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CENTER MANIFOLDS FOR EVOLUTION EQUATIONS ASSOCIATED WITH THE STEFAN PROBLEM

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Abstract. Evolution equations can be used for solving the Stefan problem. We show the existence of a center manifold for an evolution equation that is associated with a quasilinear Stefan problem with variable surface tension and undercooling. This generalizes previous result for existence of center manifold for a Stefan problem where the relaxation coefficient is constant.

1. Introduction. In recent years the study of evolution equations has been the major field of research of many scientists from different communities. In dynamical systems in particular, important is the question of existence of center manifolds for these equations. One way of solving free boundary problems is to transform them to a evolution equation on the boundary of a fixed domain. A special kind of a free boundary problem which models phase transition phenomena of two or more materials is the Stefan problem, named after the physicist J. Stefan who has originally designed a model that describes ice formation in polar seas, [15], [16]. We consider the one phase quasi stationary Stefan problem where

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the heat equation is replaced by the Laplace equation $\Delta u = 0$. The interface is actually the free boundary $\partial\Omega_t$ which varies with the time variable t. On the interface the boundary condition

$$v_n + \partial_\nu u = 0,$$

where v_n is the normal velocity of $\partial\Omega_t$ is known as Stefan condition. In the classical formulation of the Stefan problem it is assumed that on the interface $\partial\Omega_t$ there is always an equilibrium, that is, the temperature is constant and equal to the melting temperature of the material but in phase transitions, as a consequence of the capillary effects and the geometry of the domain, the temperature on the interface can differ from the melting temperature. Physical effects such as kinetic undercooling and overheating lead to the observed states in which liquids and solids exist under the freezing, and above the melting temperature respectively. In order to theoretically understand these phenomena, several models that aim to capture these effects have been proposed, leading to different equations on the interface. One can take $u = \sigma\kappa$ on $\partial\Omega_t$, with positive constant σ, called surface tension, and the mean curvature κ. This is called Gibbs-Thomson condition and is studied in [4], [13], [3]. Another condition which comes often in modeling is $u = \alpha v_n$ on $\partial\Omega_t$, and is called Stefan problem with kinetic undercooling.

In our model we replace the classical Gibbs-Thomson condition on the boundary, by the more general $u = av_n + \kappa$, where a is a positive function. So we take both surface tension and thermal undercooling in consideration. The coefficient of surface tension σ is taken to be constant, i.e. independent of temperature, which after normalization is $\sigma = 1$. To have a model consistent with the second law of thermodynamics according to which the growth of entropy on the interface is nonnegative, one must take nonnegative function a. In the case where $a = 0$, there is no entropy production on the interface. However, the case with nonnegative function is considerably more difficult.

The Stefan problem now consists in finding the unknown free boundary Γ_t and the temperature u in the following set of equations

$$\begin{cases} \Delta u = 0 & \text{in } \Omega_t \\ v_n + \partial_\nu u = 0 & \text{on } \partial\Omega_t \\ u = av_n + \kappa & \text{on } \partial\Omega_t \\ \partial\Omega(0) = \partial\Omega_0 \end{cases} \tag{1.1}$$

The boundary condition $u = av_n + \kappa$ with positive function $a > 0$ expresses the temperature as a function of the local normal velocity v_n and the normal curvature κ of the phase boundary.

The transformed Stefan condition is a fully nonlinear parabolic evolution equation. Using the quasilinear structure of the mean curvature operator, this equation then becomes quasilinear parabolic evolution equation. Existence of solutions can be shown by using the concept of maximal regularity, which we briefly introduce in Section 3. In Section 4 we study the commutator of the Dirichlet-Neumann and Laplace-Beltrami operator and use this result to show the existence of center manifold in Section 5.

2. The Stefan problem on a fixed domain. Using the so called Hanzawa transformation [7], the free boundary problem (1.1) can be transformed to a fixed domain $D \subset \mathbb{R}^n$ with boundary $\partial D = \Sigma$,

$$\begin{cases} \mathcal{A}(\rho)v = 0 & \text{in } D \\ v + \delta\mathcal{B}(\rho)v = H(\rho) & \text{on } \Sigma \\ \partial_t\rho + L_\rho\mathcal{B}(\rho)v = 0 & \text{on } \Sigma \\ \rho(0) = \rho_0 & \text{on } \Sigma. \end{cases} \tag{2.1}$$

The first two equations form a boundary value problem, known as Oblique Derivative Problem. In the following $h^s(\Sigma)$ denotes the little Hölder space of order $s > 0$, which is defined as the closure of $BUC^\infty(\Sigma)$ in $BUC^s(\Sigma)$, the Banach space of bounded uniformly Hölder continuous functions of order s on Σ. We assume that the initial geometry $\partial\Omega_0$ is in the class of the little Hölder spaces $h^{3,\alpha}$. Then we have that the boundary describing function ρ is in the same class. $\mathcal{A}(\rho)$ is a second order uniformly elliptic operator, $\mathcal{A}(\rho) : h^{2,\alpha}(D) \to h^{0,\alpha}(D)$ and it has the representation

$$\mathcal{A}(\rho)v = \sum_{i,j} a_{ij}(\rho)\partial^2_{ij}v + \sum_i a_i(\rho)\partial_i v,$$

with $a_{ij}(\rho) \in h^{2,\alpha}(D)$, $a_i(\rho) \in h^{1,\alpha}(D)$. The boundary operator $\mathcal{B}(\rho) : h^{2,\alpha}(D) \to h^{0,\alpha}(\Sigma)$ has the representation

$$\mathcal{B}(\rho)v = \overrightarrow{b_\rho} \cdot \nabla v,$$

for a nowhere tangential and nowhere vanishing vector field

$$\overrightarrow{b_\rho} : \Sigma \to \mathbb{R}^n.$$

L_ρ is a strictly positive function. We note that δ is not a constant but also a strictly positive function. $H(\rho)$ is the transformed mean curvature and it is a $h^{1,\alpha}(\Sigma)$ function. For more details the reader is referred to [11] where the problem is posed and solved in the common Hölder spaces $C^{k,\alpha}$.

3. Maximal regularity for evolution equations. The last two equations in the Stefan problem (2.1) give the evolution equation

$$\begin{cases} \partial_t \rho + L_\rho \mathcal{B}(\rho) v = 0 & \text{on } \Sigma \\ \rho(0) = \rho_0 & \text{on } \Sigma. \end{cases} \tag{3.1}$$

Let $\mathcal{S}$ be the formal solution operator of the Oblique derivative problem

$$\begin{aligned} \mathcal{A}(\rho) u &= 0 \qquad \text{in } D, \\ u + \delta \mathcal{B}(\rho) u &= g \qquad \text{on } \Sigma. \end{aligned}$$

That is, for $g \in h^{1,\alpha}(\Sigma)$, $u = \mathcal{S} g \in h^{2,\alpha}(D)$ is the unique solution, see [6, Theorem 6.31] for a proof for the common Hölder spaces $C^{k,\alpha}$ which can be easily adapted to little Hölder space. The first two equations of (2.1) have the solution $v = \mathcal{S}(\rho) H(\rho)$, which we put into the evolution equation (3.1) to obtain the fully nonlinear evolution equation

$$\begin{cases} \partial_t \rho + L_\rho \mathcal{B}(\rho) \mathcal{S}(\rho) H(\rho) = 0 \\ \rho(0) = \rho_0. \end{cases}$$

This evolution equation can be linearized by using the quasilinear structure of the mean curvature operator $H(\rho)$. For a proof of the next theorem see Escher and Simonett [2, Lemma 3.1].

Lemma 3.1. *There exist*

$$P \in C^\infty(\mathcal{U}, \mathcal{L}(h^{3,\alpha}(\Sigma), h^{1,\alpha}(\Sigma))), \quad Q \in C^\infty(\mathcal{U}, h^{1,\beta}(\Sigma))$$

such that

$$H(\rho) = P(\rho)\rho + Q(\rho),$$

where $P(\rho)$ is a second order uniformly elliptic differential operator and $Q(\rho)$ is an analytic function depending on the first and second order derivatives of ρ.

Utilizing this representation of the mean curvature, the evolution equation becomes quasilnear

$$\begin{aligned} \partial_t \rho + A(\rho)\rho &= F(\rho) \\ \rho(0) &= \rho_0, \end{aligned} \tag{3.2}$$

with the operator $A(\rho)$ and the function $F(\rho)$ given by

$$A(\rho) := L_\rho \mathcal{B}(\rho) \mathcal{S}(\rho) P(\rho), \quad F(\rho) := -L_\rho \mathcal{B}(\rho) \mathcal{S}(\rho) Q(\rho). \tag{3.3}$$

Lemma 3.2. *Let $\mathcal{K}$ be the formal solution operator of the Dirichlet problem,*

$$\begin{aligned} \mathcal{A}(\rho)u &= 0 \quad \text{in } D \\ u &= g \quad \text{on } \Sigma, \end{aligned}$$

$u = \mathcal{K}g$. *Then for the formal solution operator $\mathcal{S}$ of the Oblique derivative problem*

$$\begin{aligned} \mathcal{A}(\rho)u &= 0 \quad \text{in } D, \\ u + \delta\mathcal{B}(\rho)u &= g \quad \text{on } \Sigma, \end{aligned}$$

it holds

$$\delta\mathcal{B}(\rho)\mathcal{S}(\rho) = I - \gamma\mathcal{S}(\rho),$$

and

$$\gamma\mathcal{S} = (I + \delta\mathcal{B}\mathcal{K})^{-1}.$$

P r o o f. For the first assertion

$$\begin{aligned} \delta\mathcal{B}(\rho)\mathcal{S}(\rho)g &= g - \gamma\mathcal{S}(\rho)g \\ &= [I - \gamma\mathcal{S}(\rho)]g. \end{aligned}$$

We put $u := \gamma\mathcal{S}\varphi$ and calculate

$$\begin{aligned} (I + \delta\mathcal{B}\mathcal{K})u &= u + \delta\mathcal{B}\mathcal{K}u = u + \delta\mathcal{B}u \\ &= \gamma\mathcal{S}\varphi + \delta\mathcal{B}\gamma\mathcal{S}\varphi = \varphi. \end{aligned}$$

Hence $(I + \delta\mathcal{B}\mathcal{K})\gamma\mathcal{S}\varphi = \varphi$, that is

$$\gamma\mathcal{S} = (I + \delta\mathcal{B}\mathcal{K})^{-1}.$$

□

One particular representation of the operator $A(\rho)$ will be used in the next section.

Lemma 3.3. *It holds*

$$A(\rho) = \frac{L_\rho}{\delta}P(\rho) - \frac{L_\rho}{\delta}\gamma\mathcal{S}(\rho)P(\rho).$$

Proof. By the previous Lemma 3.2

$$A(\rho) = \frac{L_\rho}{\delta}[\delta\mathcal{B}(\rho)\mathcal{S}(\rho)P(\rho)] = \frac{L_\rho}{\delta}(I - \gamma\mathcal{S}(\rho))P(\rho)$$
$$= \frac{L_\rho}{\delta}P(\rho) - \frac{L_\rho}{\delta}\gamma\mathcal{S}(\rho)P(\rho). \qquad \square$$

In the following, we recall the concept of continuous maximal regularity. For two Banach spaces E_0 and E_1 with E_1 densely embedded in E_0, $E_1 \hookrightarrow E_0$, and $0 < \theta \leq 1$, Da Prato and Grisvard introduced the following interpolation spaces, with $J = [0, T)$ and $\dot{J} = (0, T)$.

$$\mathbb{E}_0^\theta(J) := \{u \in C(\dot{J}, E_0) : \lim_{t\to 0+} \|t^{1-\theta}u(t)\|_{E_0} = 0\},$$

$$\mathbb{E}_1^\theta(J) := \{u \in C^1(\dot{J}, E_0) \cap C(\dot{J}, E_1) : \lim_{t\to 0+} t^{1-\theta}\left(\|u'(t)\|_{E_0} + \|u(t)\|_{E_1}\right) = 0\},$$

$$\gamma\mathbb{E}_1^\theta(J) := \{u(0) : u \in \mathbb{E}_1^\theta(J)\}.$$

With $\mathcal{H}(E_1, E_0)$ we denote the set of all analytic generators from E_1 to E_0.

Definition 3.4. The operator $A \in \mathcal{H}(E_1, E_0)$ has continuous maximal regularity if the inhomogeneous Cauchy problem

$$\begin{cases} \dot{u}(t) = Au(t) + f(t), & \text{on J} \\ u(0) = u_0. \end{cases} \tag{3.4}$$

has a unique solution $u \in \mathbb{E}_1^\theta$ for all $f \in \mathbb{E}_0^\theta$ and $u_0 \in \gamma\mathbb{E}_1^\theta$.
The set of all operators with maximal regularity is denoted by $\mathcal{M}(E_1, E_0)$.

For maximal regularity in the context of L^p spaces see the excellent survey by Kunstmann and Weis [9].

Remark 3.5. Uniformly elliptic operators have maximal regularity. This property is inherited by operators which are lower order perturbations of operators with maximal regularity.

Remark 3.6. The maximal regularity of the uniformly elliptic operator $A(\rho)$ follows from its representation as a lower order perturbation of $P(\rho)$ given in Lemma 3.3 and the previous Remark 3.5.

4. The fully nonlinear evolution equation and spherical harmonics. We consider the fully nonlinear evolution equation

$$\begin{aligned} \partial_t \rho + \Phi(\rho) &= 0 \\ \rho(0) &= \rho_0, \end{aligned}$$

and the operator L, which is the linearization of the operator $\Phi(\rho)$ in zero

$$L := \dot{\Phi}(0).$$

Stefan Problem is reduced to solving this evolution equation. The operator $\Phi(\rho)$ is composition of the three operators

$$\Phi(\rho) = L_\rho \mathcal{B}(\rho)\mathcal{S}(\rho)H(\rho),$$

where $H(\rho)$ is the transformed mean curvature operator that comes from the Stefan condition. $\mathcal{B}(\rho)$ is the transformed oblique derivative and $\mathcal{S}(\rho)$ is the solution operator of the boundary value problem. For the investigation of the spectrum of the linearization L, we need its representation, contained in the following result.

Theorem 4.1. *Let* $\mathcal{B} := \mathcal{B}(0)$, $\mathcal{S} := \mathcal{S}(0)$, $H = H(0)$ *and* $D := \dot{H}(0)$. *Then*

$$L = a^{-1}D - a^{-1}(I + a\mathcal{B}\mathcal{K})^{-1}D. \tag{4.1}$$

P r o o f. First we show that $\dot{\Phi}(0) = \mathcal{B}\mathcal{S}\dot{H}$. Calculation gives

$$\dot{\Phi}(0) = \dot{L_0}\mathcal{B}\mathcal{S}H + L_0(\mathcal{B}(\rho)\dot{\mathcal{S}}(\rho))H + \mathcal{B}\mathcal{S}\dot{H},$$

and only the last term does not vanish. $\dot{L_0} = 0$ since $L_0 \equiv 1$. The mean curvature of the sphere $H(0)$ is $H(0) = 1/R$. Hence $\mathcal{S}(\rho)H(0) = 1/R$ and $\mathcal{B}(\rho)\mathcal{S}(\rho)H = 0$ for all $\rho \in U$, where U is a small neighborhood of zero. It follows that $(\dot{\mathcal{B}\mathcal{S}})H = 0$.

Utilizing Lemma 3.2

$$\delta\mathcal{B}(\rho)\mathcal{S}(\rho) = I - \gamma\mathcal{S}(\rho),$$

and

$$\gamma\mathcal{S} = (I + \delta\mathcal{B}\mathcal{K})^{-1},$$

we compute

$$L = \mathcal{B}\mathcal{S}D = \delta^{-1}(\delta\mathcal{B}\mathcal{S}D)$$

$$\begin{aligned} &= \delta^{-1}(I - \gamma\mathcal{S})D \\ &= \delta^{-1}D - \delta^{-1}(I + \delta\mathcal{B}\mathcal{K})^{-1}D. \end{aligned}$$

It is clear that for $\rho \equiv 0$, $\delta = a$ and so the theorem is proved. □

Let Σ denote the sphere in $\mathbb{R}^n$ and let

$$\mathcal{H}_m(\Sigma) := \{\psi : \psi = u|_\Sigma \text{ for some } u \in \mathcal{H}_m(\mathbb{R}^n)\},$$

where $\mathcal{H}_m(\mathbb{R}^n)$ is the subspace of solid spherical harmonics of degree m. The spherical harmonics are eigenfunctions for $\Lambda := \mathcal{B}\mathcal{K}$, the Dirichlet-Neumann operator, and Δ_{Σ_R}, the Laplace-Beltrami operator, with eigenvalues m and $m(m+n-2)$ accordingly. The operators Λ and Δ_{Σ_R} commute on $\mathcal{H}_m(\Sigma)$, and the orthogonal direct sum $\bigoplus_{m=0}^{\infty} \mathcal{H}_m(\Sigma)$ is dense in $L^2(\Sigma)$, see [5, Theorem 2.53]. Hence the following result holds for the commutator $[\Lambda, \Delta_{\Sigma_R}]$. It plays an important role in the proof of the theorem in the next section.

Theorem 4.2. *The commutator* $[\Lambda, \Delta_{\Sigma_R}] = \Lambda\Delta_{\Sigma_R} - \Delta_{\Sigma_R}\Lambda$ *vanishes on* $L^2(\Sigma_R)$.

5. The center manifold. The Stefan problem is reduced to a single quasilinear evolution equation (3.2), and it can be shown that it has a unique local solution, see [10]. The existence of a center manifold for the evolution equation associated with the special case of Stefan problem for constant function $a = 1$ in (1.1) is given in [8].

The following result for the commutator $[\Lambda, \Delta_{\Sigma_R}]$ will be used in the next Theorem. It says that the spectrum of the operator $-L$ is not complicated under particular boundedness condition on the commutator.

Theorem 5.1. *Let* $a > 0$ *be positive function such that for the norm of the commutator* $[a^{1/2}\Lambda, a^{1/2}]$ *holds* $\left\|[a^{1/2}\Lambda, a^{1/2}]\right\| \leq 1$. *Then the spectrum of the operator* $-L$ *consists only of eigenvalues and* $\sigma_p(-L) \subset (-\infty, 0]$.

P r o o f. The operator L has a compact resolvent because

$$L \in \mathcal{H}(h^{3,\alpha}(\Sigma_R), h^{1,\alpha}(\Sigma_R))$$

and the domain $h^{3,\alpha}(\Sigma_R)$ is compactly embedded in $h^{1,\alpha}(\Sigma_R)$. Hence its spectrum consists only of isolated eigenvalues with finite multiplicities.

Now we use the formula for the derivative D of the normal curvature operator H from [1] to show that these eigenvalues are non positive.

$$D := \dot{H}(0) = -\frac{1}{R^2}I - \frac{1}{n-1}\Delta_{\Sigma_R}.$$

Let λ be an eigenvalue of $-L$,

$$\lambda f + Lf = 0.$$

We apply the operator $I + a\mathcal{BK}$ to this equation and get with Theorem 4.1 after short calculation

$$\lambda f + \lambda a\mathcal{BK}f + \mathcal{BK}Df = 0.$$

Next we multiply the equation by $\bar{f}$ and integrate over Σ_R to get:

$$\lambda\langle f, f\rangle + \langle \lambda a\mathcal{BK}f, f\rangle + \langle \mathcal{BK}Df, f\rangle = 0. \tag{5.1}$$

The third term is $\langle \mathcal{BK}Df, f\rangle \geq 0$, because the operators $\mathcal{BK}$ and D are positive and by Theorem 4.2 commute on the sphere. Since $a > 0$, the second term can be written in the form

$$\langle a\mathcal{BK}f, f\rangle = -\left\langle [a^{1/2}\mathcal{BK}, a^{1/2}]f, f\right\rangle + \langle a^{1/2}\mathcal{BK}a^{1/2}f, f\rangle.$$

Then (5.1) is

$$\lambda\left[\langle f, f\rangle - \left\langle [a^{1/2}\mathcal{BK}, a^{1/2}]f, f\right\rangle + \langle a^{1/2}\mathcal{BK}a^{1/2}f, f\rangle\right] = -\langle \mathcal{BK}Df, f\rangle. \tag{5.2}$$

Note that by the assumption for the commutator and the Cauchy-Schwarz inequality follows

$$\left\langle [a^{1/2}\mathcal{BK}, a^{1/2}]f, f\right\rangle \leq \langle f, f\rangle,$$

and that

$$\langle a^{1/2}\mathcal{BK}a^{1/2}f, f\rangle = \langle \mathcal{BK}a^{1/2}f, a^{1/2}f\rangle \geq 0.$$

Therefore

$$\langle f, f\rangle - \left\langle [a^{1/2}\mathcal{BK}, a^{1/2}]f, f\right\rangle + \langle a^{1/2}\mathcal{BK}a^{1/2}f, f\rangle \geq 0.$$

Since $-\langle \mathcal{BK}Df, f\rangle \leq 0$, it follows from (5.2) that $\lambda \leq 0$. The proof is complete. □

We denote by $X^c := \ker L$ the kernel of the operator L, which is a finite dimensional subspace of $h^{3,\alpha}(\Sigma_R)$ and can be complemented. There is a projection

$\pi^c : h^{3,\alpha}(\Sigma_R) \to X^c$. In particular π^c can be taken as the corresponding spectral projection to the eigenvalue 0 of L defined by the Dunford integral formula

$$\pi^c = \frac{1}{2\pi i} \int_\gamma (\lambda I - L)^{-1} \, d\lambda.$$

Let $\pi^s := I - \pi^c$. With $X^s := \operatorname{ran} \pi^s$, the complement of X^c in $h^{3,\alpha}(\Sigma_R)$, there is the direct sum

$$h^{3,\alpha}(\Sigma_R) = X^c \oplus X^s.$$

We will use a simplified version of the next result from [14, Section 4]. It is an abstract existence theorem for center manifolds.

Theorem 5.2 (Existence of center manifolds)**.** *Let X_0 and X_1 be two Banach spaces with X_1 densely embedded in X_0 and let the autonomous quasilinear evolution equation*

$$\dot{u} + A(u)u = F(u), \quad t > 0$$

satisfy the following conditions

(1) $(A, F) \in \mathrm{Lip}(\mathcal{U}, \mathcal{L}(X_1, X_0) \times X_0)$, *where F is Frechet differentiable in zero with Frechet derivative $\partial F(0)$ and $F(0) = 0$*

(2) $A(x) \in \mathcal{M}(X_1, X_0)$ *for each $x \in \mathcal{U}$*

(3) *The spectrum of the operator $L := A(0) - \partial F(0)$ consists only of eigenvalues*

(4) $\sigma_p(-L) \subset (-\infty, 0]$

where $\mathcal{U}$ is a neighborhood of zero of the interpolation space $X_\gamma = (X_0, X_1)_\gamma$ of exponent γ, $0 < \gamma < 1$, given by an arbitrary interpolation method. Then there exist a unique mapping σ, $\sigma : X^c \to X^s$, which is differentiable at 0 with

$$\sigma(0) = \partial\sigma(0) = 0,$$

and the graph

$$\mathcal{M}^c = graph(\sigma)$$

of σ is a center manifold for the equation.

We use this result to prove the main theorem.

Theorem 5.3. *Consider the evolution equation*

$$\begin{aligned} \partial_t \rho + A(\rho)\rho &= F(\rho) \\ \rho(0) &= \rho_0, \end{aligned}$$

associated with the Stefan problem (2.1), *with* A *from* (3.3). $\rho = 0$ *is an isolated eigenvalue of the operator* L *with representation* (4.1). *Let* $a > 0$ *be a positive function, with* $\left\| [a^{1/2}\Lambda, a^{1/2}] \right\| \leq 1$. *Then there is* $\epsilon > 0$, *an open neighborhood* $\mathcal{O}$ *of* 0 *in* X^c *and a function* $g : \mathcal{O} \to X^s$ *with* $g(0) = \partial g(0) = 0$, *such that the graph* $\mathcal{M}^c(0)$ *of this function,*

$$\mathcal{M}^c(0) = \{(x, g(x)) : \|x\|_{3,\alpha} < \epsilon\}$$

is a center manifold for the evolution equation.

Proof. We now proceed to show that the evolution equation satisfies all the assumptions from Theorem 5.2. With the same notation as in the theorem we put $X_0 = h^{1,\alpha}(\Sigma)$ and $X_1 = h^{3,\alpha}(\Sigma)$. The little Hölder spaces can be realized as interpolation spaces. The assumption (1) from Theorem 5.2 follows from (3.2) and Lemma (3.1). By the Remark (3.6) the assumption (2) is satisfied. Finally, the Theorem (5.1) corresponds to the last two spectral assumptions (3) and (4) in Theorem 5.2. □

Remark 5.4. For the constant function $a \equiv 0$, which corresponds to the Hele-Shaw problem, is obviously $\left\| [a^{1/2}\Lambda, a^{1/2}] \right\| = 0$ and we have again existence of a center manifold.

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