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ADEQUATE COMPACTA WHICH ARE GUL'KO OR TALAGRAND

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Abstract. We answer positively a question raised by S. Argyros: Given any coanalytic, nonalytic subset Σ' of the irrationals, we construct, in Mercourakis space $c_1(\Sigma')$, an adequate compact which is Gul'ko and not Talagrand. Further, given any Borel, non F_σ subset Σ' of the irrationals, we construct, in $c_1(\Sigma')$, an adequate compact which is Talagrand and not Eberlein.

0. Introduction. On the last Sunday of August 1998, the first named author, Petr Čížek died at a car accident in the U.S.A. This paper was prepared on the basis of his Diploma Thesis [2] by the second named author, his supervisor.

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In [11], Talagrand constructed a Talagrand compact space which is not Eberlein. In [12], he constructed an example of a Gul'ko compact space which is not Talagrand. His example is based on the fact that the set of all well founded trees is not analytic. In this note, we suggest a method of constructing a nontrivial compact set in Mercourakis space $c_1(\Sigma')$ where Σ' is any coanalytic subset of a 0-dimensional Polish space Σ. This is done via an adequate family of subsets in Σ'. In such a way we get, in Theorem 3.4, a *Gul'ko compact which is not Talagrand* (if Σ' is not analytic) and, in Theorem 3.6, a *Talagrand compact which is not Eberlein* (if Σ' is Borel non F_σ). We use a fact that Σ can be continuously injected into the space of trees in such a way that the preimage of the well-founded trees is Σ'. Our adequate family on Σ' is then obtained as the preimage of an adequate family in the set of all well-founded trees, which was constructed in [12].

1. Preliminaries. A compact space is called *Eberlein* if it is homeomorphic to a weakly compact subset of a Banach space. Put

$$\mathcal{S} = \emptyset \cup \mathbb{N} \cup \mathbb{N}^2 \cup \cdots$$

where $\mathbb{N}$ denotes the set of positive integers. For $\sigma \in \mathbb{N}^{\mathbb{N}}$ and $n \in \mathbb{N}$ we put $\sigma|n = (\sigma(1), \ldots, \sigma(n))$. A topological space X is called *$\mathcal{K}$-analytic ($\mathcal{K}$-countably determined)* if X is a subspace of a compact space C and there are closed subsets $K_s \subset C,\ s \in \mathcal{S}$, such that

$$X = \bigcup_{\sigma \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} K_{\sigma|n}$$

$$\left(X = \bigcup_{\sigma \in \Sigma'} \bigcap_{n=1}^{\infty} K_{\sigma|n} \quad \text{for some subset } \Sigma' \subset \mathbb{N}^{\mathbb{N}} \right).$$

It is known (and can be shown without much effort) that these concepts do not depend on which compact superspace C is considered.

Proposition 1.1 ([11, Proposition 1.1], [4, Proposition 7.1.1]). *A completely regular space X is $\mathcal{K}$-analytic ($\mathcal{K}$-countably determined) if and only if there is an upper semicontinuous and compact valued mapping from $\mathbb{N}^{\mathbb{N}}$ (from a subset of $\mathbb{N}^{\mathbb{N}}$) onto X.*

A compact space K is called *Talagrand (Gul'ko)* if the space $C(K)$ of continuous functions on K endowed with the topology p of the pointwise convergence on K is $\mathcal{K}$-analytic ($\mathcal{K}$-countably determined).

In what follows, we shall focus on a special class of compacta consisting of characteristic functions of a family of subsets of a given set. Let Γ be a nonempty set. A family $\mathcal{A}$ of subsets of Γ is called *adequate* if

(i) for every $\gamma \in \Gamma$ the singleton $\{\gamma\}$ belongs to $\mathcal{A}$,

(ii) whenever $A \in \mathcal{A}$ and $B \subset A$, then $B \in \mathcal{A}$, and

(iii) if $A \subset \Gamma$ and $B \in \mathcal{A}$ for every finite set $B \subset A$, then $A \in \mathcal{A}$.

If $\mathcal{A}$ is such a family, we put

$$K_{\mathcal{A}} = \{\chi_A : \ A \in \mathcal{A}\};$$

then it is easy to check that $K_{\mathcal{A}}$ is a compact subset in the space $\{0,1\}^{\Gamma}$. The compacta constructed in this way will be a main objective of this paper. For $\gamma \in \Gamma$ put

$$\delta(\gamma)(\chi_A) = \chi_A(\gamma), \quad \chi_A \in K_{\mathcal{A}};$$

then, obviously, $\delta(\gamma) \in C(K_{\mathcal{A}})$. Put

$$\Gamma^* = \delta(\Gamma) \cup \{0\}.$$

Proposition 1.2 ([11]). *Let $\mathcal{A}$ be an adequate family of subsets of some set Γ. Then:*

(i) The set Γ^* separates the points of the compact $K_{\mathcal{A}}$.

(ii) The set Γ^* is closed in $(C(K_{\mathcal{A}}), p)$.

(iii) The set $\delta(\Gamma)$ is discrete in (Γ^*, p).

(iv) The mapping $\delta : \Gamma \to C(K_{\mathcal{A}})$ is injective.

(v) The sets $\Gamma^* \backslash \delta(A)$, $A \in \mathcal{A}$, form a subbase of neighbourhoods of 0 in the subspace (Γ^*, p).

P r o o f. It can be found in the proof of [11, Théorème 4.2]. □

Theorem 1.3 (see [11, Théorème 4.2], [4, Theorem 4.3.2]). *Let $\mathcal{A}$ be an adequate family consisting of at most countable subsets of a set Γ. Then the corresponding compact $K_{\mathcal{A}}$ is Eberlein if and only if there exist subsets $\Gamma_n \subset \Gamma,\ n \in \mathbb{N}$, such that $\Gamma = \bigcup_{n=1}^{\infty} \Gamma_n$ and for every $A \in \mathcal{A}$ and every $n \in \mathbb{N}$ the set $A \cap \Gamma_n$ is finite.*

Theorem 1.4 (see [11, Théorème 4.2]). *Let $\mathcal{A}$ be an adequate family consisting of subsets of a set Γ. Then $K_{\mathcal{A}}$ is Talagrand compact if and only if there exist subsets $\Gamma_s \subset \Gamma,\ s \in \mathcal{S}$, such that $\Gamma = \bigcup_{\sigma \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \Gamma_{\sigma|n}$ and for every $A \in \mathcal{A}$ and every $\sigma \in \mathbb{N}^{\mathbb{N}}$ there is $n \in \mathbb{N}$ such that the set $A \cap \Gamma_{\sigma|n}$ is finite. Moreover, the system $\{\Gamma_s :\ s \in \mathcal{S}\}$ can be considered monotone in the sense that $\Gamma_s \subset \Gamma_t$ whenever $s, t \in \mathcal{S}$ and $s \prec t$.*

Theorem 1.5 (see [7, Theorem 1.2]). *Let X be a $\mathcal{K}$-analytic ($\mathcal{K}$-countably determined) topological space and let $\mathcal{A}$ be an adequate family of subsets of X such that each $A \in \mathcal{A}$ is closed and discrete. Then the corresponding compact $K_{\mathcal{A}}$ is Talagrand (Gul'ko).*

2. Talagrand's adequate family on well founded trees. We shall introduce some more notations and concepts. For $s = (s(1), \dots, s(m)) \in \mathcal{S}$ we put $|s| = m$, $[s] = s(1) + \dots + s(m)$, $s|k = (s(1), \dots, s(k))$ if $k \in \{1, \dots, m\}$, and $s\hat{\ }k = (s(1), \dots, s(m), k)$ if $k \in \mathbb{N}$. For $\sigma \in \mathbb{N}^{\mathbb{N}}$ and $k \in \mathbb{N}$ we put $\sigma|k = (\sigma(1), \dots, \sigma(k))$. For $s = (s(1), \dots, s(m)) \in \mathcal{S}$ and $t = (t(1), \dots, t(n)) \in \mathcal{S}$ we write, by definition, $s \prec t$ if $m < n$ and $s(1) = t(1), \dots, s(m) = t(m)$. A nonempty subset T of the set $\mathcal{S}$ is called a *tree* if $s \in T$ whenever $t \in T$ and $s \prec t$. We shall not consider the tree $\{\emptyset\}$. The set of all trees is denoted by $\mathcal{T}$. For a tree T we denote by $[T]$ the set of all $\sigma \in \mathbb{N}^{\mathbb{N}}$ such that $\sigma|n \in T$ for every $n \in \mathbb{N}$. A tree T is called *ill-founded* if $[T]$ is nonempty. The set of ill-founded trees is denoted by $\mathcal{P}$. We put $\mathcal{L} = \mathcal{T} \backslash \mathcal{P}$ and the elements of $\mathcal{L}$ are called *well-founded* trees. On $\mathcal{T}$, we consider the topology of the pointwise convergence on $\mathcal{S}$; thus $\mathcal{T}$ is a subspace of the metric compact $\{0,1\}^{\mathcal{S}}$. For $n \in \mathbb{N}$ we put $I_n = \{s \in \mathcal{S} :\ [s] \le n\}$ and

$$V_n(Y) = \{X \in \mathcal{T} :\ X \cap I_n = Y \cap I_n\}$$

for $Y \in \mathcal{T}$. Note that the sets $V_n(Y)$ are clopen and form a basis of the topological space $\mathcal{T}$.

Let $\mathcal{A}_0$ be a family consisting from all finite subsets B of $\mathcal{L}$ such that we can write $B = \{Y_1, \ldots, Y_n\}$ and there exist $X \in \mathcal{T}$ and $s \in X$, with $|s| \geq n$, so that $Y_i \in V_{[s|i]}(X)$, $i = 1, \ldots, n$. Let $\mathcal{A}$ be the smallest adequate family of subsets of $\mathcal{L}$ which contains $\mathcal{A}_0$.

Lemma 2.1. *Consider $A \in \mathcal{A}$ and let X be a cluster point of A. Then $X \in \mathcal{P}$.*

Proof. Let $\mathcal{J}$ denote the set consisting of the empty set $\emptyset$ and of all strictly increasing sequences of positive integeres. We observe that the mapping $\psi : \mathcal{S} \to \mathcal{J}$ defined by

$$\psi(\emptyset) = \emptyset, \quad \psi(n_1, \ldots, n_k) = (n_1, n_1 + n_2, \ldots, n_1 + n_2 + \cdots + n_k), \ (n_1, \ldots, n_k) \in \mathcal{S},$$

is a bijection. Using this observation, we can translate our Lemma to [12, lemma 1]. □

3. Construction of counterexamples in $c_1(\Sigma')$. Given a topological space X, we define *Mercourakis' space* $c_1(X)$ by

$$c_1(X) = \{f \in \mathbb{R}^X : \ \{x \in X : \ |f(x)| \geq \epsilon\} \text{ is closed and discrete for every } \epsilon > 0\}$$

and consider the topology of the pointwise convergence on it [7], [4, page 127]. We note that if $\mathcal{A}$ is an adequate family consisting of closed discrete subsets of X, then the corresponding $K_{\mathcal{A}}$ is a subspace of $c_1(X)$.

Adequate families for our compacta will be constructed in coanalytic subsets of 0-dimensional Polish spaces. Such subsets can be continuously sent into the set $\mathcal{L}$ of well founded trees, see for instance [6]. Using a simple trick, we arrange this mapping injective:

Proposition 3.1. *Let Σ be a 0-dimensional Polish space (for instance $\mathbb{N}^{\mathbb{N}}$) and Σ' its coanalytic subset. Then there exists a continuous injective mapping $H : \Sigma \to \mathcal{T}$ such that $\Sigma' = H^{-1}(\mathcal{L})$.*

Proof. The set $\Sigma \backslash \Sigma'$ is analytic. Hence it can be written in the form $\bigcup_{\sigma \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} F_{\sigma|n}$ where $\{F_s : \ s \in \mathcal{S}\}$ is a monotone system of closed subsets of

Σ. Since the space Σ is 0-dimensional, we can assume that all the sets F_s are clopen. Further we can assume that the system $\{F_s : \ s \in \mathcal{S}\}$ forms a base for the topology in Σ (If not, then we can add some countable base of clopen sets to the beginning of it.) We define a mapping $H : \Sigma \to \mathcal{T}$ by

$$H(x) = \{s \in \mathcal{S} : \ x \in F_s\}, \quad x \in \Sigma.$$

H is well defined, continuous, and injective. This is so since the system $\{F_s : \ s \in \mathcal{S}\}$ is monotone, consists of clopen sets, and separates the points of the space Σ (as it is a base for the topology of Σ).

Now, $x \in \Sigma \backslash \Sigma'$ if and only if there exists $\sigma \in \mathbb{N}^{\mathbb{N}}$ such that $\sigma | n \in H(x)$ for every $n \in \mathbb{N}$, which means that $H(x)$ is an ill-founded tree. Therefore $\Sigma \backslash \Sigma' = H^{-1}(\mathcal{P})$ and so $\Sigma' = H^{-1}(\mathcal{L})$. □

Proposition 3.2. *Let Σ, Σ', and H be as in Proposition 3.1, let $\mathcal{A}$ be the family defined in Section 2, and put*

$$\mathcal{A}_1 = \{A \subset \Sigma' : \ H(A) \in \mathcal{A}\}.$$

Then the family $\mathcal{A}_1$ is adequate and its elements are closed and discrete in Σ'.

Proof. If $x \in \Sigma'$, then $H(x) \in \mathcal{L}$, hence $\{H(x)\} \in \mathcal{A}$, and so $\{x\} \in \mathcal{A}_1$. If $A \in \mathcal{A}$ and $B \subset A$, we have $H(A) \in \mathcal{A}_0$ and $H(B) \subset H(A)$; hence $H(B) \in \mathcal{A}$ and so $B \in \mathcal{A}_1$. Consider a set $A \subset \Sigma'$ such that $B \in \mathcal{A}_1$ for every finite $B \subset A$. Let $C \subset H(A)$ be any finite set. Find a finite set $B \subset A$ such that $H(B) = C$. But then $B \in \mathcal{A}_1$, i.e., $H(B) = C \in \mathcal{A}$. Thus $H(A) \in \mathcal{A}$, i.e., $A \in \mathcal{A}_1$.

Take any $A \in \mathcal{A}_1$ and assume that it is not closed or is not discrete in Σ'. Then there exists a one to one sequence (x_n) in A converging to an $x \in \Sigma'$. But then $\{x_1, x_2, \ldots\} \in \mathcal{A}_1$ and so $\{H(x_1), H(x_2), \ldots\} \in \mathcal{A}$. Hence $(H(x_n))$ is a one to one sequence converging to $H(x)$ in the space $\mathcal{L}$ because H is injective and continuous. However, this is impossible since the elements of $\mathcal{A}$ are closed and discrete in $\mathcal{L}$. □

Proposition 3.3. *Let Σ, Σ', H, and $\mathcal{A}_1$ be as in Proposition 3.2, and assume there exists a monotone system $\{\Gamma_s : \ s \in \mathcal{S}\}$ of subsets of Σ' such that*

$$\Sigma' = \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \Gamma_{\rho | n}$$

and that for every $A \in \mathcal{A}_1$ and for every $\rho \in \mathbb{N}^{\mathbb{N}}$ there is $n \in \mathbb{N}$ such that the set $A \cap \Gamma_{\rho|n}$ is finite. Then

$$\Sigma' = \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \overline{\Gamma_{\rho|n}}^{\Sigma}.$$

P r o o f. Assume that there exists $y \in \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \overline{\Gamma_{\rho|n}}^{\Sigma} \setminus \Sigma'$. Then $H(y) \in \mathcal{P}$ and hence there is $\sigma \in \mathbb{N}^{\mathbb{N}}$ so that $\sigma|n \in H(y)$ for every $n \in \mathbb{N}$. Trivially, $H(y) \in \bigcap_{n=1}^{\infty} V_{[\sigma|n]}(H(y))$. Hence $y \in \bigcap_{n=1}^{\infty} H^{-1}(V_{[\sigma|n]}(H(y)))$. Find $\rho \in \mathbb{N}^{\mathbb{N}}$ so that $y \in \bigcap_{n=1}^{\infty} \overline{\Gamma_{\rho|n}}^{\Sigma}$. Hence $y \in H^{-1}(V_{[\sigma|n]}(H(y))) \cap \overline{\Gamma_{\rho|n}}^{\Sigma}$ for every $n \in \mathbb{N}$. Here each set $H^{-1}(V_{[\sigma|n]}(H(y)))$ is open. Choose $y_1 \in H^{-1}(V_{[\sigma|1]}(H(y))) \cap \Gamma_{\rho|1}$. Choose $y_2 \in (H^{-1}(V_{[\sigma|2]}(H(y))) \setminus \{y_1\}) \cap \Gamma_{\rho|2} \dots$ Choose $y_n \in (H^{-1}(V_{[\sigma|n]}(H(y))) \setminus \{y_1, \dots, y_{n-1}\}) \cap \Gamma_{\rho|n} \dots$ Then put $A = \{y_1, y_2, \dots\}$. Note that $\{H(y_1), \dots, H(y_n)\} \in \mathcal{A}_0$ for every $n \in \mathbb{N}$. Hence, by the definition of $\mathcal{A}$, we get $H(A) \in \mathcal{A}$, and therefore $A \in \mathcal{A}_1$. Thus, for every $n \in \mathbb{N}$ the set $\Gamma_{\rho|n} \cap A$ contains the infinite set $\{y_n, y_{n+1}, \dots\}$, which is a contradiction. □

Theorem 3.4. *Let Σ be a 0-dimensional Polish space (for instance $\mathbb{N}^{\mathbb{N}}$) and let Σ' be a coanalytic nonanalytic subset of Σ. Then there exists a compact subset in $c_1(\Sigma')$, which is Gul'ko and not Talagrand. Actually, the compact can be found in the form $K_{\mathcal{A}_1}$ where $\mathcal{A}_1$ is an adequate family on Σ'.*

P r o o f. Let $\mathcal{A}_1$ be the adequate family constructed in Proposition 3.2 for our Σ and Σ'. This proposition together with Theorem 1.5 guarantee that $K_{\mathcal{A}_1}$ is Gul'ko compact. It is a subspace of $c_1(\Sigma')$ as every element of $\mathcal{A}_1$ is closed and discrete. Assume that $K_{\mathcal{A}_1}$ is Talagrand compact. Then, by Theorem 1.4, there is a monotone system $\{\Gamma_s : s \in \mathcal{S}\}$ of subsets of Σ' such that

$$\Sigma' = \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \Gamma_{\rho|n}$$

and satisfying the remaining assumption of Proposition 3.3. Thus

$$\Sigma' = \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \overline{\Gamma_{\rho|n}}^{\Sigma}.$$

However, this means that Σ' is an analytic set, which is in contradiction with the assumption. □

Proposition 3.5. *Let Σ, Σ', H, and $\mathcal{A}_1$ be as in Proposition 3.2, and assume there exists a system $\{\Gamma_n : n \in \mathbb{N}\}$ of subsets of Σ' such that*

$$\Sigma' = \bigcup_{n=1}^{\infty} \Gamma_n$$

and that for every $A \in \mathcal{A}_1$ and for every $n \in \mathbb{N}$ the set $A \cap \Gamma_n$ is finite. Then

$$\Sigma' = \bigcup_{n=1}^{\infty} \overline{\Gamma_n}^{\Sigma}.$$

P r o o f. We can proceed as in the proof of Proposition 3.3. However, it is simpler to use directly this proposition. Indeed, it is enough to put $\widetilde{\Gamma}_s = \Gamma_{s(1)},\ s \in \mathcal{S}$. Then

$$\Sigma' = \bigcup_{n=1}^{\infty} \Gamma_n = \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \widetilde{\Gamma}_{\rho|n}$$

and for every $A \in \mathcal{A}_1$ and for every $\rho \in \mathbb{N}^{\mathbb{N}}$ the set $A \cap \widetilde{\Gamma}_{\rho|1} = A \cap \Gamma_{\rho(1)}$ is finite. Hence by Proposition 3.3,

$$\Sigma' = \bigcup_{\rho \in \mathbb{N}^{\mathbb{N}}} \bigcap_{n=1}^{\infty} \overline{\widetilde{\Gamma}_{\rho|n}}^{\Sigma} = \bigcup_{n=1}^{\infty} \overline{\Gamma_n}^{\Sigma}. \qquad \square$$

Theorem 3.6. *Let Σ be a 0-dimensional Polish space (for instance $\mathbb{N}^{\mathbb{N}}$) and let Σ' be a Borel non F_σ subset of Σ. Then there exists a compact subset of $c_1(\Sigma')$, which is Talagrand and not Eberlein. Actually, the compact can be found in the form $K_{\mathcal{A}_1}$ where $\mathcal{A}$ is an adequate family on Σ'.*

P r o o f. We start as in the proof of Theorem 3.4. Since Σ' is Borel, and hence $\mathcal{K}$-analytic, Theorem 1.5 guarantees that $K_{\mathcal{A}_1}$ is Talagrand compact. Assume that $K_{\mathcal{A}_1}$ is Eberlein compact. Then, by Theorem 1.3, there exist subsets $\Gamma_n \subset \Sigma',\ n \in \mathbb{N}$, such that $\Sigma' = \bigcup_{n=1}^{\infty} \Gamma_n$ and for every $A \in \mathcal{A}_1$ and every $n \in \mathbb{N}$ the set $A \cap \Gamma_n$ is finite. By Proposition 3.5, we then have that

$$\Sigma' = \bigcup_{n=1}^{\infty} \overline{\Gamma_n}^{\Sigma}.$$

Hence the set Σ' is F_σ, which is in contradiction with the assumptions. □

The above theorem, in a slightly more general form, was proved, in a different way, by Mercourakis [8].

Taking into account well known facts, see e.g. [11] or [4], we get: *The Banach space $C(K_{\mathcal{A}_1})$ where $K_{\mathcal{A}_1}$ is from Theorem 3.4 is Vašák (i.e. weakly countably determined) and not weakly $\mathcal{K}$-analytic. The Banach space $C(K_{\mathcal{A}_1})$ where $K_{\mathcal{A}_1}$ is from Theorem 3.6 is weakly $\mathcal{K}$-analytic and not a subspace of a weakly compactly generated space.*

REFERENCES

[1] D. Amir, J. Lindenstrauss. The structure of weakly compact sets in Banach spaces. *Ann. Math.* **88** (1968), 35–46.

[2] P. Čížek. O některých zvobecněních Eberleinových kompaktů. Diploma Thesis, Charles University, Prague 1996.

[3] J. Diestel. Geometry of Banach spaces – Selected topics. Lecture Notes in Math. **485**, Springer-Verlag, 1975.

[4] M. Fabian. Gâteaux Differentiability of Convex Functions and Topology – Weak Asplund spaces. A Wiley-Interscience Publ., John Wiley & Sons, Inc., New York, 1997.

[5] V. Farmaki. The structure of Eberlein, uniformly Eberlein and Talagrand compact spaces in $\Sigma(\mathbb{R}^\Gamma)$. *Fundamenta Math.* **128** (1987), 15–28.

[6] A. S. Kechris. Classical Descriptive Set Theory. GTM, vol. **156**, Springer Verlag, Berlin, 1995.

[7] S. Mercourakis. On weakly countably determined Banach spaces. *Trans. Amer. Math. Soc.* **300** (1987), 307–327.

[8] S. Mercourakis. Some characterizations of analytic metric spaces. *Pacific J. Math.* **128** (1987), 149–156.

[9] H. P. Rosenthal. The heredity problem for weakly compactly generated Banach spaces. *Compositio Math.* **28** (1974), 83–111.

[10] G. A. Sokolov. On some classes of compact spaces lying in Σ-products. *Comment. Math. Univ. Carolinae* **25** (1984), 219–231.

[11] M. Talagrand. Espaces de Banach faiblement $\mathcal{K}$-analytiques. *Annals Math.* **110** (1979), 407–438.

[12] M. Talagrand. A new countably determined Banach space. *Israel J. Math.* **47** (1984), 75–80.

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