

Acad. Lubomir Iliev  
1913 – 2000

On 6th of June 2000, following prolonged and terminal illness, Acad. L. Iliev, one of the most prominent Bulgarian scientists, died.

**Acad. Lubomir Iliev was born** on April 20, 1913 in the town of Veliko Tŕrnovo. He left his native town in 1932, having accomplished his secondary education. Prof. L. Iliev graduated in mathematics from the Faculty of Physics and Mathematics of the University of Sofia "Kliment Ohridsky" in 1936. After that he specialized for two years with Acad. N. Obreshkov, heading at that time the Chair of Algebra. In 1938 he got a doctor's degree. Acad. Iliev spend the academic year of 1940 in Germany, as a Humboldt fellow, under the tuition of the eminent mathematicians Prof. O. Peron and Prof. C. Carathéodory at the University of Munich. He worked as teacher in Sofia from 1938 up to 1941. From 1941 to 1947 he was an Assistant, from 1947 up to 1949 - a private Associate professor and from 1949 he became an Associate Professor at the Chair of Analysis at the University of Sofia, then headed by Acad. L. Tchakalov. In 1952, after the retirement of Prof. Tchakalov, he became a regular professor and head of the Chair.

In 1958 he was elected Corresponding Member of the Bulgarian Academy of Sciences and in 1967 - regular member (Fellow) of the Academy.

**Acad. Lubomir Iliev was an outstanding Bulgarian mathematician.** His research activities and scientific contributions are in the field of complex analysis, namely schlicht functions, analytic inextensibility and overconvergence of series, distribution of the zeroes of polynomials and entire functions having integral representation and inequalities of P. Turan.

Let  $S_k$ ,  $k = 1, 2, 3, \dots$ , be the class of functions

$$f^{[k]}(z) = z + a_1^{[k]}z^{k+1} + a_2^{[k]}z^{2k+1} + a_3^{[k]}z^{3k+1} + \dots,$$

which are schlicht and regular in the circle  $|z| < 1$ , and let

$$f_n^{[k]}(z) = z + a_1^{[k]}z^{k+1} + a_2^{[k]}z^{2k+1} + \dots + a_n^{[k]}z^{nk+1}; \quad n = 1, 2, 3, \dots$$

be the partial sums of the functions from  $S_k$ .

Acad. Iliev proved that the partial sums  $f_n^{[2]}(z)$  are schlicht in the disk  $|z| < 3^{-1/2}$ , the partial sums  $f_n^{[3]}(z)$  are schlicht in the disk  $|z| < 3^{1/3}/2$  and these results are exact. The first of these results was open for 20 years.

The very first publications of Prof. L. Iliev in the field of analytic inextensibility and overconvergence of power and other series provoked great interest. His years-long investigations and many publications in this field served as a basis for his monograph *Analytische Nichtfortsetzbarkeit und Überkonvergenz einiger Klassen von Potenzreihen* published in Berlin in 1960 and in Sofia in 1961. Here a new method was created by generalizing a result of Szegő and a theorem of M. Riesz. As a result of its systematic application many new theorems for analytic non-extensibility and overconvergence of various classes of power and other series were obtained. The new criteria of analytic inextensibility of series without gaps should be mentioned here. Also new criteria of overconvergence in a restricted sense, namely those of convergence or boundedness of sequences of partial sums on the boundary of the disk of convergence, should be mentioned with the papers of P. Erdős, G. Piranian, M. Noble, M. A. Evgrafov, N. A. Davidov, D. Gaier, W. Mayer-König. The results were applied to series on other systems of functions. The investigations on Dirichle series should be pointed out for the rather complicated methods used there. A flavor of these results is demonstrated by the following statement proved by Acad. Iliev.

*Let for the series*

$$(1) \quad f(z) = \sum_{n=0}^{\infty} c_n z_n \quad \overline{\lim}_{n \rightarrow \infty} |c_n|^{1/n} = 1$$

*There exist two infinite sequences of indices  $\{m_\nu\}$  and  $\{n_\nu\}$ ,  $m_\nu < n_\nu$ ;  $\nu = 1, 2, 3, \dots$ , a number  $\varepsilon$ ,  $0 < \varepsilon < 1$ , such that*

$$\lim_{\nu \rightarrow \infty} |c_{n_\nu}|^{1/n_\nu} = 1;$$

$$c_{m_\nu - k} = c_{n_\nu - k}; \quad k = 1, 2, 3, \dots, [\varepsilon n_\nu], \quad m_\nu > [\varepsilon n_\nu]$$

*and*

$$\lim_{\nu \rightarrow \infty} c_{m_\nu} / c_{n_\nu} \neq 1,$$

*then the serie (1) cannot be extended analytically outside the unit disk.*

The results for the overconvergence obtained in the monograph under consideration refer both to the classical case of A. Ostrovski and to the overconvergence in a generalized sense.

Acad. Iliev applied these results also to some series on special systems of functions. He showed directly, by means of the established approach, that to every criterion of analytic inextensibility of power series there corresponds a criterion of analytic inextensibility of series on Faber polynomials. It should be pointed out that the direct proof of Szegő's theorem for power series whose coefficients take a finite number of values was extended to the series on Faber polynomials as well. Ostrovski's two theorems of overconvergence are also applied to the series on Faber polynomials.

Acad. Iliev was interested on the distribution of the zeroes of polynomials and entire functions having integral representation, in connection with the investigations of J. Jensen, G. Polya, G. Segö, G. H. Hardy, E. Titchmarsh, M. Cartwright, N. Obreshkov, L. Tchakalov and were stimulated by the integral representation of the Riemann  $\zeta$ -function. Thus, many new classes of polynomials and entire functions have been introduced and it was established that their zeroes are situated either on the real axis, in a semi-plane, or in a strip between two parallel lines, or in a disk, or in an angular region. These results of Prof. Iliev stimulated further investigations of Bulgarian and foreign mathematicians and have been cited in the papers of N. Obreshkov, A. Renyi, E. Božorov, Haradze, P. Rusev, K. Dočev, N. Minkov, D. Dimitrov, etc.

Denote by  $E(a)$  ( $\overline{E}(a)$  respectively) the set of non-negative  $R$ -integrable functions  $f(t)$  defined in the interval  $(0, a)$ ,  $a > 0$ , for which the entire function

$$F(z) = \int_0^a f(t) \cos zt \, dt$$

has only real (only real and simple respectively) zeroes.

A great number of the results of Prof. Iliev consist in finding functions  $f(t)$  decreasing in the interval  $(0, a)$  which belong to  $E(a)$  or  $\overline{E}(a)$ . For example:

Let  $q$  be a natural number and  $f_0(t) = \varphi_0(x)$ ,  $x = 1 - t^{2q}$ . If  $f_0(t) \in E(1)$ ,  $f_0(t) \in \overline{E}(1)$  respectively, then the function

$$f_1(t) = \varphi_1(x) = \int_0^x \varphi_0(u) \, du$$

will belong to  $E(1)$ , to  $\overline{E}(1)$ , respectively.

Acad. Iliev established a classification of the systems of polynomials and entire functions formed by Laguerre's functions. He proved that the inequalities of P. Turan and some more general ones are satisfied by wide classes of functions of several real variables.

There are many interesting results of Prof. Iliev in the field of spline functions. In this research he uses a logic for generation that is implied by the numerous previous investigations he accomplished on various systems of polynomials and entire functions.

The mathematical creative work of Acad. Iliev is marked by its own specific style. His contributions are remarkable for the elegance of the results and conciseness of the proofs. In many cases the proof is a clever trick, a combination of various well-known propositions, which, taken together, yield numerous new results. Many of his works are not lengthy but they are rich in results and new ideas. Due to this they have been often cited and have drawn the attention of many Bulgarian and foreign mathematicians.

**Acad. Lubomir Iliev was a pioneer and promoter of the computer science in Bulgaria.** In 1961 the Bulgarian Academy of Sciences established the first computing center in Bulgaria, as part of the Institute of Mathematics at the Academy. Acad. Iliev was assigned the major role of the first director of this center. A particularly important activity was the creation of the first Bulgarian computer. He established a team of young Bulgarian experts whose product was the first Bulgarian computer, "Vitosha."

He organized the chair of Computational Mathematics at the Faculty of Mathematics of Sofia University and proved the necessity of computer science and technology in education. Acad. Iliev contributed to introduction of Bulgaria to the international computer science community. In 1955 and 1956, thanks to his efforts, Bulgaria became part of the informal cooperation of several COMMECON countries in the field of computer science.

**Acad. Lubomir Iliev was a driving force for the international scientific and technical cooperation.** He was the first official representative of Bulgaria to IFIP, from 1965 to 1982, and he was President of the Bulgaria National IFIP Committee from 1970 to 1982. During those years, he nominated Bulgarian Technical Committee representatives to IFIP. Under his auspices and IFIP sponsorship, a number of workshops,

symposia and conferences were organized in Bulgaria. He was an IFIP vice-president from 1974 to 1977 and was awarded the IFIP Silver Core for his service. Acad. Iliev was the second scientist in the world presented IFIP Isaac L. Auerbach Award, commemorating the founding president of IFIP. He was also awarded a "Computer Pioneer" by the Computer Society IEEE.

Acad. L. Iliev was very active in the Balkan Mathematical Union and was its Vice-President from 1966 to 1974 and its President from 1974 to 1977. He was the President of the Scientific Council of the International Mathematical Center "S. Banach" in Warsaw 1972 - 1977 and served in many other international positions.

For his scientific achievements and international activities, he was elected as a Foreign Member of: the Academy of Sciences of the USSR (now the National Academy of Russian Federation) in 1976, the Academy of Sciences of the German Democratic Republic in 1977 and a Honorary Member of the Hungarian Academy of Sciences in 1983. Prof. Iliev became Doctor Honoris causa of the Technical University of Dresden in 1977.

**Acad. Iliev was a talented scientific manager.** He started his scientific administration work as Vice-Dean of the Faculty of Mathematics and Natural Sciences at Sofia University – 1949-1951. From 1959 to 1962 he was Vice-President of the State Committee for Sciences. In 1963 he became a Director of the Institute of Mathematics with Computer Center to the Bulgarian Academy of Sciences and for a quarter of a century he worked on developing this Institute according to his dreams.

Acad. L. Iliev served for a long time in the government of the Bulgarian Academy of Sciences. He was General Scientific Secretary of the Academy from 1961 to 1968 and Vice-President of the Academy from 1968 to 1973.

As a scientist with very high reputation, Acad. Iliev served as Vice-President and President of the Higher Attestation Committee from 1966 to 1980.

**Acad. Iliev was proud to be Bulgarian.** He has a live interest in the history of Bulgaria and was a passionate investigator of the ancient world and the peoples which had inhabited our land. Few people know that he was a zealous numizmatist and had in particular devoted his studies to the coins minted in the Balkan Peninsula. This historic and archeological hobby has also contributed to the ties with his native town Veliko Tŕrnovo.

Acad. L. Iliev established and was the first Editor-in-Chief of *SERDICA – Bulgaricae mathematicae publicationes* in 1974, *PLISKA – Studia mathematica bulgarica* in 1975 and *AZBUKI – Bulgarian Mathematical Monographs* in 1980. All the titles of these publications are connected with the Bulgarian history: *SERDICA* is the roman name of the Bulgarian capital Sofia, *PLISKA* is the name of an old Bulgarian capital and *AZBUKI* is the name of the Bulgarian alphabet, known as cirilic.

With Acad. Iliev demise Bulgaria lost one of her prominent citizens, outstanding scientists and gifted scientific managers, the Bulgarian mathematical community lost its doyen and wise leader, the world lost a tireless proponent of scientific, cultural and technical cooperation among peoples.

Bl. Sendov