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# OLIGOPOLY MODEL OF A DEBIT CARD NETWORK 

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#### Abstract

The paper builds an oligopoly model of a debit card network. It examines the competition between debit card issuers. We show that there is an optimal pricing for the debit card network, which maximizes all issuer's revenues. The paper also shows that establishing a link between debit card networks averages the costs provided that there is no growth in the customer's usage of the networks, resulting from the link.


1. Introduction. Debit card systems were developed to provide an electronic means for consumers to access their deposit accounts with banks to pay for purchases at point-of-sale. These point-of-sale (POS) debit card systems have proved to be popular with consumers and merchants.
[^0]Debit card systems should be generally accepted by merchants and customers in order to be successful. In fragmented banking market, it would be difficult for any bank to achieve widespread access. In such markets, multi-bank organization was developed to link the accounts of all bank depositors to a wide range of merchants, with whom the banks had business relationships. In more concentrated banking markets, single banks may sponsor a debit card system that offers widespread access to its participants.

The systems linking different banks are usually called networks. The physical arrangement of the network usually involves telecommunication lines across which electronic transaction information is transmitted, linked by computers that route the information between merchants and the card issuing bank. These computers are often called switches, as they route, or switch, the information to participants.

Banks can participate in a debit card system either by (co-)owning the switch or linking to it. If more than one bank owns a switch, the owners usually participate as issuers of debit cards.

Non-banks can also participate in the debit card systems. In case a nonbank owns the switch, bank membership in the debit card network is essential for the provision of cards that draw on deposit accounts.

Membership arrangements play an important role in specifying the legal rights and obligations of participants. Following [2], several antitrust publications have discussed the potential anticompetitive effects of the collective determination of interchange fees within payment card associations (See also [4], [10], [11], [7] and [1]). Formal models of the payment card industry have been developed recently, allowing for a more rigorous analysis of the impact of interchange fees on prices and volumes of activity in payment card networks. These models have also highlighted the existence of common patterns between this industry and other network industries (See for example [16]). The objective of this paper is to provide new oligopoly model of a debit card network, which aims at determining the optimal fees for the network. We use different arrangements for the debit card network, giving us a good platform to examine the price competition between issuers.

If there exist more than one debit card network, then the merchants in one network will not be accessible for the debit card holders belonging to another network(s) unless there are some links between these networks. This can be achieved in several ways. One solution is the switches to link together on either bilateral or multilateral basis. This is the most common form for achieving interconnectivity in systems with more than one network. The other solution is
merchants (directly or through an acquirer) and/or card issuers to participate in more than one network. In this paper we examine the first model by allowing two debit card networks to link. Furthermore, the paper shows that establishing a link between debit card networks averages the costs given that there is no growth in the customer's usage of the networks, resulting from the link.

## 2. Assumptions of the basic model

2.1. Agents. There is a very large number of debit cards, issued to customers. The debit cards are numbered $0,1,2, \ldots, n$. The debit cards are issued to the holders for free. The issuer pays all fees related to the issuance.

Debit card holders live in one country.
There is a debit card network with a profit-maximizing switch. The switch operates as a platform industry [16], because it needs two types of customers that wish to interact: merchant(s) and debit card holders.

The switch registers all transactions with debit cards in the country. All debit card transactions are settled at the switch. Running the settlement process generates costs. The unit cost of the switch per transaction is $z$.

In the network, there are two debit card issuers denoted by 1 and 2 . Every issuer has issued at least one debit card. Issuer $i$ charges customer fee $p_{i}$ per transaction, regardless of the transaction value.

There is one merchant in the network. The merchant cannot become member of foreign debit card networks. The merchant achieves a turnover via pricing of his service to debit card holders. The merchant discount is $m$.

Our analysis makes two simplifying assumptions, which fit well into the debit card industry. First, we assume that the issuers compete with each other, following the Bertrand model of price competition [3]. The second simplifying assumption is that customers have a fixed volume of transactions, normalized to one transaction, i.e. there is inelastic demand for the merchant's goods and/or services.
2.2. Order of moves. The model is a full information game, in which agents can always correctly calculate each others decisions and moves. Moves are made in the following order.

1. The debit card network switch sets its switch fee $z$, where $z>0$.
2. Debit card issuers determine their customer fees $p_{i}$. Every issuer tries to maximize his own wealth by minimizing the expected value of the sum of
fees paid out (switch fee $z$ ), and maximize the value of the fees paid in (the corresponding customer fee $p_{i}$ ). The issuers make revenues, if $p_{i}>z$.
3. The merchant discount fee $m$ is determined by the debit card network participants. The switch makes revenue if $m>z$.
4. Each customer chooses an issuer, opens a deposit account with it, and receives a debit card for accessing his deposit account. These choices are made independently and simultaneously without cooperation.
5. Customers make their payments. See Figure 2.1


Fig. 2.1
2.3. Customer's preference. The total utility of a customer is determined by the fees charged for using the debit card and the customer's preference between the two issuers.

Each consumer has to make a discrete choice between the two banks. Three factors are taken into account: First, the distance to the bank, secondly, a general preference parameter $(G)$, and finally the number of other customers who are going to use the same bank. Customers can correctly calculate others choices and the resulting market shares.

The expected utility of consumer $x$ is

$$
W_{x}= \begin{cases}G+\left(2-i_{x}\right)+s\left(p_{2}-p_{1}\right), & \text { if he chooses issuer } 1 ;  \tag{2.1}\\ -G+\left(i_{x}-1\right)+(1-s)\left(p_{1}-p_{2}\right), & \text { if he chooses issuer } 2 .\end{cases}
$$

where

- $G$ is an exogenous preference parameter. The parameter is exogenous and common to all customers. If $G>0$, issuer 1 is preferred by most customers.

If $G<0$, most customers prefer issuer 2. If $G=0$, customers are, on average, indifferent between the two issuers. This parameter does not reflect any scale or network effects, and its value is not affected by other customers' choices.

- $i_{x}$ is a customer-specific exogenous parameter, denoting the location of the customer. The issuers are located at the endpoints of the interval, issuer 1 at point 1 and issuer 2 at point 2 . Getting service from an issuer that is close to the customer provides the customer with higher utility. If $1 \leq i_{x}<1 \frac{1}{2}$, parameter $i$ favors issuer 1 , if $1 \frac{1}{2}<i_{x} \leq 2$, parameter $i$ favors issuer 2 , and if $i_{x}=1 \frac{1}{2}$, the parameter is neutral as between the two issuers. Because the common parameter $G$ may differ from zero, $i_{x}<1 \frac{1}{2}$ does not necessarily imply that consumer $x$ would prefer issuer 1 .
- $s$ is the endogenously determined market share of issuer 1.
- $p_{2}-p_{1}$ is the difference between the customer fees, charged by the two issuers. If $p_{2}-p_{1}>0$, the customer would rather prefer issuer 1 , which has lower customer fee. This can be interpreted as the customer prefers the more cost-efficient debit card. Also, we should take into account the market share of the debit card issuer as a measure of how willing is the customer to use this card. Therefore, the expected utility of making a payment with a debit card from issuer 1 is $s\left(p_{2}-p_{1}\right)$. Analogically, the expected utility of making a payment with debit card from issuer 2 is $(1-s)\left(p_{1}-p_{2}\right)$.

With the exception of the preference parameter $i_{x}$, all parameters are common to all customers.

We suppose that the issuers and the switch are making revenue, so we require

$$
p_{i}>z>0 \quad \text { and } \quad m>z>0 .
$$

As we will see later, parameters $G, p_{1}$ and $p_{2}$ must satisfy inequality (3.4)

$$
-1<2 G+p_{2}-p_{1}<1 .
$$

There is a pure network externality in the model. Using the same issuer as the majority of customers provides the consumer with utility. This network effect is a direct externality and is not caused by the effects of other customers choices on any prices.

Issuers cannot practice price discrimination, because they cannot observe the exact location of different consumers on the interval $[1,2]$.
2.4. Issuers' revenues. The revenue of issuer 1 , when the cost of debit card network operation is not taken into account, is

$$
\Pi_{1}=n s\left(p_{1}-z\right)
$$

and the revenue of issuer 2 is

$$
\Pi_{2}=n(1-s)\left(p_{2}-z\right) .
$$

## 3. Solving the basic model

3.1. Competition between issuers. The utility of customer $x$ is determined according to function (2.1).

The customer $x$ chooses issuer 1 , if

$$
\begin{align*}
& G+\left(2-i_{x}\right)+s\left(p_{2}-p_{1}\right)>-G+\left(i_{x}-1\right)+(1-s)\left(p_{1}-p_{2}\right) \Longleftrightarrow \\
& i_{x}<\left(3+2 G+p_{2}-p_{1}\right) / 2 \tag{3.1}
\end{align*}
$$

The market share of issuer 1 is determined by the number of customers for whom the condition is valid.

The density function is uniform on $[0, n]$. Hence, because $n$ is a very large integer, the market share is almost exactly

$$
s=\frac{1}{n} \int_{1}^{c} n d i,
$$

where $c$ is the point where the condition for $i_{x}$ (3.1) is no longer valid, i.e.

$$
c=\left(3+2 G+p_{2}-p_{1}\right) / 2
$$

The market share is given by

$$
\begin{equation*}
s=\frac{1+2 G+p_{2}-p_{1}}{2} . \tag{3.2}
\end{equation*}
$$

Similarly, issuer 2 market share is

$$
\begin{equation*}
(1-s)=\frac{1-2 G+p_{1}-p_{2}}{2} \tag{3.3}
\end{equation*}
$$

Since the market shares should be between $0 \%$ and $100 \%$, we derive the following inequality

$$
\begin{equation*}
-1<2 G+p_{2}-p_{1}<1 \tag{3.4}
\end{equation*}
$$

In the following, it is assumed that each issuer has a positive market share, even though a market share may be close to zero.

The situation is symmetric, the only difference between the two issuers being the eventually nonzero value of $2 G+p_{2}-p_{1}$. Therefore, all the following results are equally valid for both issuers. To simplify the notation, the analysis is in most cases presented only for issuer 1.

Formula (3.2) implies effects that are quite intuitive: if the issuer charges higher fees than its competitor, its market share declines.

We have that $d s / d p_{1}=-1 / 2$. Whenever formula (3.2) predicts positive values of issuer 2's market share, $d s / d p_{1}<0$, which is reasonable.

If issuer 1 is not popular $(G<0)$, his market share remains small. On the other hand, high prices charged by its rival increase his market share.

### 3.2. Issuers' pricing decisions.

Proposition 3.1. There exist optimal customers' fees maximizing both issuers' revenues.

Proof. An issuer can change its customer fee, so we consider the revenue function of both issuers as a function of the customer fee. The optimization conditions are

$$
\begin{aligned}
& \frac{d \Pi_{1}}{d p_{1}}=\frac{n}{2}\left(1+z+2 G+p_{2}-2 p_{1}\right)=n s-\frac{n\left(p_{1}-z\right)}{2}=0 \Leftrightarrow p_{1}=z+2 s \\
& \frac{d \Pi_{2}}{d p_{2}}=\frac{n}{2}\left(1+z-2 G+p_{1}-2 p_{2}\right)=n(1-2)-\frac{n\left(p_{2}-z\right)}{2}=0 \Leftrightarrow p_{2}=z+2(1-s)
\end{aligned}
$$

The first derivatives of $\Pi_{1}$ and $\Pi_{2}$ should be zero for one and the same market share $s$. We obtain an unique combination of customer prices

$$
\begin{align*}
p_{1}^{0} & =1+z+2 G / 3 \\
p_{2}^{0} & =1+z-2 G / 3  \tag{3.5}\\
s^{0} & =1 / 2+G / 3
\end{align*}
$$

$\Pi_{1}$ has a local extremum for $\left(p_{1}^{0}, p_{2}^{0}\right)$. The extremum is a local maximum, because the second derivative is negative.

$$
\frac{d^{2} \Pi_{1}}{d p_{1}{ }^{2}}=\frac{d}{d p_{1}}\left(\frac{n\left(1+z+2 G+2 p_{2}-p_{1}\right)}{2}\right)=-\frac{n}{2}<0
$$

Similarly, $\Pi_{2}$ has a local maximum for $\left(p_{1}^{0}, p_{2}^{0}\right)$, because

$$
\frac{d^{2} \Pi_{2}}{d p_{2}{ }^{2}}=\frac{d}{d p_{2}}\left(\frac{n\left(1+z-2 G+2 p_{1}-p_{2}\right)}{2}\right)=-\frac{n}{2}<0
$$

We proved that there exists an unique pair of customer prices $\left(p_{1}^{0}, p_{2}^{0}\right)$ for which both issuers' revenues are maximized.
3.3. Switch revenues. The switch revenue is determined by $\Phi=$ $n(z-m)$. Increase of the switch fee $z$ should cause the issuers to proportionately raise their customer fees $p_{i}$ in order to optimize them, and vice versa, lowering the switch fee $z$ should cause the issuers to proportionately lower the customers' fees $p_{i}$.

Note that any increase in the switch fees is actually paid by the customers, when the issuer's pricing is at the optimal level.
4. Bilateral model. Suppose we have two debit card networks, conforming to the described basic model. Also suppose that the issuers' pricing is at the optimal level at both networks, and the merchants in both networks are selling the same good for the same price. See Figure 4.1

We want the merchants to be accessible by all customers, which is achieved by providing a bilateral link between them. The cost of using the link is $l$, where $l>0$ and it is paid by the customer network's switch, when the customer buys from the merchant of the other network. Establishing the bilateral link should not affect the market shares of the issuers in the individual networks, and they will stay at the optimal level.


Fig. 4.1

The customer can buy the goods from either merchant for the same price, which makes both merchants indistinguishable from the customer's standpoint. This affects the merchant discount fee of the domestic network. Since the customer can buy from both merchants, then the new effective merchant fee for both networks can be written as follows:

$$
\begin{equation*}
m_{\mathrm{new}}=\frac{m^{\prime}+m^{\prime \prime}+l}{2} . \tag{4.1}
\end{equation*}
$$

The merchant fee for the given network decreases/increases if $m_{\text {domestic }}-$ $m_{\text {foreign }}$ is greater/lower than $l$, and stays the same if $m_{\text {domestic }}-m_{\text {foreign }}=l$. Note that since $l>0$, in any case the merchant fee in one network increases, while the merchant fee in the other network decreases.

The change in the effective merchant fee results in a change in the switch revenue. Since the switch is profit-maximizing one, the switch fee should increase/ decrease in the case when the merchant fee increases/decreases, so that the switch makes a profit at least equal to the profit before establishing the link. If the customers buy as many goods as before, then the switch fee should change with the difference between the new and old merchant fees.

The above can be summarized as follows:
Proposition 4.1. Bilaterally interlinking two debit card networks results in new merchant fee

$$
m_{\mathrm{new}}=\frac{m^{\prime}+m^{\prime \prime}+l}{2}
$$

thus, at least in one of the networks, the merchant fee will increase as a result of establishing the link.

Corollary 4.2. The switch fee of the bilaterally linked debit card networks increases/decreases in the case where the merchant fee increases/decreases. If the customers buy as many goods as before, then the switch fee will change with the difference between the new and old merchant fees.
5. Conclusions. This paper presents an oligopoly model of the debit card industry. The main focus is the competition between two debit card issuers, participating in a monopolistic national debit card network, and the impact of establishing a link between two such debit card networks.

The paper shows that there exist optimal customer fees, maximizing both issuer's revenues. The paper also shows that establishing a link between debit card networks averages the costs provided that there is no growth in the customer's usage of the networks, resulting from the link.

The model used in this paper approximates a specific class of debit card networks. Real life networks are much more complex in nature, yet the main results of the paper still give us some good ideas what actually could happen in practice.

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