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HERMITIAN MANIFOLDS OF POINTWISE CONSTANT ANTIHOLOMORPHIC SECTIONAL CURVATURES

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ABSTRACT. In dimension greater than four, we prove that if a Hermitian non-Kaehler manifold is of pointwise constant antiholomorphic sectional curvatures, then it is of constant sectional curvatures.

1. Introduction. Let (M, g, J) ($\dim M = 2n \geq 4$) be an almost Hermitian manifold. Any two-plane (section) E in the tangential space $T_p M$, $p \in M$ determines an angle $\theta = \sphericalangle (E, JE)$, $\theta \in [0, \frac{\pi}{2}]$. Two types of planes with respect to the angle θ are remarkable: *holomorphic* sections – characterized by the condition $\theta = 0$ or $E = JE$; *antiholomorphic* sections – characterized by the condition $\theta = \frac{\pi}{2}$ or $E \perp JE$. The latter are also known as *totally real* in view of the condition $E \perp JE$.

If Φ is the fundamental Kähler form of the manifold, then any antiholomorphic section E is characterized by the condition $\Phi_{|E} = 0$. Because of this characterization, these tangent planes are also known as *Lagrangian*.

An almost Hermitian manifold is said to be of pointwise constant antiholomorphic sectional curvature ν if the Riemannian sectional curvature $K(E;p)$ does not depend on the antiholomorphic section E in T_pM, $p \in M$, i.e. $K(E;p) = \nu(p)$ is only a function of the point $p \in M$.

A tensor characterization for an almost Hermitian manifold of pointwise constant antiholomorphic sectional curvature in dim $M \geq 4$ has been found in [2].

In [6] it has been proved that the antiholomorphic sectional curvature $\nu(p)$ is a constant on the manifold under the condition dim $M > 4$.

A complete classification of compact Hermitian surfaces (dim $M = 4$) with pointwise constant antiholomorphic sectional curvature has been given in [1]. Four-dimensional almost Hermitian manifolds of pointwise constant antiholomorphic sectional curvature have been studied in [8].

In this paper we consider the class of Hermitian manifolds and prove our main

Theorem A. *If a Hermitian non-Kähler manifold with a real dimension greater than four is of pointwise constant antiholomorphic sectional curvature, then the manifold is of constant sectional curvature.*

2. Preliminaries. Let (M, g, J) (dim $M = 2n \geq 4$) be an almost Hermitian manifold with metric g and almost complex structure J. The tangent space to M at an arbitrary point $p \in M$ is denoted by T_pM and the algebra of all differentiable vector fields on M is denoted by $\mathfrak{X}M$. The Kähler form Φ of the structure (g, J) is defined by the equality

$$\Phi(X, Y) = g(JX, Y), \quad X, Y \in T_pM, p \in M.$$

The Levi-Civita connection of the metric g is denoted by ∇ and the Riemannian curvature tensor R of type (1,3) is given by $R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]}Z$, $X, Y, Z \in \mathfrak{X}M$. The corresponding curvature tensor of type (0,4) is given by $R(X, Y, Z, U) = g(R(X, Y)Z, U)$, for all vector fields X, Y, Z, U.

Let $\{e_1, \ldots, e_{2n}\}$ be an orthonormal basis at a point $p \in M$. The Ricci tensor ρ and the scalar curvature τ of the metric g are determined as follows

$$\rho(X, Y) = \sum_{i=1}^{2n} R(e_i, X, Y, e_i), \quad \tau = \sum_{i=1}^{2n} \rho(e_i, e_i); \quad X, Y \in T_pM.$$

The almost complex structure (g, J) gives rise to the $*$-Ricci tensor ρ^* and to the $*$-scalar curvature τ^* defined by the formulas

$$\rho^*(X, Y) = \sum_{i=1}^{2n} R(e_i, X, JY, Je_i), \quad \tau^* = \sum_{i=1}^{2n} \rho^*(e_i, e_i); \quad X, Y \in T_pM.$$

While the Ricci tensor is symmetric, the $*$-Ricci tensor has the property

$$(2.1) \qquad \rho^*(JX, JY) = \rho(Y, X), \quad X, Y \in T_pM.$$

The following tensor of type (0,3)

$$F(X, Y, Z) = g((\nabla_X J)Y, Z), \quad X, Y, Z \in \mathfrak{X}M$$

is closely related to the structure (g, J). This tensor satisfies the following properties

$$(2.2) \qquad F(X, Y, Z) = -F(X, Z, Y), \quad F(X, JY, JZ) = -F(X, Y, Z).$$

The only trace of this tensor determines the Lee form θ of the manifold in the following way

$$\theta(X) = -\sum_{i=1}^{2n} F(e_i, JX, e_i), \quad X \in \mathfrak{X}M.$$

The well known classes of almost Hermitian manifolds have been obtained in terms of the properties of the tensor F in [5].

In this section we consider Hermitian manifolds, which are characterized by the following property of the tensor F [5]:

$$(2.3) \qquad (\nabla_{JX} J)Y = J(\nabla_X J)Y \iff F(JX, Y, Z) = -F(X, JY, Z).$$

Let $T_p^{\mathbb{C}}M$ be the complexification of the tangent space T_pM at any point $p \in M$. By $\mathfrak{X}^{\mathbb{C}}M$ we denote the algebra of complex differentiable vector fields on M. The complex structure J generates the standard splittings

$$T_p^{\mathbb{C}}M = T_p^{1,0}M \oplus T_p^{0,1}M, \quad \mathfrak{X}^{\mathbb{C}}M = \mathfrak{X}^{1,0}M \oplus \mathfrak{X}^{0,1}M.$$

If $\{e_1, \ldots, e_n; Je_1, \ldots, Je_n\}$ is an orthonormal frame at a point $p \in M$, then the vectors $Z_\alpha = \dfrac{e_\alpha - iJe_\alpha}{2}$ and $Z_{\bar{\alpha}} = \bar{Z}_\alpha = \dfrac{e_\alpha + iJe_\alpha}{2}$; $\alpha = 1, \ldots, n$ form a basis for $T_p^{1,0}M$ and $T_p^{0,1}M$, respectively. Further, we call these bases $\{Z_\alpha; Z_{\bar{\alpha}}\}$ $\alpha = 1, \ldots, n$ *orthogonal complex bases*.

For an arbitrary tensor T we denote $T_{\alpha\ldots} = T(Z_\alpha \ldots)$ and $T_{\bar{\alpha}\ldots} = T(Z_{\bar{\alpha}} \ldots)$.

In what follows, the summation convention is assumed and Greek indices $\alpha, \beta, \gamma, \ldots$ run from 1 to n.

It follows that the components of the metric tensor with respect to an orthogonal complex basis satisfy the conditions

$$g_{\alpha\beta} = 0 \,(\text{for all } \alpha, \beta); \quad g_{\alpha\bar{\beta}} = 0, \,(\text{for } \alpha \neq \beta); \quad g_{\alpha\bar{\alpha}} = \frac{1}{2} \,(\text{for all } \alpha).$$

We have the following

Lemma 2.1. *Let (M, g, J) be a Hermitian manifold. If $(\nabla_{\bar{Z}}J)Z = 0$ for an arbitrary $Z \in T_p^{1,0}M$, then $\nabla J = 0$ at the point p.*

P r o o f. Indeed, the condition $(\nabla_{\bar{Z}}J)Z = 0$ implies that $(\nabla_X J)X = 0$ for all $X \in T_pM$. Therefore M satisfies the condition characterizing a nearly Kähler manifold at p. Since M is Hermitian, then M is Kählerian, i.e. $\nabla J = 0$ at p [5]. $\square$

Now, let (M, g, J) be a Hermitian manifold with pointwise constant antiholomorphic sectional curvature. This means that for any orthonormal antiholomorphic frame $\{X, Y\}$, $(g(X, X) = g(Y, Y) = 1, g(X, Y) = g(X, JY) = 0)$ at an arbitrary point $p \in M$ the sectional curvature $R(X, Y, Y, X)$ does not depend on the antiholomorphic section span$\{X, Y\}$, i.e. $R(X, Y, Y, X)$ is only a function of the point p. We denote this function by $\nu(p)$.

Let $Q(X, Y)$ be a tensor on M having the symmetry (2.1), i.e.

$$(2.4) \qquad\qquad Q(JX, JY) = Q(Y, X).$$

The following tensor construction $\Psi(Q)$ is relevant to the considerations in this paper:

$$\begin{aligned}
\Psi(Q)(X, Y, Z, U) = {} & g(Y, JZ)Q(X, JU) - g(X, JZ)Q(Y, JU) \\
& - 2g(X, JY)Q(Z, JU) + g(X, JU)Q(Y, JZ) \\
& - g(Y, JU)Q(X, JZ) - 2g(Z, JU)Q(X, JY).
\end{aligned}$$

We also recall the basic invariant tensors π_1 and π_2 only formed by the fundamental tensors g and Φ:

$$\pi_1(X, Y, Z, U) = g(Y, Z)g(X, U) - g(X, Z)g(Y, U),$$

$$\pi_2(X, Y, Z, U) = g(Y, JZ)\, g(X, JU) - g(X, JZ)\, g(Y, JU) - 2\, g(X, JY)\, g(Z, JU).$$

The first author has proved the following tensor characterization for an almost Hermitian manifold of pointwise constant antiholomorphic sectional curvatures.

Theorem [2]. *An almost Hermitian manifold with $\dim M = 2n \geq 4$ is of pointwise constant antiholomorphic sectional curvature $\nu(p)$ if and only if its curvature tensor satisfies the identity*

$$(2.5) \qquad R - \frac{1}{2(n+1)}\, \Psi(\rho^*) + \frac{\tau^*}{2(n+1)(2n+1)}\, \pi_2 = \nu \left(\pi_1 - \frac{1}{2n+1}\, \pi_2 \right).$$

We introduce the tensor

$$Q = \frac{1}{2(n+1)}\, \rho^* - \frac{\tau^* + 2(n+1)\nu}{4(n+1)(2n+1)}\, g,$$

which in view of (2.1) has the property (2.4). Then the condition (2.5) can be written as follows:

(2.6) $$R = \Psi(Q) + \nu\,\pi_1.$$

The second author has proved in [6] that in $\dim M \geq 6$ the function $\nu(p)$ in (2.5) is constant. Thus, we shall speak about almost Hermitian manifolds of constant antiholomorphic sectional curvature instead of "pointwise constant" antiholomorphic sectional curvature.

3. Proof of Theorem A. In this section we prove Theorem A on the base of the following statement.

Proposition 3.1. *Let (M, g, J) ($\dim_{\mathbb{C}} M \geq 3$) be a Hermitian manifold of constant antiholomorphic sectional curvature. Then any non-Kähler point of M has a neighborhood in which (M, g, J) is of constant sectional curvature.*

P r o o f. Let p_0 be a point in M with $F \neq 0$ at p_0. We consider a neighborhood U of p_0, such that $F \neq 0$ at any point of U. We shall prove that (M, g, J) is of constant sectional curvature in U.

For any $p \in U$, we consider an orthogonal complex basis $\{Z_\alpha, Z_{\bar\alpha}\}$ $\alpha = 1, \ldots, n$ at the point p.

The property (2.4) of the tensor Q implies that

(3.1) $$Q_{\alpha\bar\beta} = Q_{\bar\beta\alpha}, \qquad Q_{\alpha\beta} = -Q_{\beta\alpha}.$$

Taking into account the property (2.3) of the covariant derivative of the complex structure and the symmetry (2.4) of the tensor Q, we compute

(3.2) $$(\nabla_X Q)(JY, JZ) = (\nabla_X Q)(Z, Y) - Q((\nabla_X J)Y, JZ) - Q(JY, (\nabla_X J)Z)$$

for arbitrary $X, Y, Z \in \mathfrak{X}U$.

Since the tensor F has the symmetries (2.2) and (2.3), then its essential components (those which may not be zero) with respect to an orthogonal complex basis $\{Z_\alpha, \; Z_{\bar\alpha}\}$ are only $F_{\bar\alpha\beta\gamma}$ and their conjugates. These components satisfy the condition $F_{\bar\alpha\beta\gamma} = -F_{\bar\alpha\gamma\beta}$. These properties of the tensor F can be expressed in terms of the covariant derivative $(\nabla_X J)Y$ as follows

(3.3) $$\nabla_\alpha J_\beta^\gamma = \nabla_{\bar\alpha} J_\beta^\gamma = \nabla_\alpha J_\beta^{\bar\gamma} = 0.$$

The equalities (3.2) and (3.3) imply that

$$(3.4) \qquad \nabla_\alpha Q_{\bar\gamma\beta} = \nabla_\alpha Q_{\beta\bar\gamma} + i\nabla_\alpha J_{\bar\gamma}^\sigma Q_{\beta\sigma};$$

$$(3.5) \qquad \nabla_\alpha Q_{\beta\gamma} = -\nabla_\alpha Q_{\gamma\beta}, \quad (\text{especially} \quad \nabla_\alpha Q_{\beta\beta} = 0);$$

$$(3.6) \qquad \nabla_{\bar\alpha} Q_{\beta\beta} = i\nabla_{\bar\alpha} J_{\bar\beta}^{\bar\sigma} Q_{\bar\sigma\beta}.$$

First we prove the following statement

Lemma 3.2. *Let* $Z, W \in T_p^{1,0}M$ *and* $g(Z, \bar W) = 0$. *If* $F(\bar Z, Z, W) \neq 0$, *then* $Q(Z, W) = 0$.

P r o o f. Since $g(Z, \bar W) = 0$, then we can find an orthogonal complex basis $\{Z_\alpha, Z_{\bar\alpha}\}$ $\alpha = 1, \ldots, n$ such that the vectors Z and W are collinear with Z_α and Z_β, respectively, for some $\alpha \neq \beta$.

Applying the Bianchi identity for the curvature tensor R in the form

$$\nabla_\alpha R_{\beta\gamma\beta\bar\gamma} + \nabla_\beta R_{\gamma\alpha\beta\bar\gamma} + \nabla_\gamma R_{\alpha\beta\beta\bar\gamma} = 0,$$

we find

$$(3.7) \qquad \nabla_\beta Q_{\alpha\beta} = 0.$$

Further we apply the Bianchi identity in the form

$$\nabla_{\bar\alpha} R_{\alpha\beta\alpha\beta} + \nabla_\alpha R_{\beta\bar\alpha\alpha\beta} + \nabla_\beta R_{\bar\alpha\alpha\alpha\beta} = 0$$

and taking into account (3.7), we obtain

$$(3.8) \qquad F_{\bar\alpha\alpha\beta} Q_{\alpha\beta} = 0.$$

Under the conditions of the lemma we have $F_{\bar\alpha\alpha\beta} \neq 0$. Then it follows from (3.8) that $Q_{\alpha\beta} = 0$. $\square$

Next we prove

Lemma 3.3. *The tensor* Q *is symmetric at any point* $p \in U$.

P r o o f. Since the tensor $F \neq 0$ at the point p, then because of the Lemma 2.1 there exist indices $\alpha \neq \beta$ so that $F_{\bar\alpha\alpha\beta} \neq 0$. Applying Lemma 3.2, it follows that $Q_{\alpha\beta} = 0$.

Let $\gamma \neq \alpha, \beta$. Since $F_{\bar\alpha\alpha\beta} \neq 0$, then the complex function $w(t) = F(Z_{\bar\alpha}, Z_\alpha, Z_\beta + tZ_\gamma) \neq 0$ for all sufficiently small $t \in \mathbb{R}$. It follows from Lemma 3.2 that $Q(Z_\alpha, Z_\beta + tZ_\gamma) = 0$. Hence, $Q(Z_\alpha, Z_\gamma) = 0$, i.e. $Q_{\alpha\gamma} = 0$.

Similarly, the inequality $F(Z_\alpha + tZ_\gamma, Z_\alpha + tZ_\gamma, Z_\beta) \neq 0$ for all sufficiently small real t and Lemma 3.2 imply that $Q(Z_\alpha + tZ_\gamma, Z_\beta) = 0$. Hence, $Q_{\gamma\beta} = 0$. So far, we obtained

$$Q_{\alpha\beta} = Q_{\alpha\gamma} = Q_{\beta\gamma} = 0.$$

In dim $M > 6$, let $\delta \neq \alpha, \beta, \gamma$. As in the above, we find

$$Q_{\alpha\delta} = Q_{\alpha\delta} = Q_{\beta\delta} = 0.$$

On the other hand, the inequality $F(Z_{\bar\alpha} + tZ_{\bar\gamma}, Z_\alpha + tZ_\gamma, Z_\beta + tZ_\delta \neq 0)$, which is valid for sufficiently small real t, implies that $Q_{\gamma\delta} = 0$.

Thus we obtained $Q_{\lambda\mu} = 0$ for all $\lambda, \mu = 1, \ldots, n$, which proves the assertion. $\square$

Finally, we shall prove that the tensor Q is proportional to the metric tensor g in U.

For that purpose it is sufficient to prove that

$$(3.9) \qquad\qquad Q_{\lambda\bar\mu} = 0,$$

for all different indices λ and μ.

We consider two cases for the tensor $F \neq 0$:

1) There exist three different indices α, β, γ, such that $F_{\bar\gamma\alpha\beta} \neq 0$;

2) $F_{\bar\gamma\alpha\beta} = 0$ for all different indices α, β, γ with respect to any orthogonal complex basis.

The case 1). Applying the second Bianchi identity in the form

$$\nabla_\alpha R_{\beta\bar\gamma\beta\bar\gamma} + \nabla_\beta R_{\bar\gamma\alpha\beta\bar\gamma} + \nabla_{\bar\gamma} R_{\alpha\beta\beta\bar\gamma} = 0,$$

we get the equality $F_{\bar\gamma\alpha\beta}Q_{\beta\bar\gamma} = 0$, which implies that $Q_{\beta\bar\gamma} = 0$.

Now, arguments similar to those in Lemma 3.3 show (3.9).

The case 2). According to Lemma 2.1 there exist two different indices α and β such that $F_{\bar\alpha\alpha\beta} \neq 0$. Applying the second Bianchi identity in the form

$$\nabla_\alpha R_{\bar\gamma\beta\beta\bar\alpha} + \nabla_{\bar\gamma} R_{\beta\alpha\beta\bar\alpha} + \nabla_\beta R_{\alpha\bar\gamma\beta\bar\alpha} = 0,$$

and taking into account the equalities $Q_{\alpha\beta} = 0$, $F_{\bar\gamma\alpha\beta} = 0$, we find

$$-\nabla_{\bar\gamma} Q_{\beta\beta} + iQ(Z_\beta, (\nabla_{\bar\gamma} J)Z_\beta) + \nabla_\beta Q_{\bar\gamma\beta} = 0.$$

The last equality in view of (3.6) implies

$$(3.10) \qquad\qquad \nabla_\beta Q_{\bar\gamma\beta} = 0.$$

Applying the second Bianchi identity in the form

$$\nabla_{\bar{\alpha}} R_{\alpha\beta\beta\bar{\gamma}} + \nabla_{\alpha} R_{\beta\bar{\alpha}\beta\bar{\gamma}} + \nabla_{\beta} R_{\bar{\alpha}\alpha\beta\bar{\gamma}} = 0,$$

we find

$$3i F_{\bar{\alpha}\alpha\beta} Q_{\beta\bar{\gamma}} + 2\nabla_{\beta} Q_{\beta\bar{\gamma}} + 2i Q(Z_{\beta}, (\nabla_{\beta} J) Z_{\bar{\gamma}}) = 0,$$

which together with (3.4) gives

$$3i F_{\bar{\alpha}\alpha\beta} Q_{\beta\bar{\gamma}} + 2\nabla_{\beta} Q_{\bar{\gamma}\beta} = 0.$$

The last equality and (3.10) imply that

$$F_{\bar{\alpha}\alpha\beta} Q_{\beta\bar{\gamma}} = 0.$$

Hence, $Q_{\beta\bar{\gamma}} = 0$. Applying again the scheme of the proof of Lemma 3.3, we obtain the conditions (3.9).

Thus, in both cases 1) and 2), we obtained the conditions (3.9), which are equivalent to the identity

(3.11) $Q(X, Y) = 0,$ whenever $X, Y \in T_p M,$ $g(X, Y) = 0.$

Applying standard arguments for the symmetric tensor $Q(X, Y)$, we obtain that the tensor Q is proportional to the metric tensor g, i.e.

$$Q = \frac{\operatorname{tr} Q}{2n} g, \quad \operatorname{tr} Q = \frac{\tau^* - 2n\nu}{2(2n+1)}.$$

Hence

$$R = \nu \pi_1 + \frac{\operatorname{tr} Q}{n} \pi_2.$$

Further we use the following statement

Theorem [9]**.** *Let M be a connected almost Hermitian manifold with real dimension $2n \geq 6$ and Riemannian curvature tensor of the following form:*

$$R = f \pi_1 + h \pi_2,$$

where f and h are C^∞ functions on M such that h is not identical zero. Then M is a complex space form (i.e. a Kähler manifold with constant holomorphic sectional curvature).

Applying the above mentioned theorem, we obtain that the function $\operatorname{tr} Q = 0$, i.e. $\tau^* - 2n\nu = 0$. Hence M is of constant sectional curvature ν in U. $\square$

Remark 3.4. If the curvature tensor of an almost Hermitian manifold has the form $R = \nu \, \pi_1$, then

$$\nu = \frac{\tau}{2n(2n-1)} = \frac{\tau^*}{2n}$$

and $\tau = (2n-1)\tau^*$.

To complete the proof of Theorem A, denote by H the set of points in M, in which $R = \nu \, \pi_1$. Then H is closed and the set $M \setminus H$ is open.

If $M \setminus H$ is empty, then M is of constant sectional curvatures ν.

Let $M \setminus H$ be nonempty. According to Proposition 3.1 $\nabla J = 0$ in $M \setminus H$ and consequently $R = \dfrac{\nu}{4}(\pi_1 + \pi_2)$. Since the set of points in which $R = \dfrac{\nu}{4}(\pi_1 + \pi_2)$ is closed and M is connected, then there exists a point $p \in H$ such that $R = \dfrac{\nu}{4}(\pi_1 + \pi_2)$ at p. Therefore, $\nu = 0$ and (M, g, J) is flat, thus proving Theorem A. $\square$

Finally, Theorem A implies the following statement.

Corollary 3.5. *Let M be a compact non-Kählerian Hermitian manifold with* $\dim M \geq 6$. *If M is of constant antiholomorphic sectional curvatures, then it is a flat balanced Hermitian manifold.*

Proof. Let (M, g, J) be of constant antiholomorphic sectional curvatures ν.

Applying Theorem A and the result of Le Brun [7]: S^6 *has no integrable complex structure* we obtain that *there are no compact non-Kählerian Hermitian manifolds of constant antiholomorphic sectional curvature $\nu > 0$.*

Further, denote by $\delta\theta$ the co-differential of the Lee form θ. Applying Theorem A and the formula [3]

$$\tau - \tau^* = 2\,\delta\theta + \|\theta\|^2$$

we obtain

(3.12) $$\int_M \{\|\theta\|^2 - 4n(n-1)\nu\}\, dv = 0,$$

which implies that *there are no compact non-Kählerian Hermitian manifolds of constant antiholomorphic sectional curvature $\nu < 0$.* For the non-existence of compact non-Kählerian Hermitian real space forms of hyperbolic type see also [4].

Finally, if $\nu = 0$, then the formula (3.12) implies that $\theta = 0$, i.e. M is a flat balanced Hermitian manifold, which completes the proof. $\square$

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