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PARACOMPACT SPACES AND RADON SPACES

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Abstract. We prove that if E is a subset of a Banach space whose density is of measure zero and such that $(E, weak)$ is a paracompact space, then $(E, weak)$ is a Radon space of type $(\mathcal{F})$ under very general conditions.

Recall that a cardinal α is said to be of measure zero if, for any set A with this cardinal, every finite measure on the subsets of A which vanishes on the singletons is zero. The density of a topological space E is the smallest cardinal of the dense subsets of E.

A topological space E is said to have the α-property of Lindelöf, where α is a transfinite cardinal, if for every family $(G_i)_{i\in I}$ of open subsets of E, there exists $J \subseteq I$ such that card $J \le \alpha$ and

$$\bigcup_{i\in I} G_i = \bigcup_{i\in J} G_i.$$

The smallest cardinal α such that E has the α-property of Lindelöf is called the L-weight (or hereditary degree) of E.

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Similarly, we define the weak L-weight of E considering only open covering $(G_i)_{i\in I}$ of E.

A Borel measure is *weakly τ-additive* if whenever $(G_i)_{i\in I}$ is a covering of open sets of E there exists a set countable $J \subseteq I$ such that $\mu\left(\underset{i\in J}{\cup} G_i\right) = \mu(E)$. The measure μ is *τ-additive* if for every open set $G \subseteq E$ the restriction μ_G is weakly τ-additive.

Let E be a regular topological space and let $\mathcal{H}$ be a family of closed sets of E. Then, a finite Borel measure μ on E is said to be a Radon measure of type $(\mathcal{H})$ if μ is a τ-additive measure and it is innerly $\mathcal{H}$-regular [14, 18]. In particular, here we consider the case in which $\mathcal{H}$ is the family $\mathcal{F}$ of all the closed sets of E. A regular topological space E is said to be Radon space of type $(\mathcal{H})$ if every finite Borel measure on E is a Radon measure of type $(\mathcal{H})$ [13]. In particular, when $\mathcal{H}$ is the class $\mathcal{K}$ of the compact sets, the Radon measures of type $(\mathcal{K})$ coincide with Radon measures and the Radon spaces of type $(\mathcal{K})$ coincide with the Radon spaces.

A topological space E is said to be Borel measure-compact if every innerly $\mathcal{F}$-regular (or innerly regular) finite Borel measure on E is τ-additive.

The support F of a Borel measure μ is the closed set consisting of the points for which every open neighborhood has a positive measure. The measure μ is said to have a proper support if the complementary set F^c of the support has measure zero.

Proposition 1. *Every τ-additive measure μ has a proper support. If $\mu \neq 0$ is a finite weakly τ-additive measure then μ has a non-empty support an $\mu(H) = 0$ if H is a closed set disjoint with the support of μ.*

Proof. If μ is a τ-additive measure and if (G_i) is a family of open sets such that $\mu(G_i) = 0$ for every $i \in I$, then $\mu\left(\underset{i\in I}{\cup} G_i\right) = 0$ and the support of μ is proper. If $\mu \neq 0$ is weakly τ-additive and if H is a closed set disjoint with the support F of μ, then immediately $\mu(H) = 0$ and the support $F \neq \emptyset$. □

Theorem 2 [11, 10.2]**.** *Let E be a paracompact topological space whose weak L-weight is of measure zero. Then every finite Borel measure μ on E is weakly τ-additive.*

Corollary 3. *Let E be a paracompact regular topological space whose weak L-weight is of measure zero. Then every regular finite Borel measure μ on*

E is τ-additive and the support of μ is proper.

Proof. It follows from Theorem 2 because: a) Every regular weakly τ-additive measure μ is τ-additive [8, 4.3]. □

Corollary 4. *Let E be an hereditarily paracompact regular topological space whose L-weight has measure zero. Then every finite Borel measure μ on E is τ-additive, and hence, E is a Radon space of type ($\mathcal{F}$).*

Proof. b) If E is regular then every τ-additive measure on E is regular [8, 5.4]. Then Corollary 4 is a immediate consequence of b) and Corollary 3 applied to every open subset of E. □

In particular, taking into consideration A. H. Stone's theorem, the following Marcewski-Sikorski result [15] follows:

Corollary 5. *If E is a metrizable topological space whose density is of measure zero, then E is a Radon space of type ($\mathcal{F}$).*

Corollary 6. *Let E be a paracompact regular topological space whose L-weight is of measure zero and such that every open subset is $\mathcal{F}_\sigma$. Then E is a Radon space of type ($\mathcal{F}$).*

Proof. It follows from Corollary 4 and the fact that every $\mathcal{F}_\sigma$ subset of E is paracompact [16, X.I.8]. □

Remark. Theorem 2 and Corollary 4 are known result less general than [1, Th.10 and Cor.3.11], and [8, Th.3.9 and Th.6.1], [11, Th.10.2 and Th.10.3] and [5] for weakly ϑ-refinable spaces.

Theorem 7. *Let E be a subset of the Banach space X. Then every finite Borel measure μ on $(E, weak)$ with proper support F is τ-additive.*

Proof. We can suppose that X is the closed linear span of the support F. Since $Z = \{x^*|_F : x^* \in X_1^*\}$, where X_1^* is the unit ball of X^*, is a convex set of μ-measurable functions, compact for the topology τ_p of pointwise convergence, and Hausdorff for the topology τ_m of convergence in measure, it follows from a theorem of A. Bellow [23, 12.3.3] that Z is metrizable for $\tau_p = \tau_m$. Hence, $(F, norm)$ is separable and it follows immediately that μ is a τ-additive measure. □

Theorem 8. *Let E be a subset of the Banach space X. Then the support F of every finite weakly τ-additive measure μ on $(E, weak)$ is separable.*

Proof. It follows from Tortrat's theorem [23, 2.3.2]. □

Corollary 9. *Let E be a subset of the Banach space X, whose weak L-weight is of measure zero and such that $(E, weak)$ is a paracompact space. Then, $(E, weak)$ is a Borel measure-compact space.*

P r o o f. It follows from Corollary 3. □

Corollary 10. *Let E be a subset of a Banach space such that $(E, weak)$ is a Lindelöf space. Then $(E, weak)$ is a Borel measure-compact space.*

P r o o f. It suffices to take into consideration that any regular Lindelöf space is a paracompact space [16, X.1.3] whose weak L-weight is $\leq \aleph_0$. □

Corollary 11. *Let E be an $\mathcal{F}_\sigma$ subset in the weak topology of a WCG Banach space (or weakly $\mathcal{K}$-analytic) X. Then $(E, weak)$ is a Borel measure-compact space.*

P r o o f. According to [23, 2.7.2] (or [22]), $(X, weak)$ is a Lindelöf space. Then it follows from Corollary 10 that both $(X, weak)$ and $(E, weak)$ are Borel measure-compact spaces.

This result is due essentially to G. Choquet [12].

Corollary 12. *Let E be a subset of a Banach space, whose L-weight is of measure zero and such that $(E, weak)$ is a σ-paracompact (or weakly ϑ-refinable) space. Then $(E, weak)$ is a Borel measure-compact space.*

Theorem 13. *Let E be a subset of l_∞ whose weak L-weight is of measure zero and such that $(E, weak)$ is a paracompact space. Then $(E, weak)$ is a Radon space of type $(\mathcal{F})$.*

P r o o f. Let μ be a finite Borel measure on $(E, weak)$ and let F be the support of μ. Then, by Theorem 8, it turns out that F is separable, from which it follows that $F \in Ba(E, weak)$ since the closed unit ball of l_∞ belongs to $Ba(l_\infty, weak)$. Then, for any $\varepsilon > 0$, there exists a closed set $H \subseteq F^c = E \setminus F$ such that $\mu(F^c \setminus H) < \varepsilon$, and since $\mu(H) = 0$ by the Proposition 1, it turns out that $\mu(F^c) = 0$. Finally, it follows from Theorem 7 that μ is a τ-additive measure and a Radon measure of type $(\mathcal{F})$. □

Remark. In general, the results that we give for paracompact spaces hold also if we substitute that condition by the property that all its finite Borel measures are weakly additive, without any restriction neither on the weak L-weight nor on the density. In particular, if E is a subset of l_∞, then $(E, weak)$

is a Radon space if and only if every finite Borel measure $\mu \neq 0$ on it has non-empty support. Similarly, a Banach space $(X, weak)$ is a Radon space if and only if every finite Borel measure μ on it is weakly τ-additive and the closed unit ball X_1 is outerly regular for each one of the measures μ, that is, for any $\varepsilon > 0$ there exists an open set $G \supseteq X_1$ such that $\mu(G \setminus X_1) < \varepsilon$. On the other hand, the property of paracompacity can be substitute by the property that $(E, weak)$ be weakly ϑ-refinable space.

Theorem 14. *Let E be a subset of a Banach space X, whose weak L-weight is of measure zero and such that $(E, weak)$ is a paracompact space. Suppose that there exists an injective continuous mapping $T : (X, weak) \to Y$, where Y is a Hausdorff space with the property that every finite Borel measure on Y is regular. Then $(E, weak)$ is a Radon space of type $(\mathcal{F})$.*

Proof. Let μ be a finite Borel measure on $(E, weak)$ with support F. Then, by Theorem 8, F is separable and its closure $\overline{F}$ (in X) is a Polish space as well as a Lusin space in $(X, weak)$. Therefore, by [20, I.II.1], $T\overline{F}$ is a Borel set of Y.

Let ν be the Borel measure on Y defined by

$$\nu(B) = \mu(E \cap T^{-1}(B)).$$

Then, since ν is a regular measure and $T\overline{F}$ is a Borel set, for every $\varepsilon > 0$ there exists an open set $G \supseteq T\overline{F}$ such that

$$\mu(E \cap T^{-1}(G) \setminus F) = \mu(E \cap T^{-1}(G \setminus T\overline{F})) = \nu(G \setminus T\overline{F}) < \varepsilon.$$

As $\mu(E \setminus T^{-1}(G)) = 0$ by Proposition 1, it follows that F is a proper support and, hence, according to Theorem 7, μ is a τ-additive measure as well as a Radon measure of type $(\mathcal{F})$. □

Corollary 15. *Let E be a subset of a Banach space X, whose density is of measure zero and such that $(E, weak)$ is a σ-paracompact space. Suppose that there exists an injective continuous linear mapping $T : X \to c_0(I)$. Then, $(E, weak)$ is a Radon space of type $(\mathcal{F})$.*

Proof. It follows from Theorem 14 and the fact that $(c_0(I), weak)$ is a Radon space of type $(\mathcal{F})$ when card I is of measure zero [4]. □

Corollary 16. *Let E be a subset of a WCG Banach space X, whose density is of measure zero. Then, $(E, weak)$ is a Radon space of type $(\mathcal{F})$.*

P r o o f. It suffices to prove that if X_0 is the closed linear span of E, then $(X_0, weak)$ is a Radon space of type $(\mathcal{F})$. Indeed, the latter follows from Corollary 15 and the fact that, according to the theorem of Amir-Lindenstrauss [2], there exists an injective continuous linear mapping $T : X \to c_0(I)$. □

We have proved this corollary previously (in 1989) by using a different approach in [4].

Corollary 17. *Let φ be a non-degenerated Orlicz function and let E be a subset of $X = l_\varphi(I)$, whose density is of measure zero and such that $(E, weak)$ is σ-paracompact. Then, $(E, weak)$ is a Radon space of type $(\mathcal{F})$.*

P r o o f. It follows from Corollary 15. □

Remark. If φ is an Orlicz function verifying the Δ_2-condition and card I is of measure zero, then $(l_\varphi(I), weak)$ is a Radon space by [4], and it is not necessary to assume $(E, weak)$ to be σ-paracompact.

Remark. If card I is not of measure zero, it follows from Theorem 14 that there exists a finite Borel measure on $(c_0(I), weak)$ which is not regular. Indeed, if X is a WCG Banach space whose density is card I (for instance, $l_2(I)$), then according to Corollary 11 there exists a finite Borel measure on $(X, weak)$ which is not neither Radon nor regular. On the other hand, by Amir-Lindenstraus's theorem, there exists an injective continuous linear mapping $T : X \to c_0(I)$. Then it follows from Theorem 14 that there exists a finite Borel measure on $(c_0(I), weak)$ which is not regular (this space is WCG as well [12]).

Remark. I thank W. Schachermayer for the following remark in 1989: If X is a Banach space whose density is of measure zero, endowed with a Kadec-Klee norm, then by a theorem of G. A. Edgar [6] we have that $Bo(X, norm) = Bo(X, weak)$ and it follows that $(X, weak)$ is a Radon space. In particular, we can apply a S. L. Troyanski's result [24] stating that any WCG Banach space has a Kadec-Klee norm, to the case where X is a WCG space. Similarly, Corollary 16 holds also for weakly K-analytic spaces since these spaces have a Kadec-Klee norm [12, 24]. Therefore, Theorem 14 holds when Y is a weakly K-analytic Banach space or, more generally when Y is a Banach space such that $Bo(Y, norm) = Bo(Y, weak)$.

To complete the latter remark, we are going to prove several results.

Let X and Y be two topological spaces. A mapping $f : X \to Y$ is said to be universally measurable (respectively, regular) Borel mapping if, for every

Borel subset B of Y, $f^{-1}(B)$ is measurable for all finite (resp. regular) Borel measure μ on X.

Theorem 18. *Let X be a Banach space such that the unit sphere $(S_1, weak)$ is a Radon space of type $(\mathcal{F})$ and f the mapping $x \to x/\|x\|$ from $(X \setminus \{0\}, weak)$ to $(S_1, weak)$. Then $(X, weak)$ is a Borel measure-compact space if and only if f is a universally measurable regular Borel mapping.*

Proof. Let $\mathcal{U}$ be an open covering of $(E, weak)$, where $E = \{x \in X : r_1 \leq \|x\| \leq r_2\}$ and $0 < r_1 < r_2 < \infty$. We have that $E \cap f^{-1}(x')$ is a compact set for every $x' \in S_1$. Hence, there exists a finite subfamily $\mathcal{U}_{x'}$ of $\mathcal{U}$ which covers $E \cap f^{-1}(x')$ and

$$V_{x'} = S_1 \setminus f(E \setminus \cup\, \mathcal{U}_{x'})$$

is an open neighbourhood of x' in $(S_1, weak)$. Indeed, $f_{|E}$ is a closed mapping. To see this we just have to take a net (x_α) in a closed subset F of $(E, weak)$ in such a way that $x_\alpha/\|x_\alpha\|$ converges in the weak topology to x in S_1. By passing to a subnet if needed, we can assume that the limit $\lim_\alpha \|x_\alpha\| = r$ exists and hence, (x_α) converges in the weak topology to $rx \in E$, $rx \in F$ and $x \in f(F)$.

Let μ be a regular finite Borel measure on $(E, weak)$ and define a Borel measure ν on $(S_1, weak)$ by

$$\nu(B) = \mu^*(E \cap f^{-1}(B)).$$

Then, since $(S_1, weak)$ is a Radon space of type $(\mathcal{F})$, there exists a sequence (x'_n) such that $\nu\left(S_1 \setminus \bigcup_{n\in\mathbb{N}} V_{x'_n}\right) = 0$.

On the other hand, if $x \in E \cap f^{-1}(V_{x'})$, we have that $x \in \cup\, \mathcal{U}_{x'}$ and therefore, $\mu\left(E \setminus \bigcup_{n\in\mathbb{N}} \mathcal{U}_{x'_n}\right) = 0$. Then, μ is a weakly τ-additive regular measure and, by a), it is τ-additive for every $E = E(r_1, r_2)$ with $0 < r_1 < r_2 < \infty$, from which it follows that $(X, weak)$ is a Borel measure-compact space.

Conversely, if $(X, weak)$ is a Borel measure-compact and μ is a regular finite Borel measure on $(X, weak)$, μ is τ-additive, and hence, a Radon measure. Since we also have that for every Borel B in $(S_1, weak)$, $f^{-1}(B)$ is a Borel set in $(X, norm)$, it follows from a theorem due to Phillips, Dunford-Pettis and Grothendieck [20, II.I.4] that $f^{-1}(B)$ is a μ-measurable set and therefore, f is a universally measurable regular Borel measure. $\square$

Theorem 19. *Let X be a Banach space whose density is of measure zero and such that the unit sphere $(S_1, weak)$ is a σ-paracompact space. Then $(X, weak)$ is a Borel measure-compact space if and only if the mapping f considered above is a universally measurable regular Borel mapping.*

Proof. We proceed as in Theorem 18. □

Theorem 20. *Let X be a Banach space. Then, $(X, weak)$ is a Radon space if and only if the unit sphere $(S_1, weak)$ is a Radon space of type $(\mathcal{F})$ and the mapping f, considered above, is a universally measurable Borel mapping.*

Proof. Suppose that the unit sphere $(S_1, weak)$ is a Radon space of type $(\mathcal{F})$ and that f is a universally measurable Borel mapping. Let $E = \{x \in X : r_1 \leq \|x\| \leq r_2\}$ $(0 < r_1 < r_2 < \infty)$ and suppose that μ is a finite Borel measure on $(E, weak)$ with support F. Then, as in Theorem 18, it follows that μ is weakly τ-additive.

To finish the first part, it suffices to prove that the closed unit ball X_1 is outer regular for each one of the measures μ. Indeed, for every $x \notin X_1$, there exists a closed neighbourhood $V_x \subseteq X_1^c$. Therefor , since the induced measure $\mu_{X_1^c}$ is weakly τ-additive from what we have seen above, there exists a sequence $(x_n) \subseteq X_1^c$ such that $\mu\left(\underset{n\in\mathbb{N}}{\cup} V_{x_n}\right) = \mu(X_1^c)$, from what it follows that for every $\varepsilon > 0$ there is an open set $G \supseteq X_1$ such that $\mu(G \setminus X_1) < \varepsilon$.

Conversely, let $(X, weak)$ be a Radon space. Then, since S_1 is a Borel set in $(X, weak)$, we have that $(S_1, weak)$ is a Radon space. On the other hand, if μ is a finite Borel measure on $(X, weak)$ and B is a Borel set in $(S_1, weak)$, then $f^{-1}(B)$ is a Borel set in $(X \setminus \{0\}, norm)$ and, by [20, II.I.4], $f^{-1}(B)$ is a μ-measurable set.

We can prove the following theorem in a similar way:

Theorem 21. *Let X be a Banach space whose density is of measure zero and such that the unit sphere $(S_1, weak)$ is a σ-paracompact space. Then, $(X, weak)$ is a Radon space if and only if the mapping f, described above, is universally measurable Borel.*

Remark. Proceeding as in Theorem 18, we can prove that if X is a Banach space such that $(S_1, weak)$ is a Lindelöf space, then $(X, weak)$ is also Lindelöf.

Problem. Let X be a Banach space such that $(S_1, weak)$ is a paracompact space. Then, if X_1 is the closed unit ball of X, is $(X_1, weak)$ a paracompact space?

It is true indeed that if f is not a universally measurable regular Borel mapping and the density of X is of measure zero, from Corollary 12 it follows that the space $(X_1, weak)$ is not paracompact.

Remark. If the cardinal c is of measure zero, the closed unit ball X_1 in l_∞/c_0 is not a paracompact space for the weak topology. Indeed, in [3] we have constructed a Borel measure $\mu \neq 0$ on $(l_\infty/c_0, weak)$ taking the values 0 and 1 and whose support is empty. Therefore μ is not weakly τ-additive, which contradicts the fact that the restriction of μ on every closed ball is weakly τ-additive as seen in Corollary 3, if the unit ball $(X_1, weak)$ were a paracompact space.

We denote by $z(I)$ the subspace of $l_\infty(I)$ consisting of the elements $x = (x_i)_{i\in I}$ with countable support. When I is not countable, Talagrand has constructed [21] a Borel measure $\mu \neq 0$ on $(z(I), weak)$ taking the values 0 and 1, concentrated on the set F of the elements $x \in z(I)$ with coordinates 0 and 1, and with empty support.

Theorem 22. *Let X be a Banach lattice that is an ideal in $z(I)$. Suppose that the injection $X \to z(I)$ is continuous and that every finite Bore1 measure on $(X, weak)$ with support $\{0\}$, is concentrated on it. Then $(X, weak)$ is a Radon space.*

Proof. Let μ be a finite Borel measure on $(X, weak)$ and let F be the support of μ. Then, since μ is weakly τ-additive by [9, 5.3], by Theorem 8, F is separable and there exists a countable set $J \subseteq I$ such that $x_i = 0$ for every $i \notin J$ and $x = (x_i)_{i\in I} \in F$. Let S be the linear continuous mapping $X \to X$ defined by setting $Sx = (y_i)_{i\in I}$ with $y_i = 0$ for $i \in J$ and $y_i = x_i$ for $i \notin J$. Let ν is the Borel measure defined on $(X, weak)$ by $\nu(B) = \mu(S^{-1}(B))$. This measure ν has support $\{0\}$. Therefore, it is concentrated on $\{0\}$ and μ is concentrated on the subspace X_0 formed by the elements $x = (x_i)_{i\in I} \in X$ with $x_i = 0$ for $i \notin J$. To finish, it suffices to proceed as in Theorem 14, taking into consideration that both μ and the induced measure μ_{X_0} are weakly τ-additive by [9, 5.3], since every finite Borel measure with empty support on $(X, weak)$ is null, and there exists an injective continuous linear mapping $X_0 \to c_0$. □

Theorem 23. *Let E be a subset of a Banach space such that every sphere*

$S_r(a) = \{x \in E \ : \ \|x - a\| = r\}$ *($a \in E$) is a Radon space of type ($\mathcal{F}$) (or σ-paracompact and with density of measure zero) for the weak topology. Then every Borel measure $\mu \neq 0$ on $(E, weak)$ with values 0 and 1 is concentrated on a single point.*

Proof. Let F be the support of μ. It is easy to check that it contains at most a single point. Let $F = \{a\}$ or $F = \emptyset$. If μ was not concentrated on a, there would be an $r > 0$ such that $\mu(B_s(a)) = 0$ for $s < r$ and $\mu(B_s(a)) = 1$ for $s \geq r$, where $B_s(a) = \{x \in E \ : \ \|x - a\| \leq s\}$. Therefore, $\mu(S_r(a)) = 1$, which contradicts the fact that $(S_r(a), weak)$ is a Radon space of type $(\mathcal{F})$ and $F \cap S_r(a) = \emptyset$ (if we assume the other hypothesis we arrive at the same contradiction). $\square$

Corollary 24. *Let E be a Banach space such that the unit sphere $(S_1, weak)$ is a Radon space of type ($\mathcal{F}$) (or σ-paracompact and with density of measure zero). Then, every Borel measure $\mu \neq 0$ with values 0 and 1 is concentrated on a single point.*

Corollary 25. *Let E be a Banach space such that the unit sphere $(S_1, weak)$ is a Radon space of type ($\mathcal{F}$) (or σ-paracompact and with density of' measure zero). Then, every diffused finite Borel measure (that is, $\mu(\{a\}) = 0$ for all $a \in E$) on $(E, weak)$ is non-atomic.*

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