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GROUPS WITH THE MINIMAL CONDITION ON NON-"NILPOTENT-BY-FINITE" SUBGROUPS

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Abstract. We characterize the groups which do not have non-trivial perfect sections and such that any strictly descending chain of non-"nilpotent-by-finite" subgroups is finite.

0. Let $\mathfrak{X}$ be a property pertaining to subgroups. One of approaches to study the structure of groups is to investigate the groups in which the set of non-$\mathfrak{X}$-subgroups is small in some sense (or in other words, the groups which have many $\mathfrak{X}$-subgroups). There is increasing interest in groups with many nilpotent subgroups. Problems of this type have been considered in several papers. For instance, the examples of non-nilpotent groups with nilpotent and subnormal proper subgroups (the so-called groups of Heineken-Mohamed type) were constructed by Heineken and Mohamed [15], Hartley [14], Menegazzo [16] and others. In this way Bruno [6]–[8], Bruno and Phillips [9] and Asar [4] have studied the minimal non-"nilpotent-by-finite" groups (i.e. non-"nilpotent-by-finite"

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groups with nilpotent-by-finite proper subgroups). Their results imply that any minimal non-"nilpotent-by-finite" group G is either a p-group in which any two proper subgroups generate a proper subgroup or $G = V \rtimes H$ is a semidirect product of a normal subgroup V and a quasicyclic p-subgroup H, where V is a special q-group, V' is centralized by H and V/V' is a minimal normal subgroup of G/V' (p and q are distinct primes).

One of the possible interpretations of the requirement that "many subgroups of G satisfy $\mathfrak{X}$" is that "the set of subgroups in G not having $\mathfrak{X}$ satisfies the minimal condition". In this direction the aim of our article is to consider groups with the minimal condition on non-"nilpotent-by-finite" subgroups Min-$\overline{NF}$. We say that a group G satisfies Min-$\overline{NF}$ if for every strictly descending chain $\{G_n \mid n \in \mathbb{N}\}$ of subgroups in G there exists a number $n_0 \in \mathbb{N}$ such that G_n is a nilpotent-by-finite subgroup for any integer $n \geq n_0$. For the next we need the concept of an HM^*-group first introduced by Asar [3] in the class of p-groups with the normalizer condition. We extend this notion and call G an HM^*-group if its commutator subgroup G' is hypercentral and G/G' is a divisible Černikov p-group. Obviously any group of Heineken-Mohamed type and any minimal non-"nilpotent-by-finite" group are HM^*-groups and satisfy Min-$\overline{NF}$.

Notice that Černikov (see e.g. [11]) and Šunkov [20] have studied groups with the minimal condition on non-abelian subgroups; Phillips and Wilson [17] have investigated groups in which the set of non-"locally nilpotent" subgroups satisfies the minimal condition. Recently Dixon, Evans and Smith [12] have shown that a locally graded group with the minimal condition on non-nilpotent subgroups is either nilpotent or locally finite.

In this paper we characterize groups without non-trivial perfect sections and which satisfy Min-$\overline{NF}$. Namely, we prove:

Theorem. *Let G be a group without non-trivial perfect sections. Then G satisfies Min-$\overline{NF}$ if and only if it is of one of the following types:*

(1) *G is a nilpotent-by-finite group;*

(2) *G contains a normal subgroup H of finite index such that*

$$H = H_0 \cdot H_1 \cdot \ldots \cdot H_n \ (n \geq 1),$$

where H_i is an HM^*-group with nilpotent commutator subgroup $H_i' = H' \leq H_0$ $(i = 1, \ldots, n)$, H_0 is a nilpotent group with divisible Černikov quotient group H_0/H' (in particular, H_0/H' is trivial) and, furthermore, if $k \neq s$ $(1 \leq k, s \leq n)$, then $\pi(H_k/H') \cap \pi(H_s/H') = \varnothing$.

Throughout this paper p is a prime and $\mathbb{C}_{p^\infty}$ is the quasicyclic p-group. For a group G, we denote by $Z(G)$ the centre, by $G', G'', \dots, G^{(n)}, \dots$ the members of the derived series, by $\pi(G)$ the set of all primes which divide the orders of periodic elements in G and by $\mathbb{Z}_{p^n}G$ the group ring of G over the ring $\mathbb{Z}_{p^n}$ of integers modulo a prime power p^n. Recall that a section of G is a group of the form $S = H/N$ for some subgroups N and H of G, where N is normal in H. The group G is perfect if $G' = G$.

We shall also use other standard terminology from [13] and [18].

1. In the sequel we shall need the following lemmas.

Lemma 1. *Let G be a group satisfying Min-$\overline{NF}$ and let A be a subgroup of G. Then:*

(1) *A satisfies Min-$\overline{NF}$;*

(2) *if A is normal in G, then the quotient group G/A satisfies Min-$\overline{NF}$;*

(3) *if A is a normal non-"nilpotent-by-finite" subgroup, then G/A satisfies the minimal condition on subgroups.*

The proof of the lemma is immediate.
As usual, R is called a right V-ring if each of its right ideals is an intersection of maximal right ideals of R.

Lemma 2. *Let $G = A \rtimes B$ be a semidirect product of an infinite abelian subgroup A of exponent p^n $(n \geq 1)$ and a quasicyclic q-subgroup B with distinct primes p and q. If G satisfies Min-$\overline{NF}$, then G is a nilpotent group or, considered as a $\mathbb{Z}_{p^n}B$-module, the abelian group $A = S_0 \oplus S_1 \oplus \cdots \oplus \cdots \oplus S_m$ $(m \geq 1)$ is a direct sum of simple $\mathbb{Z}_{p^n}B$-submodules $S_1, \dots, S_m$ and a finite $\mathbb{Z}_{p^n}B$-submodule S_0, where $S_0 \leq Z(G)$.*

Proof. Suppose that G is a non-"nilpotent-by-finite" group. Since A is abelian, it becomes a right module over the commutative (von Neuman) regular ring $R = \mathbb{Z}_{p^n}B$ via the conjugation action on A.

Let u be any non-zero element of the module A such that $u \notin Z(G)$. Then the cyclic submodule uR is isomorphic to the right R-module $R_0 = R/\mathrm{ann}_R(u)$, where $\mathrm{ann}_R(u) = \{r \in R \mid ur = 0\}$. Moreover R_0 is a commutative (von Neuman) regular ring and each of its ideals is a right R-module. If R_0 is a non-simple module, then it contains a non-zero element b_1 such that b_1R_0 is a proper ideal in R_0. By Proposition 19.24 (d) of [13], $b_1R_0 = e_1R_0$ for some idempotent e_1 and, furthermore, $R_0 = e_1R_0 \oplus R_1$ is a direct sum (of rings) for some finitely generated ideal R_1 of R_0. If R_1 is a non-simple R-module, then it has a proper submodule e_2R_1, where $e_2^2 = e_2$. Hence $R_1 = e_2R_1 \oplus R_2$ is a direct sum for some finitely generated ideal R_2 of R_0. By a similar argument we obtain a strictly descending chain of R-submodules $R_0 > R_1 > \dots$. This implies that uR also contains a strictly descending chain of R-submodules $\{D_m \mid m \in \mathbb{N} \cup \{0\}\}$, where $D_0 = uR$ and D_m is isomorphic to R_m. By our hypothesis there exists an integer n such that D_nB is a non-"nilpotent-by-finite" subgroup, while $D_{n+1}B$ is nilpotent-by-finite and, as a consequence, $D_{n+1} \le Z(G)$.

It is also clear that R_n is a cyclic R-module. Using the same idea, as above, we deduce that D_n contains a strictly descending chain of R-submodules $\{D_{nm} \mid m \in \mathbb{N} \cup \{0\}\}$ (where $D_{n0} = D_n$) such that $D_{nk}B$ is not a nilpotent-by-finite subgroup for some integer k, while $D_{n,k+1}B$ is nilpotent-by-finite. Continuing in this manner we conclude that A contains a simple R-submodule S. By the theorem of Kaplansky [13, Corollary 19.53], R is a V-ring and S is an injective R-module. Therefore $A = S \oplus A_1$ is a direct sum of S and some submodule A_1. Repeating this argument we deduce that there exists an integer l such that

$$A = A_1 \oplus \cdots \oplus A_l \oplus A_0$$

is a direct sum of simple R-submodules $A_1 \dots, A_l$ and some R-submodule A_0, where A_0B is a nilpotent-by-finite subgroup. It is easily verified that $A_0 \le Z(G)$ and so A_0 is finite. The result follows. □

Lemma 3. *Let G be a non-perfect group with nilpotent-by-finite proper normal subgroups. Then G satisfies Min-$\overline{NF}$ if and only if it is of one of the following types:*

(1) *G is a nilpotent-by-finite group;*

(2) *G is a minimal non-"nilpotent-by-finite" group;*

(3) *$G = G' \rtimes S$, $S \cong \mathbb{C}_{p^\infty}$, $G' = S_1 \times \cdots \times S_n$ $(n \ge 1)$ is a p'-subgroup and a direct product of finitely many Sylow p_i-subgroups $S_1, \dots, S_n$, where S_i is a nilpotent subgroup of exponent $p_i^{m_i}$ and of derived length d_i. Furthermore, for any i there is an integer k_i $(1 \le k_i \le d_i)$ such that S acts trivially on $S_i^{(k_i)}$ and $S_i^{(l-1)}/S_i^{(l)} = A_0 \oplus A_1 \oplus \cdots \oplus A_m$ is a finite direct sum of simple $\mathbb{Z}_{p_i^{m_i}}S$-submodules $A_1, \dots A_m$ $(m \ge 1)$ and a finite $\mathbb{Z}_{p_i^{m_i}}S$-submodule A_0 under the conjugation action on S_i, where $A_0 \le Z(S_i^{(l-1)} \rtimes S/S_i^{(l)})$ $(i = 1, \dots, n; l = 1, \dots, k_i; k_i, m_i, d_i \in \mathbb{N}$ and $S_i^{(0)} = S_i)$;*

(4) $G = A \rtimes S$, *where* S *is a minimal non-"nilpotent-by-finite"* p*-group,* A *is a normal nilpotent* p'*-subgroup and* G/S' *is a group of type* (3).

Proof. ($\Leftarrow$) This part of the proof is evident.

($\Rightarrow$) First, if the quotient group G/G' contains two proper subgroups which generate it, then $G = AB$ is a product of two nilpotent-by-finite proper normal subgroups A and B. Since A (respectively B) contains an abelian G-invariant subgroup A_0 (respectively B_0) of finite index, we conclude that A_0B_0 is a nilpotent normal subgroup of finite index in G.

Now assume that any two proper subgroups of G/G' generate a proper subgroup in G/G'. Hence G/G' is either a cyclic p-group (and in this case G is a nilpotent-by-finite group) or the quasicyclic p-group for some prime p. Let $G/G' \cong \mathbb{C}_{p^\infty}$. If D is a proper nilpotent G-invariant subgroup of finite index in G', then G/DG'' is an abelian group, which is a contradiction. Thus G' is a nilpotent subgroup. Inasmuch as G satisfies Min-$\overline{NF}$, it contains a subgroup S which is a non-perfect minimal non-"nilpotent-by-finite" group.

If $S = G$, then G is of type (2). Therefore we may suppose that $S \neq G$. Then $G = G'S$ and, by Theorem 2.5 of [9], S is a torsion subgroup. Let $\overline{G} = G/G''(G' \cap S) = \overline{G}' \rtimes \overline{S}$. It is easy to see that $\overline{G}$ satisfies the minimal condition on normal subgroups Min-n and so, by the theorem of Baer [18, Theorem 5.25] and Theorem 2.1 of [9], G is a locally finite group. If $\overline{G}$ is a p-group, then it is Černikov (see [18, p.156, Corollary 2]). From this it follows that $\overline{G}$ is a nilpotent group and we reach a contradiction. Hence $\overline{G}'$ is a p'-subgroup. Our hypothesis and Theorem B of [5] give that G' is a π-subgroup for some finite set of primes π and, as a consequence, $G = A \rtimes Q$, where $Q = S$ is either a minimal non-"nilpotent-by-finite" p-group or $Q \cong \mathbb{C}_{p^\infty}$ and A is a p'-subgroup of G'.

Assume that Q is a quasicyclic p-subgroup and $A = S_1 \times \ldots \times S_n$ is a group direct product of the Sylow p_i-subgroups $S_1, \ldots, S_n$. Obviously $\exp(S_i) = p_i^{m_i}$ for some $m_i \in \mathbb{N}$ ($i = 1, \ldots, n$). Let d_i be the derived length of S_i.

We have seen that there exists an integer m ($1 \le m \le d_i$) such that $S_i^{(m-1)}Q$ is a non-"nilpotent-by-finite" subgroup, while $S_i^{(m)}Q$ is nilpotent-by-finite. Then $[S_i^{(m)}, Q] = 1$. By Lemma 2, $S_i^{(l-1)}/S_i^{(l)} = A_0 \oplus A_1 \oplus \cdots \oplus A_k$ is a direct sum of simple $\mathbb{Z}_{p_i^{m_i}}S$-submodules $A_1, \ldots, A_k$ ($k \ge 1$) and a finite $\mathbb{Z}_{p_i^{m_i}}S$-submodule A_0 under the conjugation action on S_i, where $A_0 \le Z(S_i^{(l-1)}Q/S_i^{(l)})$ ($1 \le l \le m$). Thus G is a group of type (3).

Finally, if Q is a minimal non-"nilpotent-by-finite" group, then it is not difficult to see that G/S' is a group of type (3). The lemma is proved. $\square$

Corollary 4. *Let G be a group without non-trivial perfect sections. If G satisfies Min-$\overline{NF}$, then it is countable and locally finite.*

Lemma 5. *Let G be an HM^*-group. Then G satisfies Min-$\overline{NF}$ if and only if G' is a nilpotent subgroup.*

P r o o f. ($\Leftarrow$) Let $\{K_n \mid n \in \mathbb{N}\}$ be any strictly descending chain of subgroups in G. Since G/G' is Černikov, there is an integer m such that $K_n \leq G'$ and so G satisfies Min-$\overline{NF}$.

($\Rightarrow$) If the commutator subgroup G' is not nilpotent-by-finite, then, in view of Lemma 2, it has a subnormal non-"nilpotent-by-finite" subgroup S with all proper normal subgroups nilpotent-by-finite. Then Lemma 3 yields that S is a non-hypercentral subgroup and we obtain a contradiction. This implies that G' is a nilpotent-by-finite subgroup and consequently it is nilpotent, as desired. □

Now we give some examples of HM^*-groups.

Examples. (i) First we recall one construction from [5]. Let p and q be distinct primes and let $\mathbb{F}_q$ be the field with q elements. We denote by $\mathbb{F}_q(\alpha)$ the subfield of the algebraic closure of $\mathbb{F}_q$ generated by α. If ε_i is a primitive p^i-th root of 1 ($i = 0, 1, 2, \ldots$), put $F_i = \mathbb{F}_q(\varepsilon_i)$ and $F = \bigcup_{i=0}^{\infty} F_i$. Let A be the additive group of F, and let B be the multiplicative group which contains the p^i-th roots of 1, where $i = 0, 1, 2, \ldots$. The rule

$$bab^{-1} = b^{p^m} \cdot a,$$

where $a \in A$, $b \in B$ and $b^{p^m} \cdot a$ is the product of the elements b^{p^m} and a in the field F, m is some non-negative integer, defines an action of B on A. The group $G_m = A \rtimes B$ constructed in this manner is called a Čarin group [5]. The groups G_0 were first considered by Čarin [10].

Since $A = G'_m$, G_m is an HM^*-group. Moreover it is a minimal non-"nilpotent-by-finite" group.

(ii) Let F be a field defined as in (i) and J a non-trivial F-F-bialgebra (it is well known that such a bialgebra J exists). We denote by A and I the subalgebra

$$\begin{pmatrix} F & J & \ldots & J \\ 0 & F & \ldots & J \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & F \end{pmatrix}$$

of the algebra of $n \times n$ matrices with the identity element 1 ($n \geq 2$) and the nilpotent ideal

$$\begin{pmatrix} 0 & J & \dots & J \\ 0 & 0 & \dots & J \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

of nilpotency index n, respectively. Then the ring $A/I \cong F \oplus \cdots \oplus F$ is a direct sum of n copies of F. It is known that a nilpotent ring I with two operations "+" and "·" forms a group under the operation $a \circ b = a + b + a \cdot b$ for all $a, b \in I$. This group is called the adjoint group of I and denoted by I°. By Lemma 2.4 of [2] I° is a q-group. Moreover, by the lemma from [21, p. 27], see also [1], I° is a nilpotent group of class n. Since the unipotent subgroup $1 + I$ is isomorphic to I° and the multiplicative group F^* of F is a q'-group, we see that the unit group

$$U(A) = (1 + I) \rtimes (F^* \times \cdots \times F^*)$$

of A is a semidirect product of the normal subgroup $1+I$ and the direct product of n copies of F^*. It is clear that $U(A)$ contains the subgroup

$$G = (1 + I) \rtimes (B \times \cdots \times B)$$

of finite index, where B is a quasicyclic p-subgroup of F^*.

We want to prove that G is an HM^*-group. First we consider the case $n = 2$. The commutator

$$\left[\begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}\right] =$$

$$\begin{pmatrix} 1 & -i \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}\begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1^{-1} & 0 \\ 0 & x_2^{-1} \end{pmatrix} = \begin{pmatrix} 1 & x_1 i x_2^{-1} - i \\ 0 & 1 \end{pmatrix} \in 1 + I$$

for all elements $i \in J$ and $x_1, x_2 \in B$ and, as a consequence,

$$(*) \qquad [1 + I, B \times B] = 1 + I.$$

Now we turn to the general case and, by using a similar argument, obtain that

$$[1 + I, \underbrace{B \times \cdots \times B}_{n \text{ times}}] = 1 + I,$$

i.e. G is an HM^*-group with nilpotent commutator subgroup $G' = 1 + I$.

(iii) Let J be a left T-nilpotent F-F-bialgebra with F and B as before. An example of a left T-nilpotent bialgebra is contained, for instance, in [19, Example 1]. We denote by J^1 the bialgebra obtained by adjoining an identity element to J. Writting A for the subalgebra

$$\begin{pmatrix} J^1 & J \\ 0 & F \end{pmatrix}$$

of the 2×2 matrix algebra with the identity element 1 and I for the left T-nilpotent ideal

$$\begin{pmatrix} J & J \\ 0 & 0 \end{pmatrix},$$

we have that $1 + I$ is a hypercentral q-subgroup by Lemma 2.4 of [2] and the lemma from [21, p. 27]. By the same argument, as in (ii), we can prove that the condition $(*)$ holds, i.e. $G = (1 + I) \rtimes (B \times B)$ is an HM^*-group.

2. Proof of Theorem. ($\Leftarrow$) This direction of the proof is immediate.

($\Rightarrow$) Assume that G is a non-"nilpotent-by-finite" group. Then G has a strictly descending subnormal series

$$G = G_0 \triangleright G_1 \triangleright \cdots \triangleright G_m = S,$$

where S is a non-"nilpotent-by-finite" group with all proper normal subgroups nilpotent-by-finite. By Lemma 1, G_j/G_{j+1} is a Černikov group ($j = 0, 1, \ldots, m-1$).

Let $z \in G_{m-1}$. Then $S^z \triangleleft G_{m-1}$ and hence $S' \lhd G_{n-1}$. We denote by D_{m-1} the subgroup of finite index in G_{m-1} such that D_{m-1}/S' is the divisible part of G_{m-1}/S'. Then $D'_{m-1} = S'$. Since $D_{m-1}D^y_{m-1}/S'$ is a Černikov group for every $y \in G_{m-2}$ and S' does not contain a proper S-invariant subgroup of finite index, we conclude that $D_{m-1} \triangleleft G_{m-2}$. Continuing this process, after a finite number steps we obtain that G has a normal subgroup D of finite index with divisible Černikov group D/D' and $D' = S'$. Consequently

$$D = D_1 \cdot \ldots \cdot D_n (n \geq 1),$$

where D_s/D' is a divisible Černikov Sylow p_s-subgroup of D/D' ($s = 1, \ldots, n$) and $p_s \neq p_l$ if $s \neq l$ ($1 \leq s, l \leq n$). If D_k is a nilpotent-by-finite subgroup for some integer k ($1 \leq k \leq n$), then it is nilpotent. This yields that D_m is not nilpotent-by-finite for some integer m, where $1 \leq m \leq n$. Then it contains a subnormal subgroup T with all proper normal subgroups nilpotent-by-finite. As above we can prove that $T' = D'_m = S'$. Hence D_m is an HM^*-group with nilpotent commutator subgroup D'_m. The theorem is proved. □

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