambiguous Baire class α.

Let PX be the set X with the topology generated by the G_δ–sets in the space X. The topology of the space PX is called the Baire topology of the space X. For every $\alpha \geq 0$ the classes $Z_\alpha(X)$, $CZ_{1+\alpha}(X)$, $A_{1+\alpha}(X)$ are open bases of the space PX.

A space X is called a P–space if $X = PX$.

A space is realcompact if it is homeomorphic to a closed subspace of a product of real lines. Denote by νX the Hewitt real-compactification of the space X. From [4, Theorem 3.11.10] we have $\nu X = \cap\{U \in CZ_0(\beta X) : X \subseteq U\} = \cap\{Y \subseteq \beta X : X \subseteq Y$ and Y is a realcompact subspace$\} = \beta X \setminus \cup\{H \in Z_0(\beta \mathbb{R}) : X \cap H = \emptyset\}$.

The discrete space of cardinality $c = 2^{\aleph_0}$ will be denoted by $D(c)$. We consider that $D(c) = P\mathbb{R}$.

Lemma 1.1. *Let X be a realcompact space. Then PX is homeomorphic to a closed subspace of the Cartesian product $D(c)^{C(X)}$.*

P r o o f. For every $f \in C(X)$ the mapping $f : PX \to P\mathbb{R} = D(c)$ is continuous. The mapping $\varphi : X \to \mathbb{R}^{C(X)}$, where $\varphi(x) = \{f(x) : f \in C(X)\}$ is an embedding and $X = \varphi(X)$ is a closed subspace of $\mathbb{R}^{C(X)}$. The mapping $\varphi : PX \to (P\mathbb{R})^{C(X)} = D(c)^{C(X)}$ is an embedding. Hence $PX = \varphi(PX)$ is a closed subspace of $D(c)^{C(X)}$. □

Corollary 1.2. *Let X be a realcompact space. Then PX is realcompact.*

Lemma 1.3. *$Z_\alpha(X) = \{X \cap H : H \in Z_\alpha(\beta(X)\}$ and $CZ_\alpha(X) = \{X \cap H : H \in CZ_\alpha(\beta X)\}$ for all $\alpha \leq \Omega$ and every space X.*

P r o o f. Follows from equality $Z_0(X) = \{X \cap H : H \in Z_0(\beta X)\}$. □

2. On distinguished subsets. A subset H of a space X is called distinguished if there exist a separable metric space Y and a continuous mapping $h : X \to Y$ such that $H = h^{-1}(h(H))$.

By $D(X)$ we denote the class of all distinguished subsets of a space X.

Let $2^{\mathbb{N}}$ be the family of all subsets of the set $\mathbb{N}$. Fix a subset $B \subseteq 2^{\mathbb{N}}$. For every sequence $\{H_n : n \in \mathbb{N}\}$ of subsets of X we define

$$\Psi_B\{H_n\} = \cup\{(\cap\{H_n : n \in \xi\}) \cap (\cap\{X \setminus H_n : n \in \mathbb{N} \setminus \xi\}) : \xi \in B\},$$

$$\Phi_B\{H_n\} = \cup\{\cap\{H_n : n \in \xi\} : \xi \in B\}.$$

The operation Ψ_B is called a ts–operation with a base B and Φ_B is called a δs–operation with a base B.

If $B \subseteq 2^{\mathbb{N}}$ and $A = \{\xi \subseteq \mathbb{N} : \eta \subseteq \xi$ for some $\eta \in B\}$, then $\Psi_A\{H_n\} = \Phi_A\{H_n\} = \Phi_B\{H_n\}$ for every sequence $\{H_n : n \in \mathbb{N}\}$ of subsets of X. Hence every δs–operation is a ts–operation (see [6, 9, 8, 3]).

For every ts–operation Ψ and family L of subsets of a space X we put $\Psi(L) = \{\Psi\{E_n\} : \{E_n : n \in \mathbb{N}\} \subseteq L\}$.

Let $\Psi(X) = \Psi(Z_0(X))$ for every space X.

Lemma 2.1. *$\Psi_B(D(X)) \subseteq D(X)$.*

P r o o f. Let $\{H_n : n \in \mathbb{N}\} \subseteq D(X)$. For every $n \in \mathbb{N}$ fix a separable metric space Y_n and a continuous mapping $f_n : X \to Y_n$ such that $H_n = f_n^{-1}(f_n(H_n))$. Consider the continuous mapping $f : X \to Y = f(x) \subseteq \prod\{Y_n : n \in \mathbb{N}\}$, where $f(x) = \{f_n(x) : n \in \mathbb{N}\}$ for every $x \in X$. By construction $f^{-1}(f(H_n)) = H_n$ for every $n \in \mathbb{N}$. Hence $f^{-1}(f(\Psi_B\{H_n\})) = \Psi_B\{H_n\}$ and $f(\Psi_B\{H_n\}) = \Psi_B\{f(H_n)\}$. $\square$

Corollary 2.2. *$\Psi_B(X) \subseteq D(X)$.*

Corollary 2.3. *$D(X)$ is a σ-algebra of open and closed subsets of the space PX.*

Corollary 2.4. *Let $\{H_n : n \in \mathbb{N}\} \subseteq \Psi_B(X)$. Then there exist a separable metric space Y and a continuous mapping $h : X \to Y$ such that $h(H_n) \in \Psi_B(Y)$ and $H_n = h^{-1}(h(H_n))$ for every $n \in \mathbb{N}$.*

Corollary 2.5. *$Z_\Omega(X) \subseteq D(X)$ for every space X.*

Theorem 2.6. *Let X be a space and $H \in D(X)$. Then there exists a unique subset $\nu H \in D(\nu X)$ such that:*

1. $\nu H \cap X = H$.

2. *If U is open in $P\nu X$ and $U \cap X = H$, then $U \subseteq \nu H$.*

3. *If U is open and closed in $P\nu X$ and $U \cap X = H$, then $U = \nu H$.*

4. *If $B \subseteq 2^N$ and $H \in \Psi_B(X)$, then $\nu H \in \Psi_B(\nu X)$.*

5. *If $H \in Z_\alpha(X)$, then $\nu H \in Z_\alpha(\nu X)$.*

Proof. Let $f : X \to Y$ be a continuous mapping onto a separable metric space Y and $f^{-1}(f(H)) = H$. There exists a continuous extension $\nu f : \nu X \to Y$ of the mapping f on νX. Let $\nu H = \nu f^{-1}(\nu f(H))$. By construction $\nu H \cap X = H$ and $\nu H \in D(\nu X)$.

Let U be open in $P\nu X$ and $U \cap X = H$. If $x \in U \setminus \nu H$, then $U \setminus \nu H$ is open in $P\nu X$ and there exists a subset $V \in Z_0(\nu X)$ such that $x \in V \subseteq U \setminus \nu H$. By construction, $V \cap X = \emptyset$. Hence U is a subset of the set νH.

Let U be closed in $P\nu X$ and $U \cap X = H$. Then $F = \nu H \setminus U$ is open in $P\nu X$ and $F \cap X = \emptyset$. Hence $F = \emptyset$ and $\nu H \subseteq F$. The assertions 1, 2 and 3 are proved. The assertions 4 and 5 follow from Corollary 2.4. The proof is complete. □

3. Factorization theorem for measurable mappings.

Definition 3.1. *A mapping $f : X \to Y$ is called:*
— B–measurable of class α if $f^{-1}(Z_0(Y)) \subseteq Z_\alpha(X)$;
— D–measurable if $f^{-1}(Z_0(Y)) \subseteq D(X)$.

Every B–measurable mapping is D-measurable.

Theorem 3.2. *Let $f : X \to Y$ be a D–measurable mapping of the space X onto a separable metric space Y. Then there exist a separable metric space Z, a continuous mapping $g : X \to Z$ and a D–measurable mapping $h : Z \to Y$ such that $f = h \circ g$. In particular, if f is B–measurable of class α, then h is B–measurable of class α as well.*

Proof. Let $\{H_n : n \in \mathbb{N}\}$ be a closed base of the space Y. Then there exist a separable metric space Z and a continuous mapping $g : X \to Z$ such that:

1. $g^{-1}(g(H_n)) = H_n$ for all $n \in \mathbb{N}$;
2. if $f^{-1}(H_n) \in Z_\alpha(X)$, then $g(f^{-1}(H_n)) \in Z_\alpha(Z)$.

Let $h(z) = f(g^{-1}(z))$ for every $z \in Z$. Then $h(h^{-1}(H_n)) = H_n$ and $h^{-1}(H_n) = g(f^{-1}(H_n))$ for every $n \in \mathbb{N}$. Hence $h : Z \to Y$ is a single-valued mapping. Every mapping of a separable metric space is D–measurable. Therefore, the mapping h is D–measurable. If f is B–measurable of class α, then $\{h^{-1}(H_n) : n \in \mathbb{N}\} \subseteq B_\alpha(Z)$ and h is B–measurable of class α. The proof is complete. □

4. On functionally countable spaces. A space X is functionally countable if the set $f(X)$ is countable for each function $f \in C(X)$ (see [11]).

R. Levy and N. D. Rice [11] have posed the following question: Let X be a Lindelöf functionally countable space. Does every Baire function on X have a countable image?

The next theorem contains a positive answer to the R. Levy and H. D. Rice question.

Theorem 4.1. *For every space X the following statements are equivalent:*

1. *X is functionally countable.*

2. *Every Baire function on X has a countable image.*

3. *For every continuous mapping $f : X \to Y$ into a metric space Y the image $f(X)$ is countable.*

4. *For every B–measurable mapping $f : X \to Y$ into a metric space Y the image $f(X)$ is countable.*

5. *For every D–measurable mapping $f : X \to Y$ into a metric space Y the image $f(X)$ is countable.*

6. *Every D–measurable image of X is functionally countable.*

P r o o f. A metric space is functionally countable if and only if it is countable. Hence, the implications $6 \to 5 \to 4 \to 3 \to 1$, $1 \to 3$ and $4 \to 2 \to 1$ are immediate.

Let X be a functionally countable space and $\varphi : X \stackrel{\text{onto}}{\longrightarrow} Y$ be a D–measurable mapping. Fix a continuous function $g \in C(Y)$. The mapping $\psi = g \circ \varphi : X \to \mathbb{R}$ is D–measurable. By virtue of Theorem 3.2, there exist a separable metric space Z, a continuous mapping $h : X \to Z$ and a mapping $f : Z \to \mathbb{R}$ such that $\psi = h \circ f$.

By implication $1 \to 3$, the image $h(X)$ is countable. Hence the set $g(Y) = \psi(X) = f(h(X))$ is countable. This proves the implication $1 \to 6$. The proof is complete. □

Corollary 4.2. *A B–measurable image of a functionally countable space is functionally countable.*

Proposition 4.3. *A space X is functionally countable if and only if the space νX is functionally countable.*

P r o o f. If $f : \nu X \to \mathbb{R}$ is a continuous function, then $f(\nu X) = f(X)$. This completes the proof. □

Proposition 4.4. *If PX is a Lindelöf space, then X and PX are functionally countable spaces.*

P r o o f. Let $f \in C(X)$. Then $f : PX \to P\mathbb{R}$ is continuous and $\gamma = \{f^{-1}(x) : x \in f(X)\}$ is a discrete open cover of PX. Hence γ is a countable cover and the image $f(X)$ is countable. The proof is complete. □

A space is scattered if every non-empty subspace contains an isolated point. A compact space is functionally countable if and only if it is scattered (see [7, 11, 12, 13]).

Proposition 4.5. *For a space X the following statements are equivalent:*

1. *X is functionally countable and pseudocompact.*
2. *βX is functionally countable.*
3. *βX is scattered.*
4. *$Z_\Omega(X) = D(X)$ and X is pseudocompact.*
5. *$Z_1(X) = D(X)$ and X is pseudocompact.*

P r o o f. The implications $2 \to 3 \to 2$ are proved in [11, 12, 13].

Let X be a functionally countable pseudocompact space. Fix a continuous mapping $f : X \to Y$ onto a metric space Y. Then Y is a compact space and, by virtue of Theorem 4.1, Y is countable. Hence $f^{-1}H \in Z_1(X)$ for every $H \subseteq Y$. Therefore, $D(X) \subseteq Z_1(X)$ and the implication $1 \to 5$ is proved.

The implication $5 \to 4$ is obvious.

Let X be a pseudocompact space, $f \in C(X)$ and $Y = f(X)$ be uncountable. Then there exists some subset $H \notin Z_\Omega(Y)$. In [1, Theorem 3.4] it is proved: $L \in Z_\Omega(Y)$ if and only if $f^{-1}(H) \in Z_\Omega(X)$. Hence $f^{-1}(H) \in D(X) \setminus Z_\Omega(X)$. This proves the implication $4 \to 1$. The proof is complete. □

Question 4.6. *Let X and Y be functionally countable spaces. Is $X \times Y$ functionally countable?*

5. Extension of mappings.

Theorem 5.1. *Let $\varphi, \psi : P\nu X \to Y$ be a continuous mappings into a space Y. If $\varphi_{|X} = \psi_{|X}$, then $\varphi = \psi$.*

P r o o f. The set X is dense in $P\nu X$. Theorem 2.1.9 [4] completes the proof. □

Corollary 5.2. *Let $\varphi, \psi : \nu X \to Y$ be D–measurable or B–measurable mappings. If $\varphi_{|X} = \psi_{|X}$, then $\varphi = \psi$.*

Theorem 5.3. *Let $\varphi : X \to Y$ be a D–measurable mapping. Then there exists a unique D–measurable mapping $\nu\varphi : \nu X \to \nu Y$ such that:*

1. $\varphi = \nu\varphi_{|X}$.

2. *If φ is a B–measurable mapping of class $\alpha \leq \Omega$, then $\nu\varphi$ is a B–measurable mapping of class α as well.*

3. *If Y is a complete separable metric space, then there exists a D–measurable mapping $\beta\varphi : \beta X \to Y$ such that $\varphi = \beta\varphi_{|X}$.*

4. *If Y is a complete separable metric space and φ is a B–measurable mapping of class $\alpha \leq \Omega$, then there exist an ordinal number $\zeta < \Omega$, a set $X_\varphi \in Z_{\zeta+1}(\beta X)$ and a B–measurable mapping $b\varphi : X_\varphi \to Y$ of class ζ such that $\zeta \leq \alpha$ and $\varphi = b\varphi_{|X}$.*

P r o o f. Suppose that Y is a separable metric space. By Theorem 3.2 there exist a separable metric space Z, a continuous mapping $f : X \to Z$ and a mapping $g : Z \to Y$ such that $\varphi = g \circ f$ and if φ is a Baire measurable mapping of class α, then g is a Baire measurable mapping of class α, too. Let $\nu f : \nu X \to Z$ be a continuous extension of f. Then $\nu\varphi = g \circ \nu f$.

Let bZ be a metrizable compactification of a space Z and $\beta f : \beta X \to bZ$ be a continuous extension of f.

Let Y be a complete separable metric space and ρ be a complete metric on Y. We fix a family $\{F_{nm} : m, n \in \mathbb{N}\}$ of closed subsets of Y such that:

1. $Y = \cup\{F_{nm} : m \in \mathbb{N}\}$.

2. $\operatorname{diam} F_{nm} < 2^{-n}$.

Let $H_{nm} = F_{nm} \setminus \cup\{F_{ni} : i < m\}$. Fix the sets $\{W_{nm} \in D(\beta X) : n, m \in \mathbb{N}\}$ with the properties:

3. $W_{nm} \cap X = \varphi^{-1}(H_{nm})$.

4. $W_{nm} \cap W_{nk} = \emptyset$ if $m < k$.

5. $W_n = \cup\{W_{nm} : m \in \mathbb{N}\} \subseteq \cup\{W_{km} : m \in \mathbb{N}\}$ if $k < n$.

By construction, $W = \cap\{W_n : n \in \mathbb{N}\} \in D(\beta X)$. Let $x \in W$. Then there exists a unique sequence $m(x) = \{m_n(x) : n \in \mathbb{N}\}$ such that $x \in \cap\{W_{nm_n(x)} : n \in \mathbb{N}\}$. We put $\beta\varphi(x) = \cap\{F_{nm_n(x)} : n \in \mathbb{N}\}$. The mapping $\beta\varphi : W \to Y$ is D–measurable and $\varphi = \beta\varphi_{|X}$. Fix a point $b \in Y$. We put $\beta\varphi(x) = b$ for all $x \in \beta X \setminus W$. Then the mapping $\beta\varphi : \beta X \to Y$ is D– measurable.

Let Y be a complete separable metric space and φ be a B–measurable mapping of class α. Then for some $\zeta \leq \alpha$, φ is a B–measurable mapping of class ζ and $\zeta < \Omega$. By virtue of the K. Kuratowski theorem [10, 35, Section VI] there exist a Baire set $H \in Z_{\zeta+1}(bZ)$ and a B–measurable mapping $\psi : H \to Y$ of class

ζ such that $Z \subseteq H$ and $g = \psi_{|Z}$. We put $X_\varphi = \beta f^{-1}(H)$ and $\beta\varphi(x) = \psi(\beta f(x))$ for every $x \in X_\varphi$.

The assertions 3 and 4 are proved.

The mapping $\varphi : PX \to Y$ is continuous. For every $f \in C(Y)$ there exists a unique D–measurable mapping $\nu_X f : \nu X \to \mathbb{R}$ such that $\nu_X f(x) = f(\varphi(x))$ for every $x \in X$ and $\nu_X f(\nu X) = f(Y)$. If φ is a B–measurable mapping of class α, then $\nu_X f \in B_\alpha(\nu X)$. Fix a pair F_1, F_2 of disjoint closed subsets of βY. There exists a continuous function $g : \beta Y \to [0,1]$, such that $F_1 \subseteq g^{-1}(0)$ and $F_2 \subseteq g^{-1}(1)$. Let $f = g_{|Y}$ and h be a continuous extension of the function $\nu_X F$ over $\beta P\nu X$. Then $\varphi^{-1}(F_1) \subseteq h^{-1}(0)$ and $\varphi^{-1}(F_2) \subseteq h^{-1}(1)$. By virtue of the A. D. Taimanov theorem [10, Theorem 3.2.1], there exists a unique continuous mapping $\psi : \beta P\nu X \to \beta Y$ such that $\varphi = \psi_{|X}$. Let $y \in \beta Y \setminus Y$. Then there exists a set $H \in Z_0(\beta Y)$ such that $y \in H \subseteq \beta Y \setminus Y$. By construction, $\psi^{-1}(H) \subseteq Z_0(\beta P\nu X)$ and $\psi^{-1}(H) \cap X = \emptyset$. Hence $\psi^{-1}(H) \cap P\nu X = \emptyset$ and $\psi^{-1}(\nu Y) \subseteq P\nu X$. Therefore $\nu\varphi = \psi_{|P\nu X} : P\nu X \to \nu Y$ is a continuous extension of the mapping $\varphi : PX \to Y$. If $h \in C(\nu Y)$ and $f = h_{|Y}$, then $\nu_X f(x) = h(\nu\varphi(x))$ for every $x \in \nu X$. Hence $\nu\varphi^{-1}(h^{-1}(0)) = \nu_X f^{-1}(0)$. In particular: if φ is D–measurable, then $\nu\varphi$ is also D–measurable; if φ is B–measurable of class α, then $\nu\varphi$ is B–measurable of class α, too. The proof is complete. □

Corollary 5.4 (P. R. Mayer [7, Theorem 7]). *Every $f \in B_\alpha(X)$ has a unique extension to an $\nu f \in B_\alpha(\nu X)$.*

Remark 5.5. For a paracompact X the assertion 4 of a Theorem 5.3 was proved in [5].

6. Extension of zero-sets.

Theorem 6.1. *Let X be a dense subspace of a realcompact space Y. Then the following statements are equivalent:*

1. $Y = \nu X$.

2. *For every $H \in Z_0(X)$ there exists a unique $\Phi \in Z_0(Y)$ such that $\Phi \cap X = H$.*

3. *For every $H \in Z_0(X)$ there exists a unique $\Phi \in D(Y)$ such that $\Phi \cap X = H$.*

4. *For every $H \in B_\Omega(X)$ there exists a unique $\Phi \in B_\Omega(Y)$ such that $\Phi \cap X = H$.*

5. *For every $H \in D(X)$ there exists a unique $\Phi \in D(Y)$ such that $\Phi \cap X = H$.*

6. *For every $f \in C(X)$ there exists a unique D–measurable extension $g : Y \to \mathbb{R}$.*

P r o o f. The implication $1 \to 6$ follows from Theorem 5.3.

Let $H \in D(X)$, $L, \Phi \in D(Y)$ and $H = L \cap X = \Phi \cap X$. Consider the functions $f : X \to \mathbb{R}$ and $\varphi, \psi : Y \to \mathbb{R}$, where $f^{-1}(0) = H$, $f^{-1}(1) = X \setminus H$, $\varphi^{-1}(0) = L$, $\varphi^{-1}(1) = Y \setminus L$, $\psi^{-1}(1) = Y \setminus \Phi$ and $\psi^{-1}(0) = \Phi$. Then $\psi_{|X} = \varphi_{|X} = f$ and f, φ, ψ are D–measurable functions. This proves the implications $6 \to 5$, $6 \to 4$, $6 \to 3$ and $6 \to 2$.

Suppose that for every $H \in Z_0(X)$ there exists a unique $\Phi \in Z_0(Y)$ such that $\Phi \cap X = H$. There exists a continuous mapping $h : \beta X \to \beta Y$ such that $h(x) = x$ for every $x \in X$. Let $y \in Y$, $x_1, x_2 \in h^{-1}(y)$ and $x_1 \neq x_2$. There exist closed subsets $H_1, H_2 \in Z_0(\beta X)$ and open subsets V_1, V_2 of βX such that $H_1 \cap H_2 = \emptyset$, $x_1 \in V_1 \subseteq H_1$ and $x_2 \in V_2 \subseteq H_2$. Let $\Phi_1, \Phi_2 \in Z_0(\beta Y)$ and $\Phi_1 \cap X = H_1 \cap X$, $\Phi_2 \cap X = H_2 \cap X$. By construction, $h(H_1) \subseteq \Phi_1$, $h(H_2) \subseteq \Phi_2$, $y \in \Phi_1 \cap \Phi_2 \in Z_0(\beta Y)$ and $\Phi \cap X = \emptyset$. Hence $\emptyset \cap X = \Phi \cap X = \emptyset \in Z_0(Y)$ and $\emptyset \neq \Phi$. Therefore the mapping h is one–to–one, $Y \subseteq \beta X = \beta Y$ and $Y = \beta X \setminus \cup\{H \in Z_0(\beta X) : H \cap X = \emptyset\} = \nu X$. The proof is complete. □

REFERENCES

[1] M. N. Choban. Baire sets in complete topological spaces. *Ukrainskii Matem. Zhurnal* **22** (1970), 330-342, (in Russian)?.

[2] M. M. Choban. Continuous images of complete spaces. *Trans. Moscow Math. Soc.* **30** (1974), 25-65, (in Russian)?.

[3] M. M. Choban. On the operations on sets. *Sibirskii Matem. Zhurnal* **16**, *6* (1975), 1332-1351.

[4] R. Engelking. General Topology. PWN, Warsaw, 1977.

[5] R. W. Hansell. On Borel mappings and Baire functions. *Trans. Amer. Math. Soc.* **194** (1974), 195-211.

[6] F. Hausdorff. Set Theory. Chelsca, New York, 1957.

[7] J. E. Jayne. Spaces of Baire functions. *Ann. Inst. Fourier (Grenoble)* **24**, *4* (1974), 47-76.

[8] L. V. Kantorovich, E. N. Livenson. Memoir on the analytical sets. *Fund. Math.* **18** (1932), 214-279.

[9] A. N. Kolmogorov. On the operations on sets. *Matem. zbornic* **35**, *3* (1928), 416-422, (in Russian)?.

[10] K. Kuratowskii. Topology, vol. 1, Academic Press, New York, 1966.

[11] R. Levy, N. D. Rice. Normal p-spaces and the G_δ–topology. *Colloq. Math.* **44**, *1* (1981), 277-240.

[12] A. Pelczynski, Z. Semadeni. Spaces of continuous functions III. *Studia Math.* **18** (1959), 211-222.

[13] W. Rudin. Continuous functions on compact spaces without perfect subsets. *Proc. Amer. Math. Soc.* **8** (1957), 39-42.

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