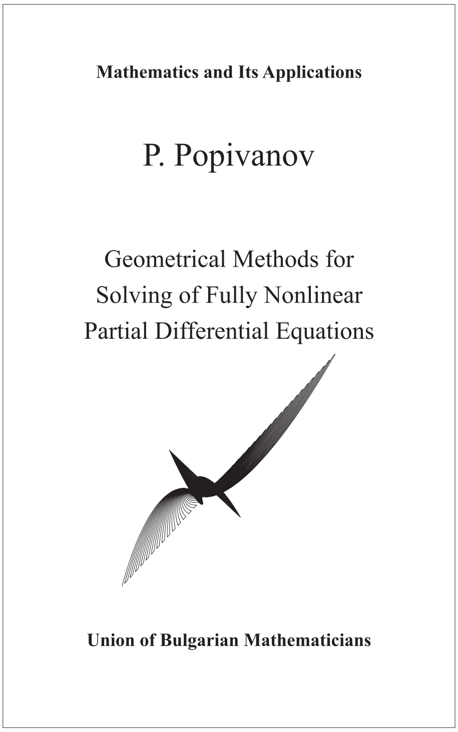
Serdica Math. J. **32** (2006), 383–386

Serdica
Mathematical Journal

Bulgarian Academy of Sciences
Institute of Mathematics and Informatics



Geometrical Methods for Solving of Fully Nonlinear Partial Differential Equations, by P. Popivanov, Mathematics and its Applications, vol. 2, Union of Bulgarian Mathematicians, Sofia, 2006, $\mathrm{X}+158$ pp., € 49, ISBN 10 954-8880-24-5, ISBN 13 978-954-8880-24-4

This is the second volume of the new series "Mathematics and its Applications" published by the Union of Bulgarian Mathematicians*

Petar Popivanov, the author of the book, is a well known expert in the theory of partial differential equations (PDE), who has published several books and over 120 research articles. Since 1969 he works at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences, Sofia,

2000 *Mathematics Subject Classification:* 35-02, 53-02, 35B65, 35C05, 35F20, 35G25, 35L60, 57R45, 58C28, 76J05.

Key words: Method of characteristics, Monge-Ampere, eikonal, Clairaut equations, propagation of singularities, anomalous and generic singularities of the integral surfaces, semilinear hyperbolic systems, Hamiltonian systems, canonical transformations.

*See Serdica Math. J. **28** (2002), 91–93 for the book review for the first issue "Multibody System Mechanics: Modelling, Stability, Control and Robustness" by V. A. Konoplev and A. Cheremenski.

Bulgaria, in 2003 he was elected for a member of the Bulgarian Academy of Sciences. For many years Prof. Popivanov has given lectures on various aspects of PDE at the Faculty of Mathematics and Informatics of the Sofia University "St. Kliment Ohridski" and at the South-West University "Neofit Rilski" in Blagoevgrad.

The goal of this book is to present different geometric methods for solvability (basically) of the Cauchy problem for fully nonlinear PDE of first and second order. This approach is very useful for geometric equations of Monge-Ampere type, for investigation of the propagation of the singularities of the solutions of nonlinear PDE and the creation of new singularities born by the collision of the initial ones.

The significant role of the nonlinear equations in physics, technics, biology, mechanics and mathematics, for example, in geometry, is well known. Since the solutions of the corresponding equations can be interpreted as surfaces, the natural question which appears is to describe their singularities. In this way the local existence results can be extended to global ones, because the singularities, as nonsmoothness and appearance of multivalued solutions, are the main difficulties for the existence of global solutions. The author connects the classical geometric approach in the spirit of Lagrange, Charpit, Pfaff, Cauchy (for first order equations), and of Goursat, Darboux, (for second order equations) with the modern investigations of the propagation of the singularities for the solutions of nonlinear hyperbolic systems and classification results for the degenerate maps in the theory of Whitney, Thom, Arnold and others (catastrophe theory).

The monograph consists of seven chapters, one appendix and an extensive list of references. To give a more detailed flavor of the monograph each chapter will be summarized now.

Chapter 1 contains preliminary results from differential geometry which are necessary for the geometric interpretation of the solutions of the Cauchy problem in the next chapters.

Chapter 2 deals with first order nonlinear PDE arising in mechanics and geometry, the Cauchy method and the method of envelopes. The results are classical and the exposition is clear. The geometric ideas are given in detail and are illustrated with many exercises, examples and figures. This is very useful for the investigation of the qualitative properties of the solutions. In this way the classical results are combined with modern ones on singularities, life span of the solutions and finding of the smooth single value branches.

Chapter 3 contains a short review on canonical transformations of Hamilton systems and the famous Jacobi and Liouville theorems for the integrability. The exposition can be used as an introduction in the theory of the canonical transformations.

Chapter 4 studies the Cauchy problem for the hyperbolic Monge-Ampere equation. The author uses the original ideas of Goursat and Darboux to reduce the problem to the solvability of the Cauchy problem for first order PDE. The proofs are elementary, clear and are based on the method of the outer differential forms. Again, the exposition is illustrated with many exercises and examples and may serve as a good introduction in the geometric theory for second order PDE.

Chapters 5, 6, 7 deal with modern results in the theory of the characteristics, propagation of singularities for semilinear hyperbolic systems. The author is one of the main contributors to the developments outlined in the book. In Chapter 5 he presents a necessary and sufficient condition for the characteristics of quasilinear hyperbolic systems in the hodograph plane to be epicycloids and gives applications to two-dimensional steady, isentropic, irrotational flows in mechanics.

Chapter 6 treats the arising of new singularities for semilinear hyperbolic equations with one space variable. Some of the characteristics transport singularities of the Cauchy data while the other ones start with smooth data. As a result of the collision of the characteristics carrying out the singularities at some points new born singularities will appear. They are weaker than the initial ones. Independently of the technicalities the proofs are clear and precise, illustrated with many pictures.

The last Chapter 7 concerns the creation of logarithmic singularities for semilinear hyperbolic systems with two space variables. This new phenomena does not appear in the one-dimensional case (Chapter 6).

The Appendix is a good survey for the readers interested in the catastrophes theory of R. Thom or the so called "generic" singularities of ruled and developable surfaces based on some recent results of S. Izumia.

The book combines in a nice way classical and modern methods for the study of some famous PDE as Monge-Ampere, Clairaut, eikonal equations and many others. The geometric approach allows the author to illustrate with many figures, including computer visualizations, the shape of the solutions, the singularities and the finding of smooth single value branches. This is a fascinating interplay between geometry and analysis. The exposition is clear, precise and the text is carefully written. The author has succeeded in distinguishing the crucial and fundamental aspects and ideas from the technicalities. The book should be of interest to (graduate) students in mathematics, physics, engineering. Even experts in analysis, PDE and differential geometry can learn a lot from the book and it would be useful to have it on the shelf.

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